(a) the maximum and minimum values of the force correspon to Rankine active and passive states. For $\varphi' = 30°$

$$K_A = \frac{1-\sin\varphi'}{1+\sin\varphi'} = \frac{1}{3} \qquad K_p = \frac{1}{K_a} = 3$$

(i) the resultant of the earth pressure on the wall has minimum and maximum values:

$$F_{min} = P_a = \frac{1}{2}\gamma K_a h^2 = \frac{1}{2} \times 20 \times \frac{1}{3} \times 3^2 = 30 \text{ kN/m}$$

$$F_{max} = P_p = \frac{1}{2}\gamma K_p h^2 = \frac{1}{2} \times 20 \times 3 \times 3^2 = 270 \text{ kN/m}$$

in both cases the earth pressure has a triangular distribution so

$$d = \frac{2}{3}h = 2 \text{ m}$$

(ii) Due to the surcharge the active and passive earth pressure distributions are increased by a constant horizontal pressure with intensitiy $K_a q = 5$ kPa and $K_p q = 45$ kPa

The active and passive thrusts due to surchauge are $P_a^q = 15$ kN/m and $P_p^q = 135$ kN/m

Thus the minimum and maximum force become

$$F_{min} = P_a + P_a^q = 45 \text{ kN/m}$$

$$F_{max} = P_p + P_p^q = 405 \text{ kN/m}$$

to compute d take moments about top of wall: $P_a \cdot \frac{2}{3}h + P_a^q \cdot \frac{h}{2} = F_{min} d$

$$d = \frac{30 \times 2 + 15 \times 1{,}5}{} = 1{,}83 \text{ m}$$

$$P_p \times \frac{2}{3}h + P_p^{q}\cdot\frac{h}{2} = F_{max}\cdot d$$

$$d = \frac{270\times 2 + 135\times 1.5}{405} = 1.83 \text{ m}$$

(b) (i) if the strut force does not vary the soil on either side must be in a limit state, with the smallest (active) thrust on one side the same as the largest (passive) thrust on the other side. Given the difference in height, it is likely that the soil on the left is in active state and the soil on the right in the passive state. If this is the case, the total thrust from the soil on the left is equal to

$$P_a = P_a' + S_w = \frac{1}{2}\gamma' k_a h^2 + \frac{1}{2}\gamma_w h^2 =$$

$$= 0.5\times 10\times \frac{1}{3}\cdot 3^2 + 0.5\cdot 9.8\times 3^2 =$$

$$= 59.1 \text{ kN}$$ (confirming our assump)

For the soil on the right side to be able to support this thrust it must be:

