Version SKH/4

EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 5 May 2021   9.00 to 10.40

Module 3D1

GEOTECHNICAL ENGINEERING I

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet and at the top of each answer sheet.*

STATIONERY REQUIREMENTS
Write on single-sided paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Attachment: 3D1 & 3D2 Geotechnical Engineering Databook, (21 pages)
You are allowed access to the electronic version of the Engineering Data Books.

10 minutes reading time is allowed for this paper at the start of the exam.

The time taken for scanning/uploading answers is 15 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers.

1 A silty soil with a D_{10} of 20 microns is to be used for construction of a 10 m high embankment across a region prone to flooding.

(a) Undisturbed samples of the soil reveal it to have a bulk unit weight of 19.0 kNm^{-3} and a dry unit weight of 16.0 kNm^{-3}. When 100 g of the dried grains were added to 100 ml of water, the total volume was found to be 138 ml. Determine the specific gravity of the grains, G_s, the in-situ voids ratio, e, and the water content of the soil. [25%]

(b) Compaction test data for the soil is shown in Table 1. Determine the optimum moisture content for compaction. [25%]

(c) The same material is available at a variety of water contents. Comment on the suitability of soils wetter and drier than optimum, explaining both their microstructure and their behaviour. [25%]

(d) The embankment is constructed with material at 12% water content, compacted as in Table 1. Plot the vertical effective stress profile at the centre of the embankment if the water table is 2 m above its base. [25%]

Table 1

Water content (%)	6	8	10	12	14	16
Bulk unit weight (kNm^{-3})	19.2	20.0	20.8	21.4	21.4	21.2

2 A construction site consists of a clay layer underlain by a permeable sand aquifer. A clay sample recovered from the site is tested in an oedometer with double drainage and yields the results shown in Table 2 for a vertical effective stress increment from 30 kPa to 50 kPa.

(a) Estimate the values of permeability, k, Young's modulus, E, and the coefficient of consolidation, C_v, for the clay. [25%]

(b) The clay layer varies gradually in thickness as shown in Fig. 1. Dewatering in the aquifer results in the water table falling by 2 m with no infiltration at the ground surface. If the oedometer results are representative of the behaviour of the entire clay layer, calculate the settlement and average rotation of the building after 1 month, 1 year and 50 years. [25%]

(c) Building damage is typically associated with curvature rather than settlement. How will the curvature of the building vary during the consolidation process? Calculate the value of T_v at which maximum curvature will be observed. [50%]

Table 2

Time (min)	0	0.5	2	8	15	30	60
Height (mm)	20	19.97	19.94	19.88	19.84	19.82	19.81

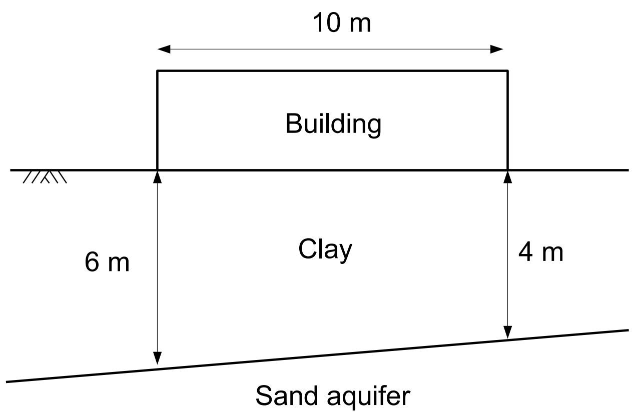

Fig. 1

3 (a) Upper and lower-bound analyses can be used to calculate the capacity of foundations. Explain what is necessary to perform valid upper and lower bound analyses. [20%]

(b) Calculate from first principles the energy dissipated in a stress fan of radius r and fan angle θ in a Tresca material with strength s_u. [20%]

(c) A 3 m × 3 m square foundation is embedded by 2 m into a clay layer with an undrained strength of 50 kPa and a unit weight of 16 kNm^{-3}.

(i) Calculate the vertical capacity of the foundation. [25%]

(ii) If the foundation is used to support a crane with a self-weight of 1 MN, up to what radius from the centre of the foundation can the crane lift a weight of 200 kN? [35%]

4 (a) Explain qualitatively how the stresses beneath a shallow foundation vary depending on the stiffness of the foundation. [20%]

(b) Why is the shape correction factor for a square foundation greater than unity for homogeneous soils, but can be less than unity for soils which increase in strength with depth? [20%]

(c) A 2 m diameter flexible circular foundation rests on a 10 m thick layer of kaolin clay with an elastic shear modulus G = 15 MPa, bulk unit weight 16 kNm^{-3} and specific volume v = 2.7 underlain by impermeable bedrock. The foundation carries a vertical load of 200 kN.

(i) Calculate the immediate elastic settlement at the centre and edge of the foundation. [15%]

(ii) Calculate the ultimate settlement of the foundation. [30%]

(iii) Estimate the time taken for 25% and 95% of the ultimate settlement to occur if C_v = 1 m^2 year^{-1}, clearly stating any assumptions made. [15%]

END OF PAPER

Engineering Tripos Part IIA

3D1 & 3D2 Geotechnical Engineering

Data Book 2019-2020

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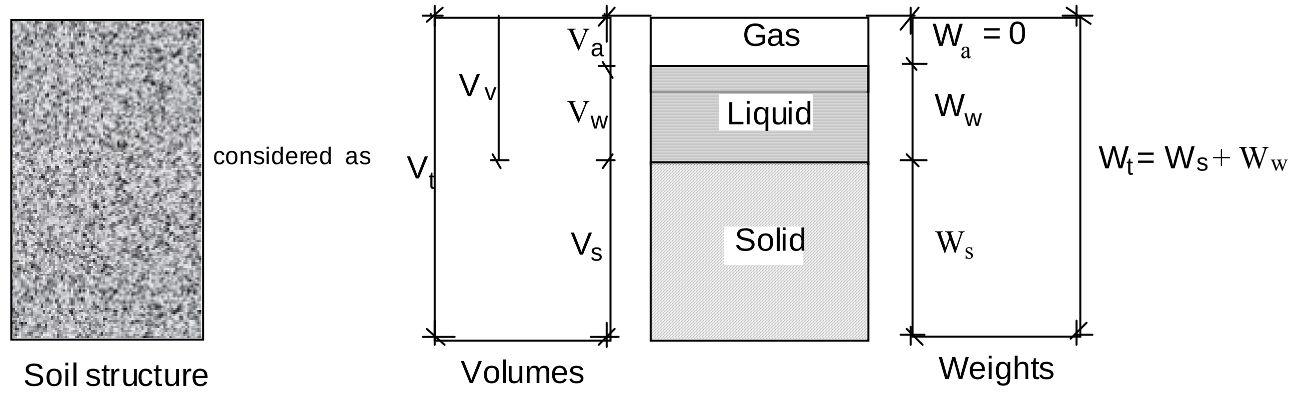
General definitions