$$59 = P_p' + S_w = \frac{1}{2}\gamma' k_p\, 1.5^2 + \frac{1}{2}\gamma_w h^2$$

$$59 = \frac{10.6}{2}\cdot k_p \times 2.25 + \frac{9.8}{2}\times 2.25$$

$$11.93\, k_p = 47.98$$

$$k_p = 4.02$$

③

$k_p = tg^2(45 + \varphi/2)$

$\sqrt{k_p} = tg(45 + \varphi/2) \qquad tg^{-1}\sqrt{k_p} = 45° + \varphi/2$

$\varphi = 2 \times (tg^{-1}\sqrt{k_p} - 45°) \cong 37°$

(ii) @ 1 m depth

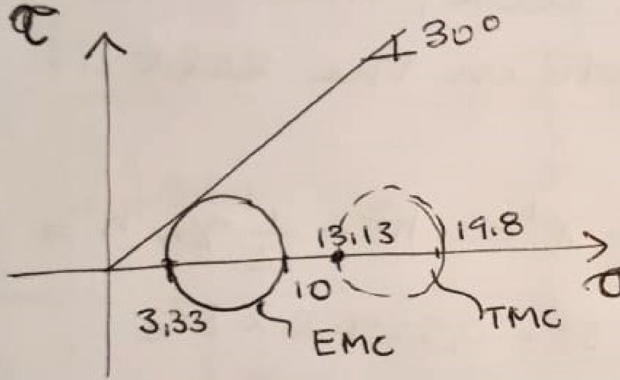
right side $\quad \sigma_v = 19.8$ kPa

$u = 9.8$ kPa

$\sigma'_v = 10$ kPa

$\sigma'_h = \sigma'_a = 3{,}33$ kPa

$\sigma_h = 3{,}33 + 9{,}8 = 13{,}13$



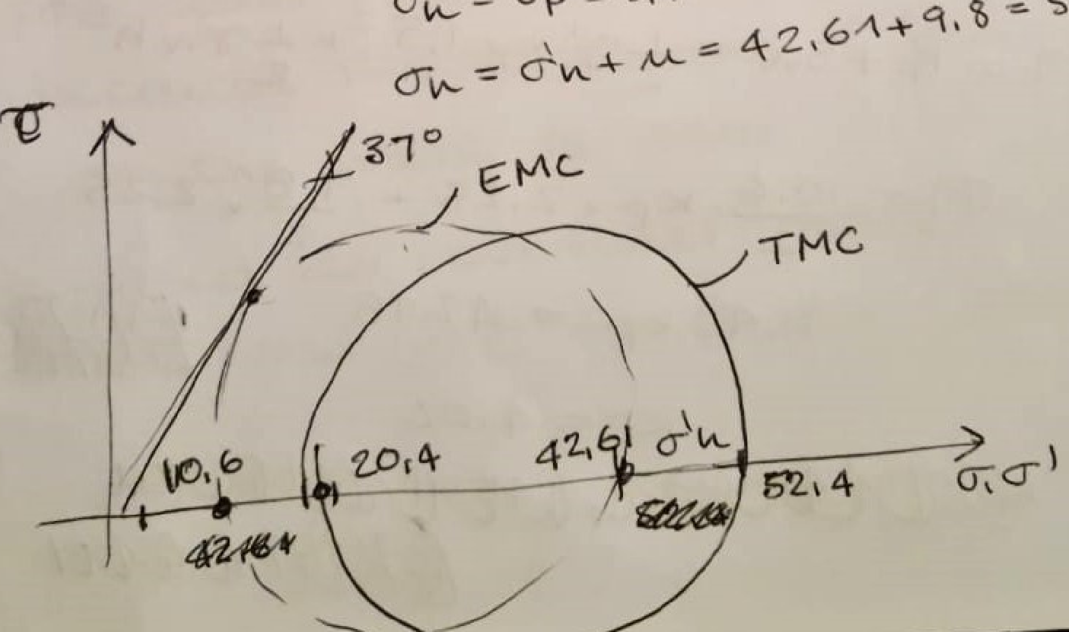
left side $\quad \sigma_v = 20.4$

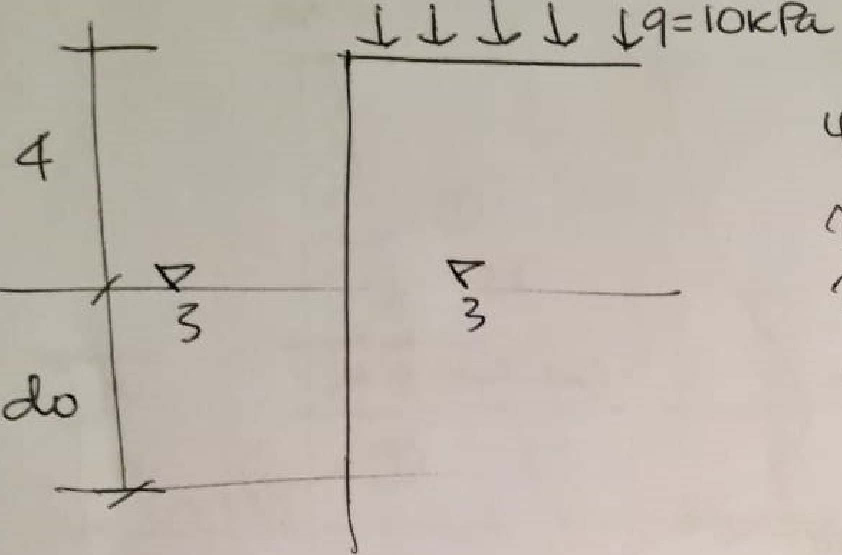
$u = 9.8$

$\sigma'_v = 10{,}6$ kPa

$\sigma'_h = \sigma'_p = 4{,}02 \times 10{,}6 = 42{,}61$ kPa

$\sigma_h = \sigma'_h + u = 42{,}61 + 9{,}8 = 52{,}4$ kPa





$\varphi' = 33°$ $\quad \delta = \frac{1}{3}\varphi' = 11°$

$\gamma_d = 15,3$ kN/m3

$\gamma_{sat} = 19.7$ kN/m3

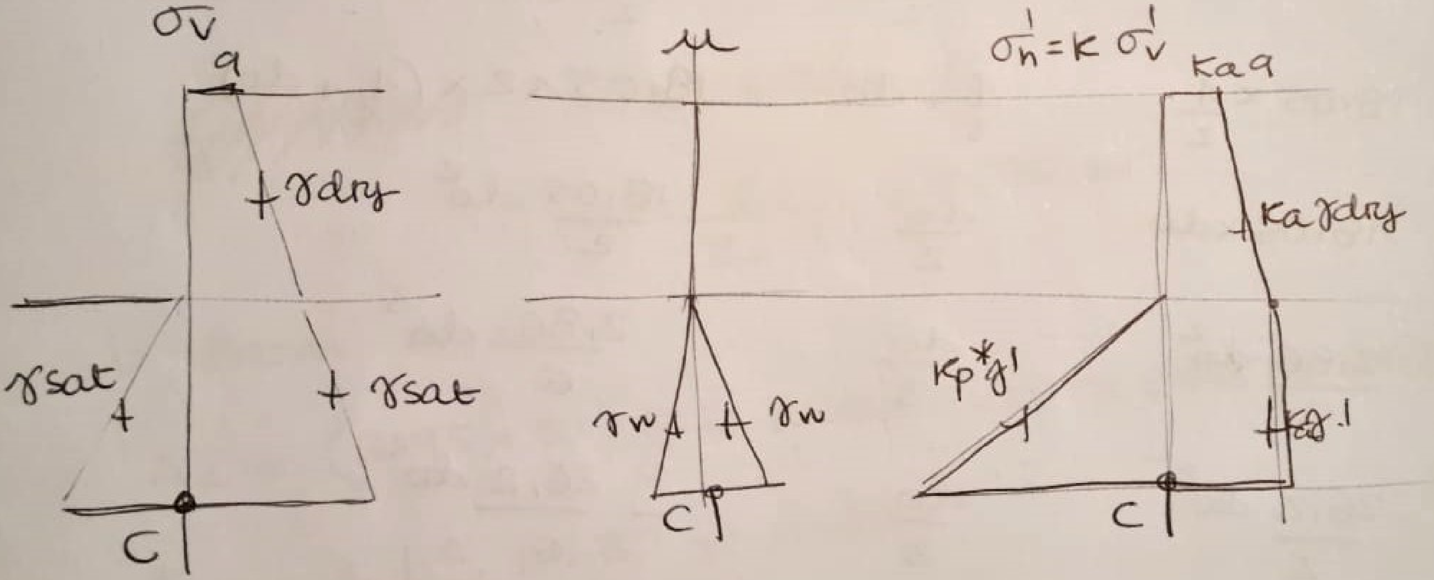
$\gamma_w \simeq 10$ kN/m3

$\gamma' \cong 9,7$ kN/m3

Ka = 0,295 (Rankine)

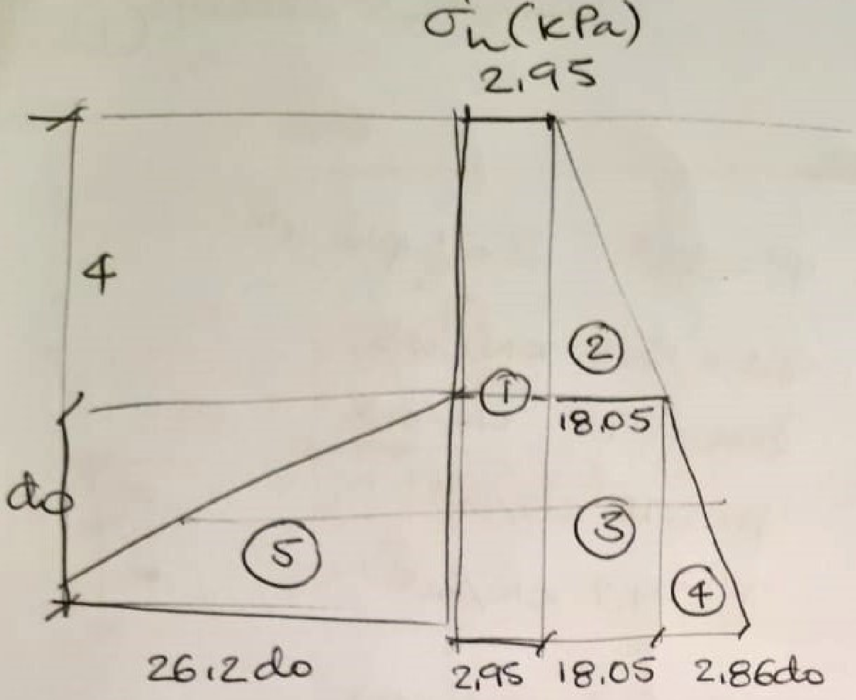
Kp = 4,596 (Lancellotta)