Specific gravity of solid	G_s	
Voids ratio	e	$= V_v / V_s$
Specific volume	v	$= V_t / V_s = 1 + e$
Porosity	n	$= V_v / V_t = e/(1 + e)$
Water content	w	$= (W_w / W_s)$
Degree of saturation	S_r	$= V_w / V_v = (w\,G_s/e)$
Unit weight of water	γ_w	$= 9.81\ \text{kN/m}^3$
Unit weight of soil	γ	$= W_t / V_t = \left(\dfrac{G_s + S_r e}{1 + e}\right) \gamma_w$
Buoyant saturated unit weight	γ'	$= \gamma - \gamma_w = \left(\dfrac{G_s - 1}{1 + e}\right) \gamma_w$
Unit weight of dry solids	γ_d	$= W_s / V_t = \left(\dfrac{G_s}{1 + e}\right) \gamma_w$
Air volume ratio	A	$= V_a/V_t = \left(\dfrac{e\,(1 - S_r)}{1+e}\right)$

Soil classification (BS1377)

Liquid limit w_L

Plastic Limit w_P

Plasticity Index $I_P = w_L - w_P$

Liquidity Index $I_L = \dfrac{w - w_P}{w_L - w_P}$

$$\text{Activity} = \frac{\text{Plasticity Index}}{\text{Percentage of particles finer than } 2\ \mu\text{m}}$$

$$\text{Sensitivity} = \frac{\text{Unconfined compressive strength of an undisturbed specimen}}{\text{Unconfined compressive strength of a remoulded specimen}}$$

(at the same water content)

Classification of particle sizes:–

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

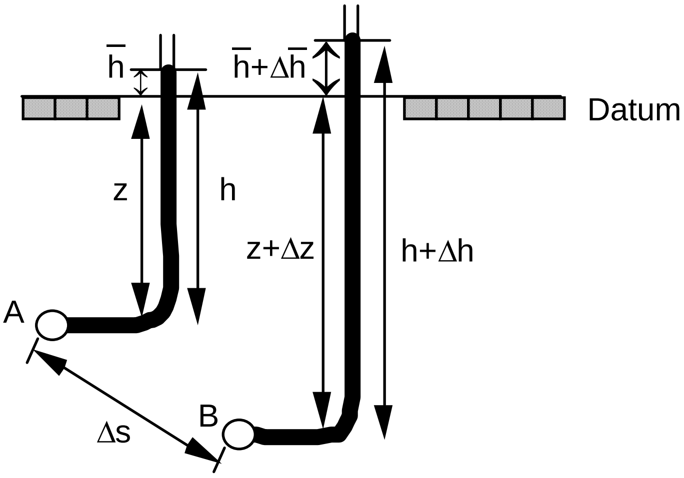
D — equivalent diameter of soil particle

D_{10}, D_{60} etc. — particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

C_U — uniformity coefficient D_{60} / D_{10}

Seepage

Flow potential:
(piezometric level)



Total gauge pore water pressure at A: $u = \gamma_w h = \gamma_w(\bar{h} + z)$

B: $u + \Delta u = \gamma_w(h + \Delta h) = \gamma_w(\bar{h} + z + \Delta\bar{h} + \Delta z)$

Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w(\bar{h} + \Delta\bar{h})$

Hydraulic gradient A → B $\quad i = -\dfrac{\Delta\bar{h}}{\Delta s}$

Hydraulic gradient (3D) $\quad i = -\nabla\bar{h}$

Darcy's law $\quad V = ki$

V = superficial seepage velocity

k = coefficient of permeability

Typical permeabilities:

$D_{10} > 10$ mm	:	non-laminar flow
10 mm $> D_{10} > 1$µm	:	$k \cong 0.01\,(D_{10}$ in mm$)^2$ m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

Saturated capillary zone

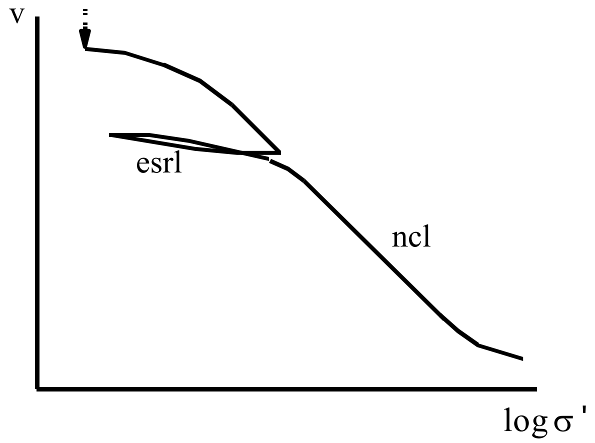
$h_c = \dfrac{4T}{\gamma_w d}$: capillary rise in tube diameter d, for surface tension T

$h_c \approx \dfrac{3\times10^{-5}}{D_{10}}$ m : for water at 10°C; note air entry suction is $u_c = -\gamma_w h_c$

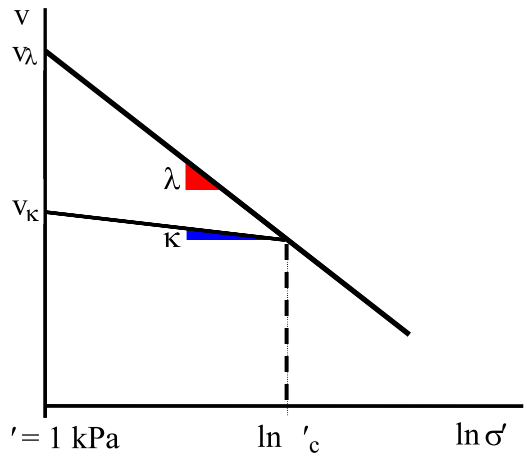
One-Dimensional Compression

- **Fitting data**

Typical data (sand or clay)



Mathematical model



Plastic compression stress σ'_c is taken as the larger of the initial aggregate crushing stress and the historic maximum effective vertical stress. Clay muds are taken to begin with $\sigma'_c \approx 1$ kPa.

Plastic compression (normal compression line, ncl): $v = v_\lambda - \lambda \ln \sigma'$ for $\sigma' = \sigma'_c$

Elastic swelling and recompression line (esrl): $v = v_c + \kappa (\ln \sigma'_c - \ln \sigma'_v)$

$= v_\kappa - \kappa \ln \sigma'_v$ for $\sigma' < \sigma'_c$

Equivalent parameters for $\log_{10}$ stress scale:

Terzaghi's compression index $C_c = \lambda \log_{10} e$

Terzaghi's swelling index $C_s = \kappa \log_{10} e$

- **Deriving confined soil stiffnesses**

Secant 1D compression modulus $E_o = (\Delta\sigma' / \Delta\varepsilon)_o$

Tangent 1D plastic compression modulus $E_o = v \sigma' / \lambda$

Tangent 1D elastic compression modulus $E_o = v \sigma' / \kappa$

One-Dimensional Consolidation

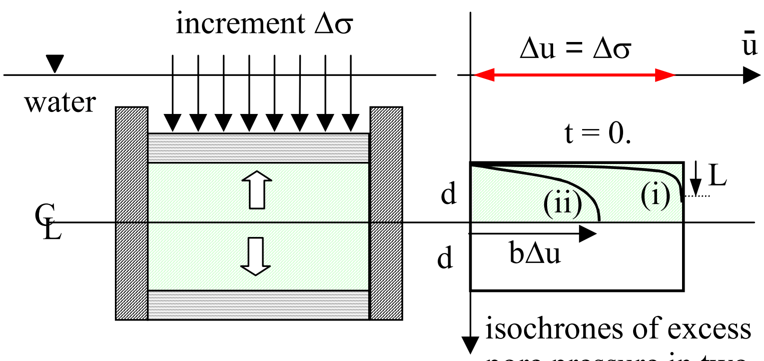
Settlement ρ $= \int m_v (\Delta u - \bar{u})\, dz = \int (\Delta u - \bar{u}) / E_o \, dz$