F = 1,7 $\quad$ $K_p^* = K_p / F = 2,704$



pore water pressures can be ignored because the resultant of the p.w.p on the wall is zero

to find do, consider moment equilibrium @ C → d = 1,2do



	F	b	M
1	$2.95 \times (4 + d_o)$	$\frac{4 + d_o}{2}$	$\frac{2.95}{2}(16 + 8d_o + d_o^2)$
2	$18.05 \times \frac{4}{2}$	$\frac{4}{3} + d_o$	$18.05 \times 2 \times (\frac{4}{3} + d_o)$
3	$18.05 \times d_o$	$\frac{d_o}{2}$	$\frac{18.05}{2} d_o^2$
4	$\frac{2.86}{2} d_o^2$	$\frac{d_o}{3}$	$\frac{2.86}{6} d_o^3$
5	$\frac{26.2}{2} d_o^2$	$\frac{d_o}{3}$	$-\frac{26.2}{6} d_o^3$

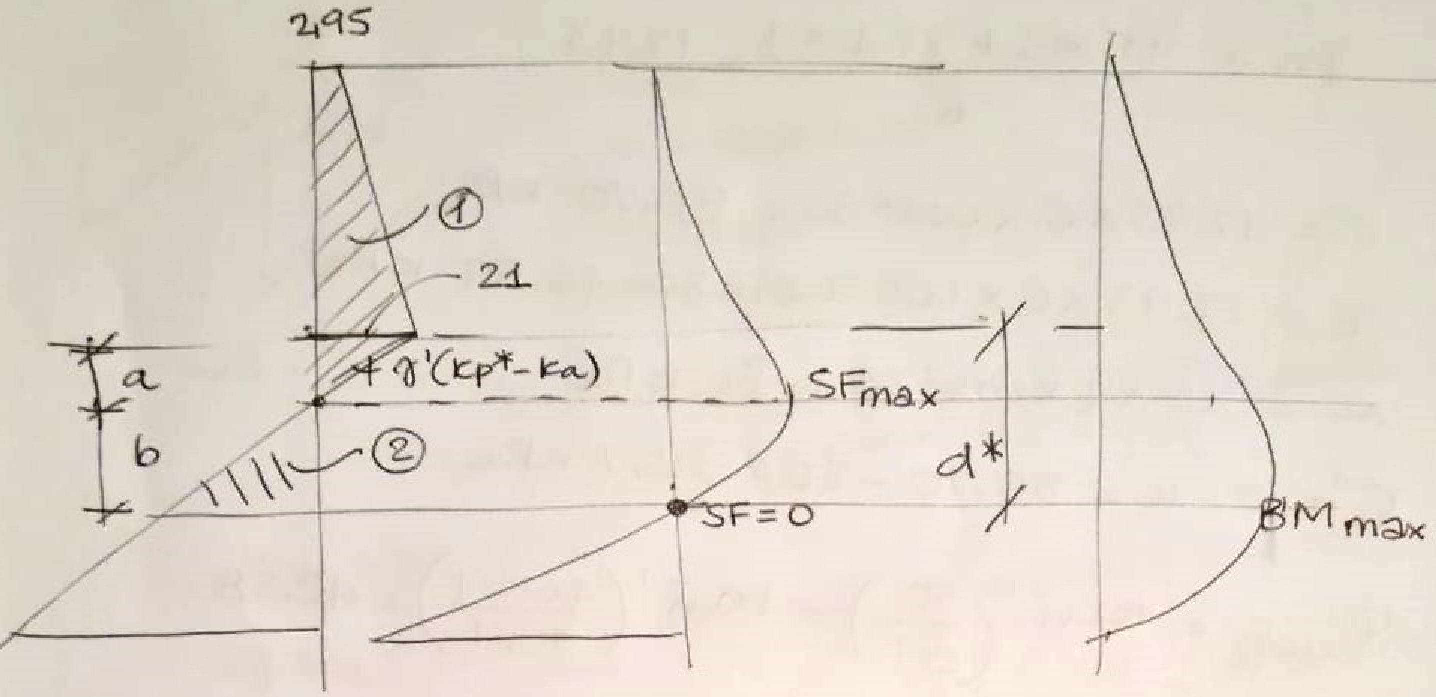
$$M = 23.60 + 11.80\, d_o + 1.48\, d_o^2 + 48.13 + 36.1\, d_o + 9.03\, d_o^2 - 3.89\, d_o^3 =$$

$$= -3.89\, d_o^3 + 10.51\, d_o^2 + 47.90\, d_o + 71.73$$

d_o	M (- is restoring)
5	81.73
6	-102.75
5.5	5.91 ✓
5.6	-13.58

very small negative value ~~this means~~ F will be only very slightly larger than 1.7.

qualitatively:



$d^* = a + b$

$\gamma'(k_p^* - k_a) = 9{,}7(2{,}704 - 0{,}295 =$
$= 23{,}37$

$a \times 23{,}37 = 21$

$$a = \frac{21}{23{,}37} = 0{,}90 \text{ m}$$

b from $A_1 = A_2$

$$A_1 = (2{,}95 + 21)\frac{4}{2} + \frac{1}{2}\, 0{,}90 \times 0{,}9 \times 23{,}37 = 57{,}36$$

$$A_2 = \frac{1}{2} b^2 \cdot 23{,}37 = 11{,}69 b^2$$

$$b = \sqrt{\frac{57{,}36}{11{,}69}} = 2{,}22 \text{ m}$$

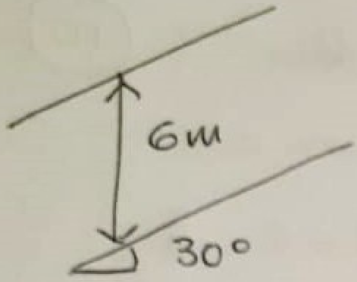
$$d^* = a + b = 2{,}22 + 0{,}90 = 3{,}12 \text{ m}$$

compute M from equation on page 5

$$M_{max} = -3{,}89 \times 3{,}12^3 + 10{,}51 \times 3{,}12^2 + 47{,}90 \times 3{,}12 + 71{,}73 =$$
$$= 205{,}34 \text{ kNm/m}$$

Q2: (a)

soils yielding on the dry side of critical state do so with dilation and softening. Therefore, these soils are brittle with their supercritical shear strength falling towards critical states with straining. Dense sands will dilate when they yield if their grains do not crush. Sands do not crush significantly at effective stress below ~ 1MPa, so if they are relatively dense (say Dr > 50%) they will dilate. The peak friction angle depends on stress level and relative density. If a sand has brittle stress strain characteristics, the stress at the point of failure is reduced and stress is transferred to adjacent points, with many elements side by side failing progressively. One should design slopes in dilatant soils as though they would only mobilise their ultimate, critical state angle of friction. For clays. both undrained and drained conditions will be relevant because the behaviour will be undrained in the short term and drained in the long term. Because excavation unloads the soil, typically with a decrease of mean effective stress and an increase of deviator stress, pore water pressures around the excavation will be depressed. With time, the soil will swell as the pore water pressure equilibrate to the long-term steady state seepage imposed by the new hydraulic boundary conditions along the excavation slopes ($u=0$ Typically the PWP decreases relative to the pre-excavation value (the PWP decreases under total stress reduction and then increases again to a smaller steady state value relative to pre-excavation) The applied shear stress increases during excavation and then remains constant. The shear strength decreases with time from initial undrained to long term drained effective strength. It follows that the safety factor along any potential failure surface $F = \frac{\tau_{lim}}{\tau}$ decreases with time, and the critical condition for the slope is the long term condition.



$\varphi'_{cs} = 36°$ $\alpha = 30°$

$e = 0.78$

$e_{max} = 0.95$

$e_{min} = 0.60$

$G_s = 2.7$

(i) assuming $\gamma_w \simeq 10$ kN/m³

$$\gamma_{dry} = \gamma_w \frac{G_s}{1+e} = 10 \times \frac{2.7}{1.78} = 15.17 \text{ kN/m}^3$$

$$\gamma_{sat} = \gamma_w \frac{G_s+e}{1+e} = 10 \times \frac{2.7+0.78}{1.78} = 19.55 \text{ kN/m}^3$$

(ii) Dry season $\bar{\gamma}_D = \dfrac{15.17 \times 4 + 19.55 \times 2}{6} = 16.63$ kN/m³

$$\sigma (= \gamma z \cos^2\alpha) = 16.63 \times 6 \times \cos^2 30 = 74.84 \text{ kPa}$$

$$\tau (= \gamma z \sin\alpha \cos\alpha) = 16.63 \times 6 \times \sin 30 \times \cos 30 = 43.21 \text{ kPa}$$

$$u (= \gamma_w z_w \cos^2\alpha) = 10 \times 2 \times \cos^2 30 = 15 \text{ kPa}$$

$$\sigma' = \sigma - u = 74.84 - 15 = 59.84 \text{ kPa}$$

$$\varphi'_{mob} = \tan^{-1}\left(\frac{\tau}{\sigma'}\right) = \tan^{-1}\left(\frac{43.21}{59.84}\right) = 35.83°$$

$\varphi'_{mob} \simeq \varphi'_{cs}$ slope only marginally stable relative to φ'_{cs}.