Coefficient of consolidation c_v $= \dfrac{k}{m_v \gamma_w} = \dfrac{kE_o}{\gamma_w}$

Dimensionless time factor T_v $= \dfrac{c_v t}{d^2}$

Relative settlement R_v $= \dfrac{\rho}{\rho_{ult}}$

- **Solutions for initially rectangular distribution of excess pore pressure**



Approximate solution by parabolic isochrones:

Phase (i) $L^2 = 12\, c_v t$

$R_v = \sqrt{\dfrac{4T_v}{3}}$ for $T_v < {}^1/_{12}$

Phase (ii) $b = \exp(\tfrac{1}{4} - 3T_v)$

$R_v = [1 - {}^2/_3 \exp(\tfrac{1}{4} - 3T_v)]$ for $T_v > {}^1/_{12}$

Solution by Fourier Series:

T_v	0	0.01	0.02	0.04	0.08	0.15	0.20	0.30	0.40	0.50	0.60	0.80	1.00
R_v	0	0.12	0.17	0.23	0.32	0.45	0.51	0.62	0.70	0.77	0.82	0.89	0.94

Stress and strain components

• **Principle of effective stress (saturated soil)**

total stress σ = effective stress σ' + pore water pressure u

• **Principal components of stress and strain**

sign convention	compression positive
total stress	$\sigma_1,\ \sigma_2, \sigma_3$
effective stress	$\sigma_1',\ \sigma_2',\ \sigma_3'$
strain	$\varepsilon_1,\ \varepsilon_2,\ \varepsilon_3$

• **Simple Shear Apparatus (SSA)** ($\varepsilon_2 = 0$; other principal directions unknown)

The only stresses that are readily available are the shear stress τ and normal stress σ applied to the top platen. The pore pressure u can be controlled and measured, so the normal effective stress σ' can be found. Drainage can be permitted or prevented. The shear strain γ and normal strain ε are measured with respect to the top platen, which is a plane of zero extension. Zero extension planes are often identified with slip surfaces.

work increment per unit volume $\delta W = \tau\,\delta\gamma + \sigma'\delta\varepsilon$

• **Biaxial Apparatus - Plane Strain (BA-PS)** ($\varepsilon_2 = 0$; rectangular edges along principal axes)

Intermediate principal effective stress σ_2', in zero strain direction, is frequently unknown so that all conditions are related to components in the 1-3 plane.

mean total stress	$s = (\sigma_1 + \sigma_3)/2$
mean effective stress	$s' = (\sigma_1' + \sigma_3')/2 = s - u$
shear stress	$t = (\sigma_1' - \sigma_3')/2 = (\sigma_1 - \sigma_3)/2$
volumetric strain	$\varepsilon_v = \varepsilon_1 + \varepsilon_3$
shear strain	$\varepsilon_\gamma = \varepsilon_1 - \varepsilon_3$
work increment per unit volume	$\delta W = \sigma_1'\delta\varepsilon_1 + \sigma_3'\delta\varepsilon_3$
	$\delta W = s'\delta\varepsilon_v + t\delta\varepsilon_\gamma$

providing that principal axes of strain increment and of stress coincide.

• **Triaxial Apparatus – Axial Symmetry (TA-AS)** (cylindrical element with radial symmetry)

total axial stress	$\sigma_a = \sigma'_a + u$
total radial stress	$\sigma_r = \sigma'_r + u$
total mean normal stress	$p = (\sigma_a + 2\sigma_r)/3$
effective mean normal stress	$p' = (\sigma'_a + 2\sigma'_r)/3 = p - u$
deviatoric stress	$q = \sigma'_a - \sigma'_r = \sigma_a - \sigma_r$
stress ratio	$\eta = q/p'$
axial strain	ε_a
radial strain	ε_r
volumetric strain	$\varepsilon_v = \varepsilon_a + 2\varepsilon_r$
triaxial shear strain	$\varepsilon_s = \frac{2}{3}(\varepsilon_a - \varepsilon_r)$
work increment per unit volume	$\delta W = \sigma_a'\delta\varepsilon_a + 2\sigma_r'\delta\varepsilon_r$
	$\delta W = p'\delta\varepsilon_v + q\delta\varepsilon_s$

Types of triaxial test include:

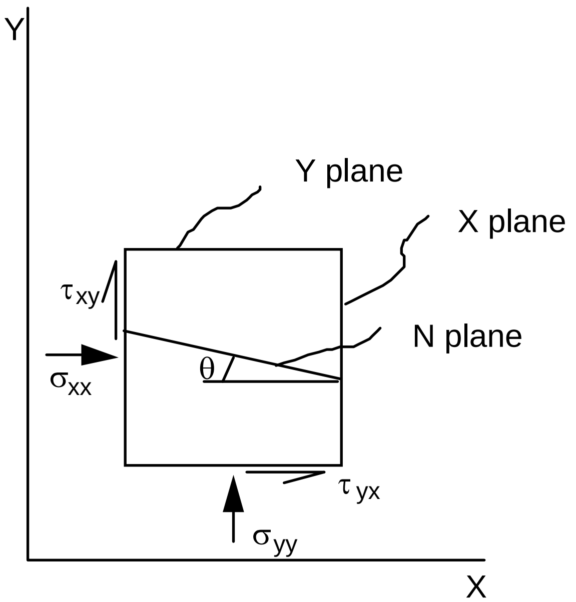
isotropic compression in which p' increases at zero q

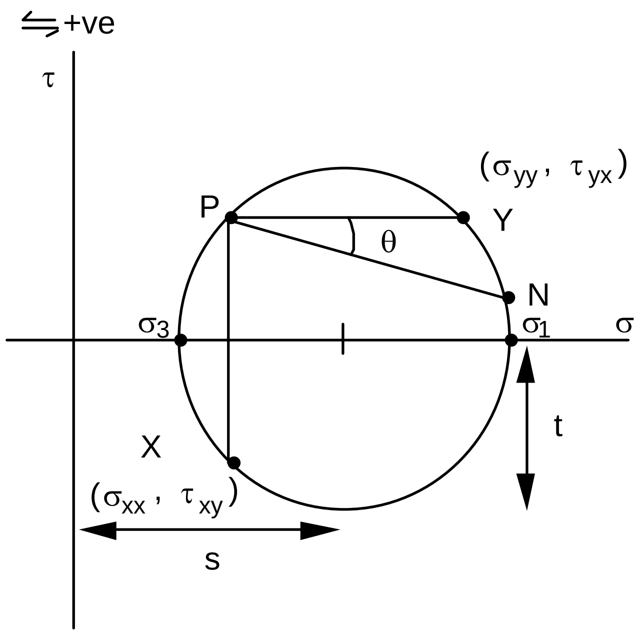
triaxial compression in which q increases *either* by increasing σ_a *or* by reducing σ_r

triaxial extension in which q reduces *either* by reducing σ_a *or* by increasing σ_r

• **Mohr's circle of stress (1–3 plane)**

Sign of convention: compression, and counter-clockwise shear, positive





Poles of planes P : the components of stress on the N plane are given by the intersection N of the Mohr circle with the line PN through P parallel to the plane.