Wet season $\bar{\gamma}_D = \dfrac{15.17 \times 2 + 19.55 \times 4}{6} = 18.09$ kN/m³

$$\sigma = 18.09 \times 6 \times \cos^2 30° = 81.41 \text{ kPa}$$

$$\tau = 18.09 \times 6 \times \sin 30° \times \cos 30° = 47.00 \text{ kPa}$$

$$u = 10 \times 4 \times \cos^2 30° = 30 \text{ kPa}$$

$$\sigma' = \sigma - u = 81.41 - 30 = 51.41 \text{ kPa}$$

$$\varphi'_{mob} = \tan^{-1}\left(\frac{47.00}{51.41}\right) = 42.43°$$

$\varphi'_{mob} > \varphi'_{cs}$ slope unstable wrt φ'_{cs}

However slope may be stable if $\varphi'_{mob} < \varphi'_{peak}$

assuming $p' \simeq \sigma' = 51.41\ kPa$

$$I_C = \ln \frac{20'000}{51.41} = 5.96$$

$$I_D = \frac{0.95 - 0.78}{0.95 - 0.60} = 0.49$$

$$I_R = I_D \cdot I_C - 1 = 1.89$$

$$\Delta\varphi = \varphi_{peak} - \varphi_{cs} = I_R \times 5 = 9.47^\circ$$

$$\varphi_{peak} = 36^\circ + \Delta\varphi = 45.47^\circ$$

$\varphi'_{mob} < \varphi_{peak}$ slope "may be" stable.

(iii)

z_w (m)	$\bar{\gamma}$ (kN/m³)	σ (kPa)	τ (kPa)	u (kPa)	σ' (kPa)	φ'_{mob} (°)
4.0	18.1	81.41	47.00	30.0	51.41	42.44
4.5	18.5	83.05	47.95	33.8	49.30	44.20
5.0	18.8	84.69	48.90	37.5	47.19	46.02
5.5	19.2	86.33	49.84	41.2	45.08	47.87
6.0	19.6	87.98	50.79	45.0	42.98	49.77

z_w (m)	I_C (–)	I_R (–)	$\Delta\varphi$ (°)	φ_P (°)	
4.0	5.96	1.92	9.61	45.61	✓
4.5	6.01	1.94	9.71	45.71	✓
5.0	6.05	1.96	9.82	45.82	✗
4.9	6.04	1.96	9.80	45.80	✓ ($\varphi'_{mob} = 45.65$)

(iv) Not a good idea

as a matter of fact

if $z = z_w = 4m$ and $\gamma = \gamma_{sat} = 19.55\ kN/m^3$

$\sigma = 53.65$ kPa

$\tau = 35.86$ kPa

$u = 30$ kPa

$\sigma' = 28.62$ kPa

$$\varphi'_{mob} = \tan^{-1} \frac{33.86}{28.62} = 49.79°$$

$$I_C = \ln \frac{20'000}{28.62} = 6.55$$

$$I_R = 6.55 \times 0.49 - 1 = 2.21$$

$$\Delta\varphi = 5 \times 2.21 = 11.05°$$

$$\varphi_{peak} = 36 + 11.05 \simeq 47°$$

$\varphi_{peak} < \varphi'_{mob}$ slope would fail

quite obvious - taken away stabilizing weight of dry soil above water table.