Elastic stiffness relations

These relations apply to tangent stiffnesses of over-consolidated soil, with a state point on some swelling and recompression line (κ-line), and remote from gross plastic yielding.

One-dimensional compression (axial stress and strain increments $d\sigma'$, $d\varepsilon$)

compressibility $m_v = d\varepsilon / d\sigma'$

constrained modulus $E_o = 1 / m_v$

Physically fundamental parameters

shear modulus $G' = dt / d\varepsilon_\gamma$

bulk modulus $K' = dp' / d\varepsilon_v$

Parameters which can be used for constant-volume deformations

undrained shear modulus $G_u = G'$

undrained bulk modulus $K_u = \infty$ (neglecting compressibility of water)

Alternative convenient parameters

Young's moduli E' (effective), E_u (undrained)

Poisson's ratios ν' (effective), $\nu_u = 0.5$ (undrained)

Typical value of Poisson’s ratio for small changes of stress: $\nu' = 0.2$

Relationships:

$$G = \frac{E}{2(1+\nu)}$$

$$K = \frac{E}{3(1-2\nu)}$$

$$E_o = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

Cam Clay

• Interchangeable parameters for stress combinations at yield, and plastic strain increments

System	Effective normal stress	Plastic normal strain	Effective shear stress	Plastic shear strain	Critical stress ratio	Plastic normal stress	Critical normal stress
General	σ^*	ε^*	τ^*	γ^*	μ^*_{crit}	σ^*_c	σ^*_{crit}
SSA	σ'	ε	τ	γ	$\tan\phi_{crit}$	σ'_c	σ'_{crit}
BA-PS	s'	ε_v	t	ε_γ	$\sin\phi_{crit}$	s'_c	s'_{crit}
TA-AS	p'	ε_v	q	ε_s	M	p'_c	p'_{crit}

• General equations of plastic work

Plastic work and dissipation $\quad \sigma^* \delta\varepsilon^* + \tau^* \delta\gamma^* = \mu^*_{crit} \sigma^* \delta\gamma^*$

Plastic flow rule – normality $\quad \dfrac{d\tau^*}{d\sigma^*} \cdot \dfrac{d\gamma^*}{d\varepsilon^*} = -1$

• General yield surface

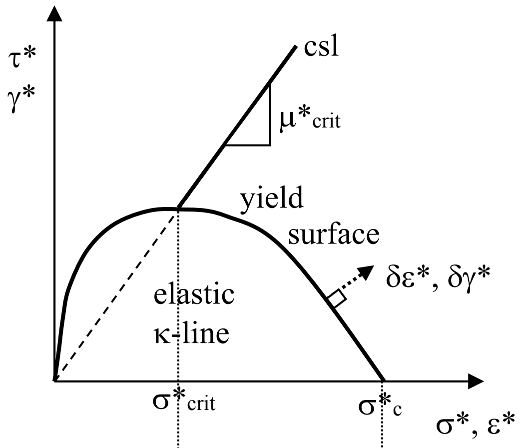
$$\frac{\tau^*}{\sigma^*} = \mu^* = \mu^*_{crit} \,.\, \ln\left[\frac{\sigma_c^*}{\sigma^*}\right]$$

• Parameter values which fit soil data

	London Clay	Weald Clay	Kaolin	Dog's Bay Sand	Ham River Sand
λ^*	0.161	0.093	0.26	0.334	0.163
κ^*	0.062	0.035	0.05	0.009	0.015
Γ^* at 1 kPa	2.759	2.060	3.767	4.360	3.026
$\sigma^*_{c,\ virgin}$ kPa	1	1	1	Loose 500	Loose 2500
				Dense 1500	Dense 15000
ϕ_{crit}	23°	24°	26°	39°	32°
M_{comp}	0.89	0.95	1.02	1.60	1.29
M_{extn}	0.69	0.72	0.76	1.04	0.90
w_L	0.78	0.43	0.74	--------------	--------------
w_P	0.26	0.18	0.42	--------------	--------------
G_s	2.75	2.75	2.61	2.75	2.65

Note: 1) parameters λ^*, κ^*, Γ^*, σ^*_c should depend to a small extent on the deformation mode, e.g. SSA, BA-PS, TA-AS, etc. This may be neglected unless further information is given.
2) Sand which is loose, or loaded cyclically, compacts more than Cam Clay allows.

• **The yield surface in (σ*, τ*, v) space**



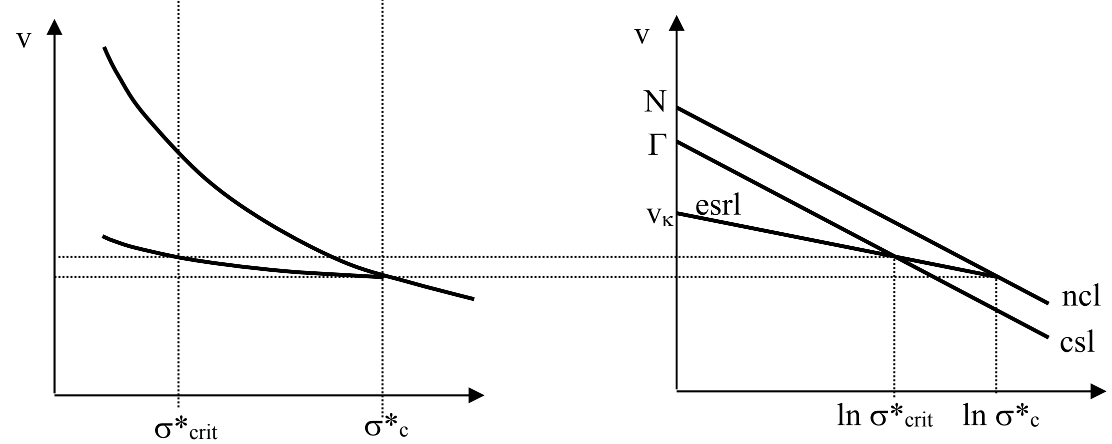
ncl: normal compression line

$$v = N - \lambda \ln \sigma^*$$

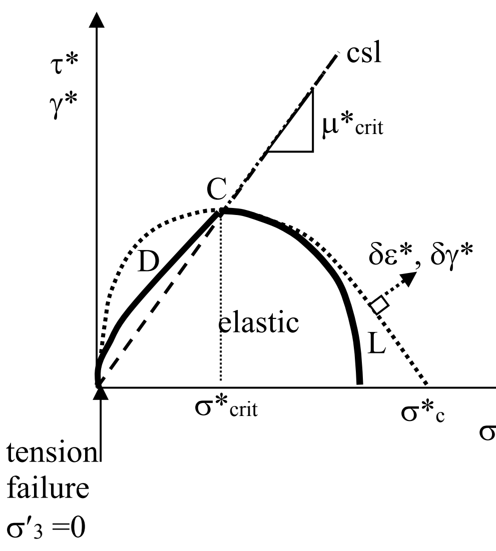
csl: critical state line

$$v = \Gamma - \lambda \ln \sigma^*$$

where $N = \Gamma + \lambda - \kappa$



• **Regions of limiting soil behaviour**



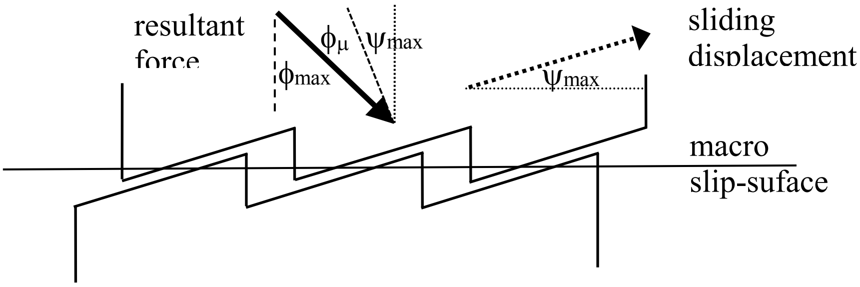
Variation of Cam Clay yield surface

Zone D: denser than critical, “dry”,
dilation or negative excess pore pressures,
Hvorslev strength envelope,
friction-dilatancy theory,
unstable shear rupture, progressive failure