Q4

$\lambda = 0{,}26 \qquad \varphi' = 26°$

$\kappa = 0{,}05 \qquad M_c = 1.02$

$\Gamma = 3{,}767 \qquad M_e = 0{,}76$

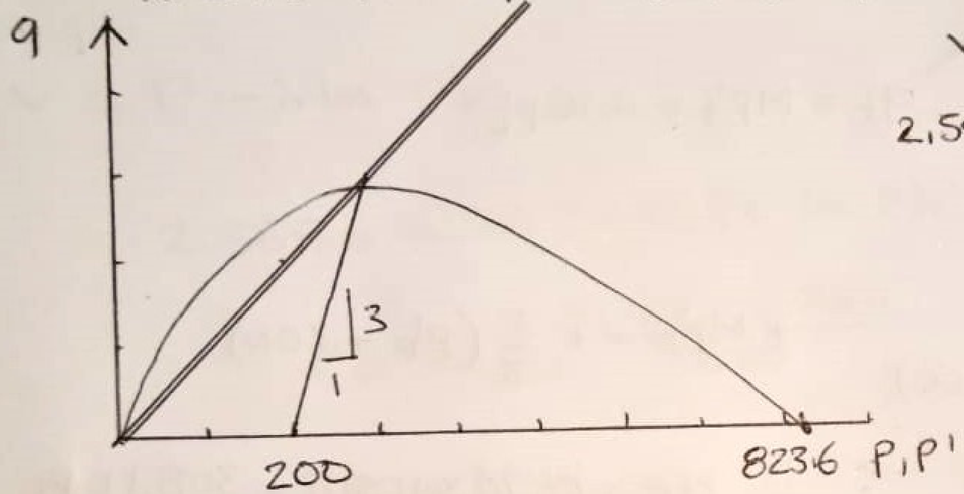
(12)

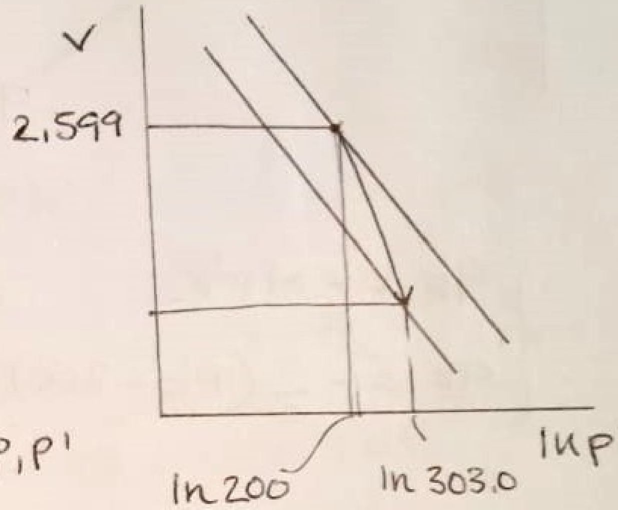
$N = \Gamma + \lambda - \kappa = 3{,}767 + 0{,}26 - 0{,}05 = 3{,}977$

v when normally consolidated to 200 kPa

(a)

$v = N - \lambda \ln p' = 3{,}977 - 0{,}26 \ln 200 = 2{,}599$





(i) triaxial compression $\Delta\sigma_r = 0$

$\Delta q = \Delta\sigma_a - \Delta\sigma_r = \Delta\sigma_a \qquad \dfrac{\Delta q}{\Delta p} = 3$

$\Delta p = \dfrac{\Delta\sigma_a + 2\Delta\sigma_3}{3} = \dfrac{\Delta\sigma_a}{3}$

$\begin{cases} q_F = M p'_F \\ q_F = 3(p'_F - 200) \end{cases} \qquad 1{,}02 p'_F = 3p'_F - 600$

$p'_F = 303$ kPa

$q_F = 1{,}02 \times 303 = 309{,}1$ kPa

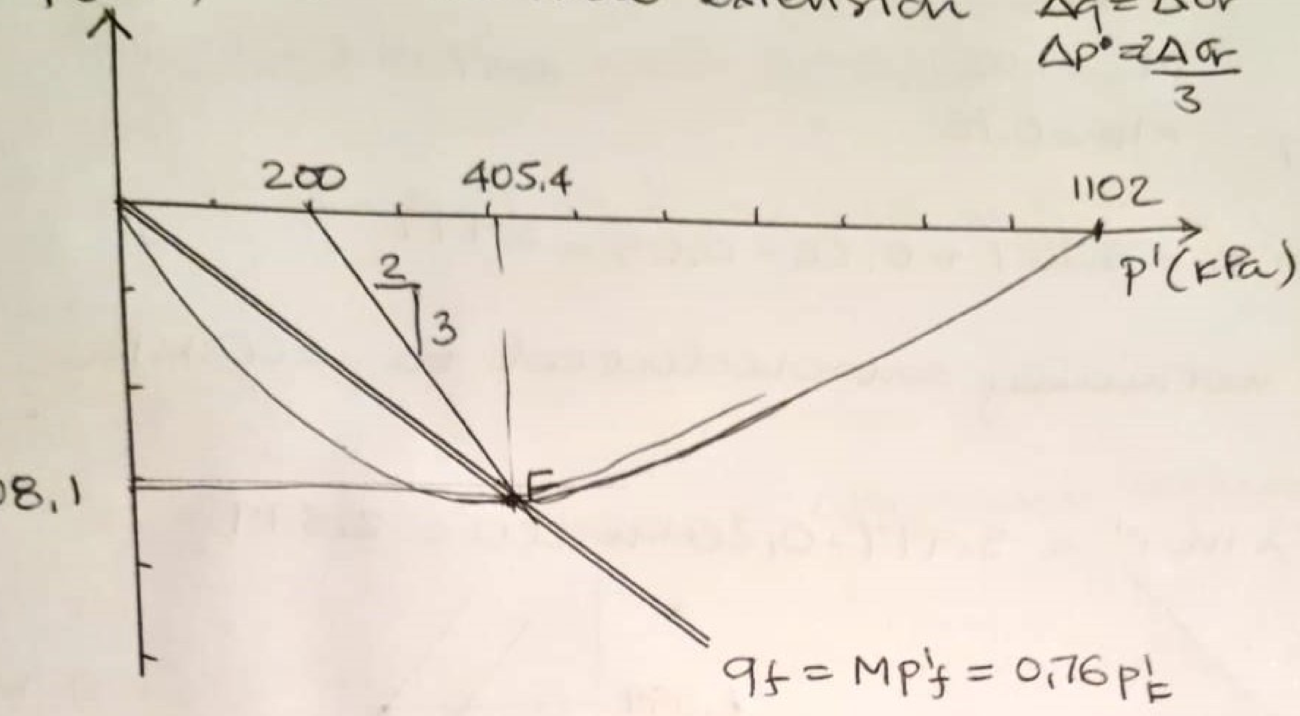
$v = \Gamma - \lambda \ln 303.0 = 3{,}767 - 0{,}26 \ln 303.0 = 2{,}281$

$\varepsilon_v = \dfrac{2{,}599 - 2{,}281}{2{,}599} = 12{,}22\%$

$p'_F = p'_c / e \Rightarrow p'_c = p'_F \cdot e = 823.6$ kPa

(ii) triaxial extension $\Delta q = -\Delta\sigma_r$

$\Delta p' = \dfrac{2\Delta\sigma_r}{3}$ $\quad \dfrac{\Delta q}{\Delta p} = -\dfrac{3}{2}$



$$\begin{cases} q_F = -Mp'_F \\ q_F = -\dfrac{3}{2}(p'_F - 200) \end{cases}$$

$+Mp'_F = +\dfrac{3}{2}(p'_F - 200)$

$q_F = 0.76 \times 405.4 = 308.1$ kPa

$2Mp'_F = 3p'_F - 600$ $\quad p'_F = \dfrac{600}{3-2M} = 405.4$ kPa

$v_F = \Gamma - \lambda \ln p'_F = 3.967 - 0.26 \ln 405.4 = 2.206$

$\varepsilon_v = \dfrac{2.599 - 2.206}{2.599} = 15.13\%$

$p'_c = p'_F \cdot e = 1102$ kPa

(b)

$$v = N - \lambda \ln p'_{max} + \kappa \ln \frac{p'_{max}}{p'} =$$

$$= 3{,}977 - 0{,}26 \ln 600 + 0{,}05 \ln \frac{600}{200} =$$

$$= 2{,}369$$

(i)

undrained shear ≡ constant volume

$$v = \Gamma - \lambda \ln p'_F$$

$$2{,}369 = 3{,}767 - 0{,}26 \ln p'_F$$

$$p'_F = e^{\frac{3{,}767 - 2{,}369}{0{,}26}} = 216{,}58 \text{ kPa}$$

$$q_F = M p'_F = 1{,}02 \cdot 216{,}58 = 220{,}9 \text{ kPa}$$

excess pore water pressure

total pressure at failure if initial pore water pressure is zero:

$$p_F = 200 + \frac{220{,}9}{3} = 273.6 \text{ kPa}$$

$$\Delta u = p_F - p'_F = 273.6 - 216{,}58 = 57{,}03 \text{ kPa}$$

(ii) cam clay yield surface

$$\frac{q}{p'} = M \ln \frac{p'_c}{p'}$$

$$q_Y = p'_Y M \ln p'_c / p'_Y$$

$$q_Y = 200 \times 1{,}02 \times \ln(600/200) = 224{,}2 \text{ kPa}$$