Zone L: looser than critical, “wet”,
compaction or positive excess pore pressures,
Modified Cam Clay yield surface,
stable strain-hardening continuum

Strength of soil: friction and dilation

• Friction and dilatancy: the saw-blade model of direct shear

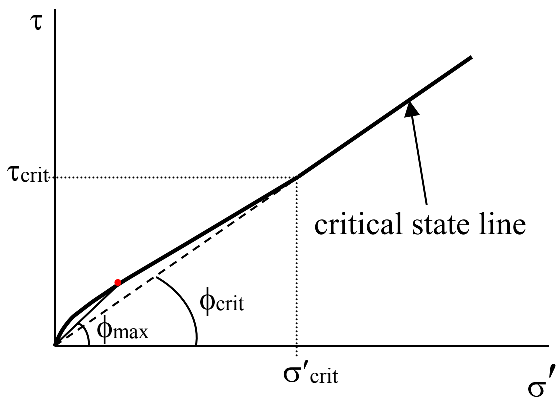


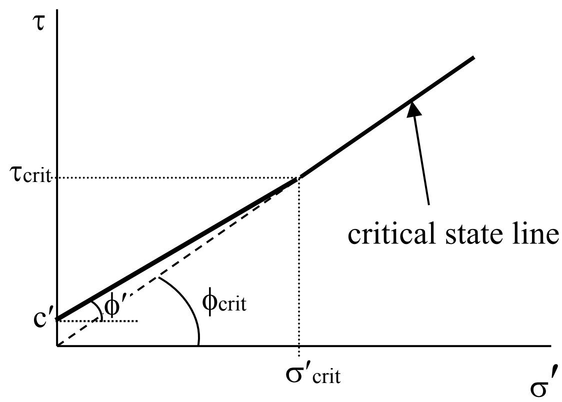
Intergranular angle of friction at sliding contacts ϕ_μ

Angle of dilation ψ_{max}

Angle of internal friction $\phi_{max} = \phi_\mu + \psi_{max}$

• Friction and dilatancy: secant and tangent strength parameters





Secant angle of internal friction

$\tau = \sigma' \tan \phi_{max}$
$\phi_{max} = \phi_{crit} + \Delta\phi$
$\Delta\phi = f(\sigma'_{crit}/\sigma')$

typical envelope fitting data:
power curve
$(\tau/\tau_{crit}) = (\sigma'/\sigma'_{crit})^{\alpha}$
with $\alpha \approx 0.85$

Tangent angle of shearing envelope

$\tau = c' + \sigma' \tan \phi'$
$c' = f(\sigma'_{crit})$

typical envelope:
straight line
$\tan \phi' = 0.85 \tan \phi_{crit}$
$c' = 0.15\, \tau_{crit}$

• Friction and dilation: data of sands

The inter-granular friction angle of quartz grains, $\phi_\mu \approx 26°$. Turbulent shearing at a critical state causes ϕ_{crit} to exceed this. The critical state angle of internal friction ϕ_{crit} is a function of the uniformity of particle sizes, their shape, and mineralogy, and is developed at large shear strains irrespective of initial conditions. Typical values of ϕ_{crit} ($\pm 2^o$) are:

well-graded, angular quartz or feldspar sands	40°
uniform sub-angular quartz sand	36°
uniform rounded quartz sand	32°

Relative density $I_D = \dfrac{(e_{max} - e)}{(e_{max} - e_{min})}$ where:

e_{max} is the maximum void ratio achievable in quick-tilt test
e_{min} is the minimum void ratio achievable by vibratory compaction

Relative crushability $I_C = \ln(\sigma_c / p')$ where:

σ_c is the aggregate crushing stress, taken to be a material constant, typical values being: 80 000 kPa for quartz silt, 20 000 kPa for quartz sand, 5 000 kPa for carbonate sand.

p' is the mean effective stress at failure which may be taken as approximately equal to the effective stress σ' normal to a shear plane.

Dilatancy contribution to the peak angle of internal friction is $\Delta\phi = (\phi_{max} - \phi_{crit}) = f(I_R)$

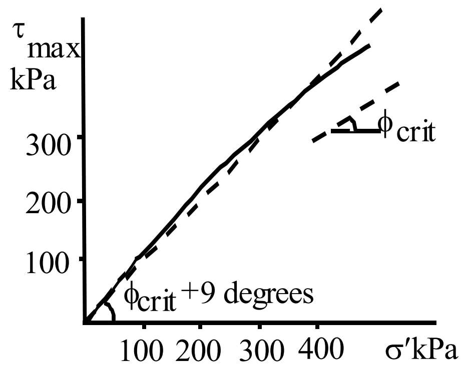
Relative dilatancy index $I_R = I_D I_C - 1$ where:

$I_R < 0$ indicates compaction, so that I_D increases and $I_R \rightarrow 0$ ultimately at a critical state
$I_R > 4$ to be limited to $I_R = 4$ unless corroborative dilatant strength data is available

The following empirical correlations are then available

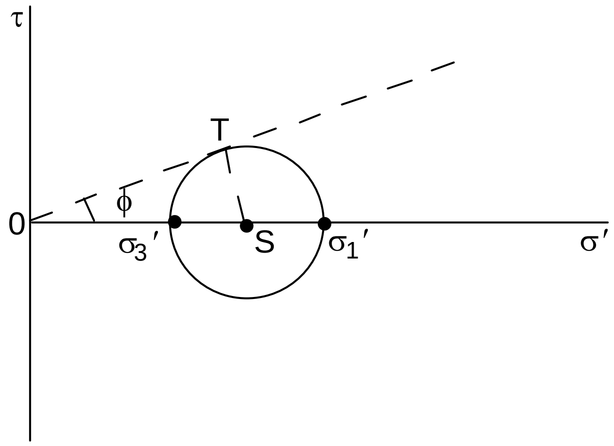
plane strain conditions	$(\phi_{max} - \phi_{crit})$	$= 0.8\ \psi_{max}$	$= 5\ I_R$ degrees
triaxial strain conditions	$(\phi_{max} - \phi_{crit})$	$= 3\ I_R$ degrees	
all conditions	$(-\delta\varepsilon_v / \delta\varepsilon_1)_{max}$	$= 0.3\ I_R$	

The resulting peak strength envelope for triaxial tests on a quartz sand at an initial relative density $I_D = 1$ is shown below for the limited stress range 10 - 400 kPa:



$\phi_{max} > \phi_{crit} + 9°$ for $I_D = 1$, $\sigma' = 400$ kPa

- **Mobilised (secant) angle of shearing ϕ in the 1 – 3 plane**



$$\sin\phi = TS/OS$$

$$= \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

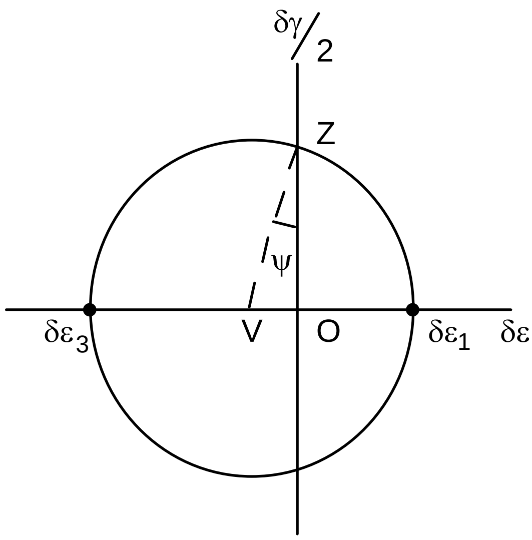
$$\left[\frac{\sigma_1'}{\sigma_3'}\right] = \frac{(1+\sin\phi)}{(1-\sin\phi)}$$

Angle of shearing resistance:

at peak strength ϕ_{max} at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state ϕ_{crit} after large shear strains

- **Mobilised angle of dilation in plane strain ψ in the 1 – 3 plane**



$$\sin\psi = VO/VZ$$

$$= -\frac{(\delta\varepsilon_1 + \delta\varepsilon_3)/2}{(\delta\varepsilon_1 - \delta\varepsilon_3)/2}$$

$$= -\frac{\delta\varepsilon_v}{\delta\varepsilon_\gamma}$$

$$\left[\frac{\delta\varepsilon_1}{\delta\varepsilon_3}\right] = -\frac{(1-\sin\psi)}{(1+\sin\psi)}$$

at peak strength $\psi = \psi_{max}$ at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state $\psi = 0$ since volume is constant

Plasticity: Cohesive material $\tau_{max} = c_u$ (or s_u)

- **Limiting stresses**

Tresca $|\sigma_1 - \sigma_3| = q_u = 2c_u$

von Mises $(\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2c_u^2$

where q_u is the undrained triaxial compression strength, and c_u is the undrained plane shear strength.

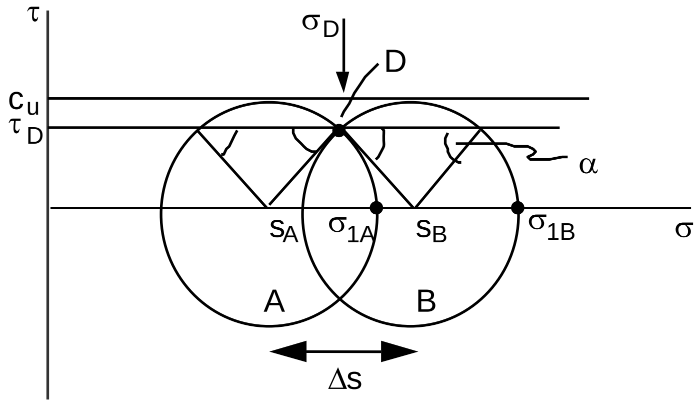
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = c_u \delta\varepsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength c_u, this becomes

$$D = Ac_u x$$

- **Stress conditions across a discontinuity**



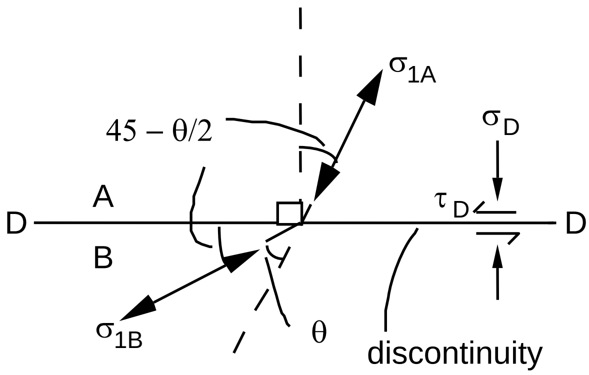
Rotation of major principal stress θ

$s_B - s_A = \Delta s = 2c_u \sin\theta$

$\sigma_{1B} - \sigma_{1A} = 2c_u \sin\theta$

In limit with $\theta \rightarrow 0$

$$ds = 2c_u d\theta$$



Useful example:

$\theta = 30^{\circ}$

$\sigma_{1B} - \sigma_{1A} = c_u$

$\tau_D / c_u = 0.87$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

Plasticity: Frictional material $(\tau/\sigma')_{max} = \tan\phi$

• Limiting stresses

$$\sin\phi = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u_s)$$

where σ'_{1f} and σ'_{3f} are the major and minor principal effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principle total stresses at failure, and u_s is the steady state pore pressure.

Active pressure:

$\sigma'_v > \sigma'_h$

$\sigma'_1 = \sigma'_v$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_h$

$K_a = (1 - \sin\phi)/(1 + \sin\phi)$

Passive pressure:

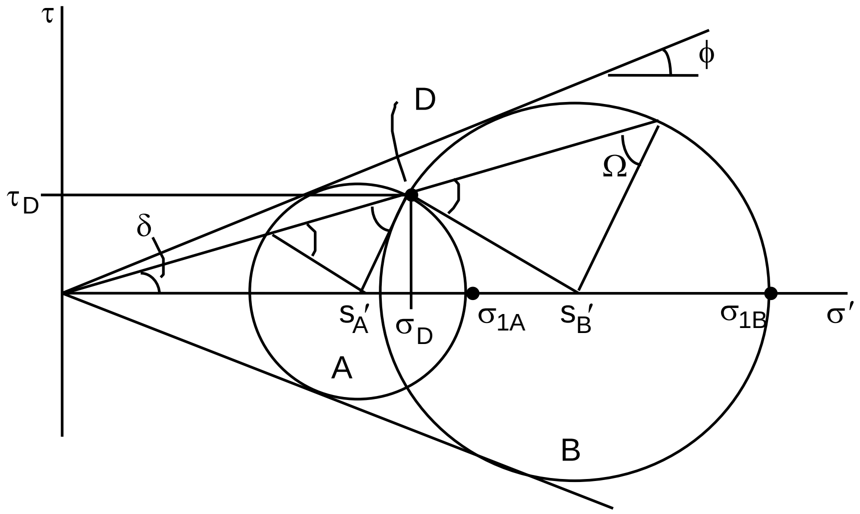
$\sigma'_h > \sigma'_v$

$\sigma'_1 = \sigma'_h$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_v$

$K_p = (1 + \sin\phi)/(1 - \sin\phi) = 1/K_a$

• Stress conditions across a discontinuity



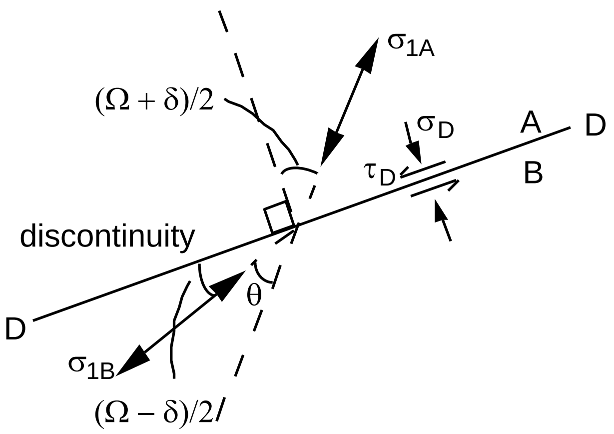
Rotation of major principal stress

$\theta = \pi/2 - \Omega$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

$\tan\delta = \tau_D / \sigma'_D$



$\sin\Omega = \sin\delta / \sin\phi$

$s'_B/s'_A = \sin(\Omega + \delta) / \sin(\Omega - \delta)$

In limit, $d\theta \to 0$ and $\delta \to \phi$

$ds' = 2s'.\, d\theta \tan\phi$

Integration gives $s'_B/s'_A = \exp(2\theta \tan\phi)$

Empirical earth pressure coefficients following one-dimensional strain

Coefficient of earth pressure in 1D plastic compression (normal compression)

$$K_{o,nc} = 1 - \sin\phi_{crit}$$

Coefficient of earth pressure during a 1D unloading-reloading cycle (overconsolidated soil)

$$K_o = K_{o,nc}\left[1 + \frac{(n-1)(n_{max}^{\alpha}-1)}{(n_{max}-1)}\right]$$

where n is current overconsolidation ratio (OCR) defined as $\sigma'_{v,max}/\sigma'_v$

n_{max} is maximum historic OCR defined as $\sigma'_{v,max}/\sigma'_{v,min}$

α is to be taken as $1.2\sin\phi_{crit}$

Cylindrical cavity expansion

Expansion $\delta A = A - A_o$ caused by increase of pressure $\delta\sigma_c = \sigma_c - \sigma_o$

At radius r: small displacement $\rho = \dfrac{\delta A}{2\pi r}$

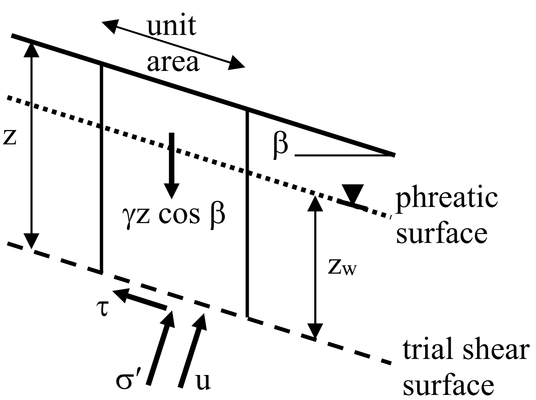
small shear strain $\gamma = \dfrac{2\rho}{r}$

Radial equilibrium: $r\dfrac{d\sigma r}{dr} + \sigma_r - \sigma_\theta = 0$

Elastic expansion (small strains) $\delta\sigma_c = G\dfrac{\delta A}{A}$

Undrained plastic-elastic expansion $\delta\sigma_c = c_u\left[1 + \ln\dfrac{G}{c_u} + \ln\dfrac{\delta A}{A}\right]$

Infinite slope analysis



$$u = \gamma_w z_w \cos^2\beta$$
$$\sigma = \gamma z \cos^2\beta$$
$$\sigma' = (\gamma z - \gamma_w z_w)\cos^2\beta$$
$$\tau = \gamma z \cos\beta \sin\beta$$

$$\tan\phi_{mob} = \frac{\tau}{\sigma'} = \frac{\tan\beta}{\left(1 - \frac{\gamma_w z_w}{\gamma z}\right)}$$

Shallow foundation design

Tresca soil, with undrained strength s_u

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

V_{ult} and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and γ (or γ') is the appropriate density of the overburden.

The exact bearing capacity factor N_c for a plane strain surface foundation (zero embedment) on uniform soil is:

$N_c = 2 + \pi$ (Prandtl, 1921)

Shape correction factor:

For a rectangular footing of length L and breadth B (Eurocode 7):

$s_c = 1 + 0.2\ B / L$

The exact solution for a rough circular foundation (D = B = L) is $q_f = 6.05 s_u$, hence $s_c = 1.18 \sim 1.2$.

Embedment correction factor:

A fit to Skempton's (1951) embedment correction factors, for an embedment of h, is:

$d_c = 1 + 0.33 \tan^{-1} (h/B)$ (or h/D for a circular foundation)

Combined V-H loading

A curve fit to Green's lower bound plasticity solution for V-H loading is:

If $V/V_{ult} > 0.5$: $\frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{H}{H_{ult}}}$ or $\frac{H}{H_{ult}} = 1 - \left(2\frac{V}{V_{ult}} - 1\right)^2$

If $V/V_{ult} < 0.5$: $H = H_{ult} = B s_u$

Combined V-H-M loading

With lift-off: combined Green-Meyerhof

Without lift-off: $\left(\frac{V}{V_{ult}}\right)^2 + \left[\frac{M}{M_{ult}}\left(1 - 0.3\frac{H}{H_{ult}}\right)\right]^2 + \left|\left(\frac{H}{H_{ult}}\right)^3\right| - 1 = 0$ (Taiebet & Carter 2000)

Frictional (Coulomb) soil, with friction angle ϕ

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation under drained loading (Coulomb soil) is:

$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + s_\gamma N_\gamma \frac{\gamma' B}{2}$$

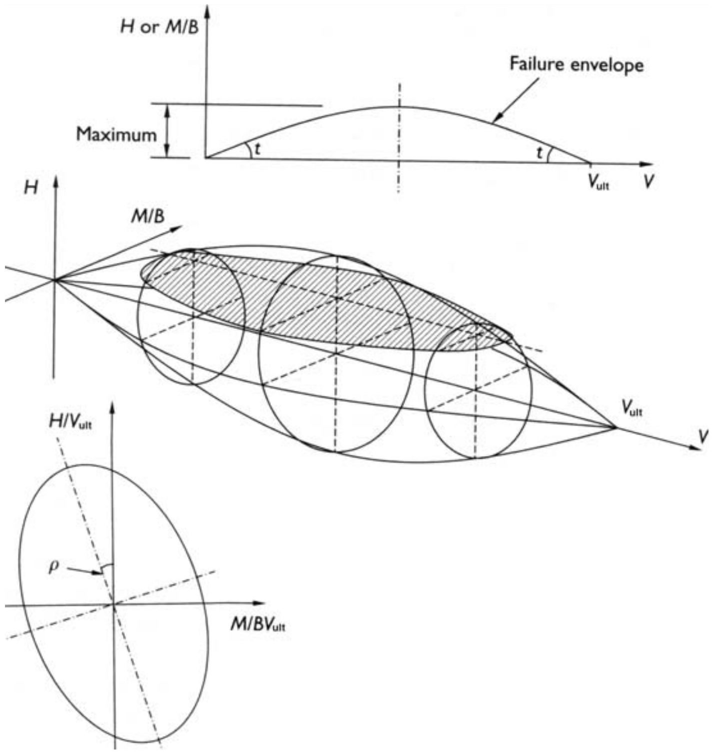
The bearing capacity factors N_q and N_γ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. σ'_{v0} is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for N_q is:

$N_q = \tan^2(\pi/4 + \phi/2)\, e^{(\pi \tan \phi)}$ (Prandtl 1921)

An empirical relationship to estimate N_γ from N_q is (Eurocode 7):

$N_\gamma = 2\,(N_q - 1) \tan \phi$



Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

Rough base: $N_\gamma = 0.1054\, e^{9.6\phi}$

Smooth base: $N_\gamma = 0.0663\, e^{9.3\phi}$

Shape correction factors:

For a rectangular footing of length L and breadth B (Eurocode 7):

$s_q = 1 + (B \sin \phi) / L$
$s_\gamma = 1 - 0.3\, B / L$

For circular footings take L = B.

Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.

Combined V-H-M loading

With lift-off- drained conditions - use Butterfield & Gottardi (1994) failure surface shown above

$$\left[\frac{H/V_{ult}}{t_h}\right]^2 + \left[\frac{M/BV_{ult}}{t_m}\right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m}\right] = \left[\frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)\right]^2$$

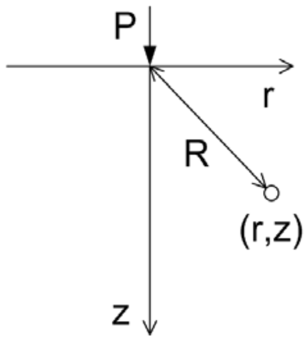
where $C = \tan\left(\frac{2\rho(t_h - t_m)(t_h + t_m)}{2 t_h t_m}\right)$ (Butterfield & Gottardi, 1994)

Typically, $t_h \sim 0.5$, $t_m \sim 0.4$ and $\rho \sim 15°$. Note that t_h is the friction coefficient, H/V= $\tan\phi$, during sliding.

Settlement of Shallow Foundations

Elastic stress distributions below point, strip and circular loads

Point loading (Boussinesq solution)



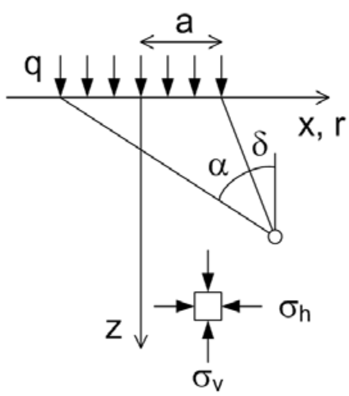
Vertical stress $\sigma_z = \dfrac{3Pz^3}{2\pi R^5}$

Radial stress $\sigma_r = \dfrac{P}{2\pi R^2}\left[\dfrac{3r^2 z}{R^3} - \dfrac{(1-2\nu)R}{R+z}\right]$

Tangential stress $\sigma_\theta = \dfrac{P(1-2\nu)}{2\pi R^2}\left[\dfrac{R}{R+z} - \dfrac{z}{R}\right]$

Shear stress $\tau_{rz} = \dfrac{3Prz^2}{2\pi R^5}$

Uniformly-loaded strip



Vertical stress $\sigma_v = \dfrac{q}{\pi}\left[\alpha + \sin\alpha\cos(\alpha+2\delta)\right]$

Horizontal stress $\sigma_h = \dfrac{q}{\pi}\left[\alpha - \sin\alpha\cos(\alpha+2\delta)\right]$

Shear stress $\tau_{vh} = \dfrac{q}{\pi}\sin\alpha\sin(\alpha+2\delta)$

Principal stresses $\sigma_1 = \dfrac{q}{\pi}(\alpha+\sin\alpha)$ $\sigma_3 = \dfrac{q}{\pi}(\alpha-\sin\alpha)$

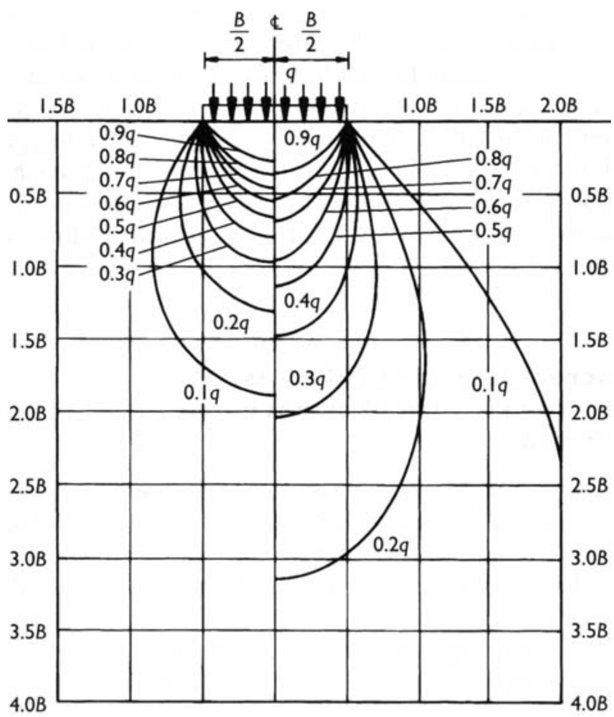
Uniformly-loaded circle radius *a* (on centerline, r=0)

Vertical stress

$$\sigma_v = q\left[1-\left(\frac{1}{1+(a/z)^2}\right)^{\frac{3}{2}}\right]$$

Horizontal stress

$$\sigma_h = \frac{q}{2}\left[(1+2\nu) - \frac{2(1+\nu)z}{(a^2+z^2)^{1/2}} + \frac{z^3}{(a^2+z^2)^{3/2}}\right]$$



Contours of vertical stress below uniformly-loaded circular (left) and strip footings (right)

Elastic solutions for surface settlement
Isotropic, homogeneous, elastic half-space (semi-infinite)

Point load (Boussinesq solution)

Settlement, w, at distance s: $w(s) = \frac{1}{2\pi}\frac{(1-\nu)}{G}\frac{P}{s}$

Circular area (radius a), uniform soil

Uniform load: central settlement: $w_o = \frac{(1-\nu)}{G}qa$

edge settlement: $w_e = \frac{2}{\pi}\frac{(1-\nu)}{G}qa$

Rigid punch: (q_{avg}= V/πa^2) $w_r = \frac{\pi}{4}\frac{(1-\nu)}{G}q_{avg}a$

Rectangular area, uniform soil

Uniform load, corner settlement:

$$w_c = \frac{(1-\nu)}{G}\frac{qB}{2}I_{rect}$$

Where I_{rect} depends on the aspect ratio, L/B:

L/B	I_{rect}	L/B	I_{rect}	L/B	I_{rect}	L/B	I_{rect}
1	0.561	1.6	0.698	2.4	0.822	5	1.052
1.1	0.588	1.7	0.716	2.5	0.835	6	1.110
1.2	0.613	1.8	0.734	3	0.892	7	1.159
1.3	0.636	1.9	0.750	3.5	0.940	8	1.201
1.4	0.658	2	0.766	4	0.982	9	1.239
1.5	0.679	2.2	0.795	4.5	1.019	10	1.272

Rigid rectangle: $w_r = \frac{(1-\nu)}{G}\frac{q_{avg}\sqrt{BL}}{2}I_{rgd}$ where I_{rgd} varies from 0.9→0.7 for L/B = 1-10.

Note: $G = \frac{E}{2(1+\nu)}$ where ν= Poisson's ratio, E= Young's modulus.