Version GV/3

EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 26 April 2023 9.30 to 11.10 am

Module 3D2

GEOTECHNICAL ENGINEERING II

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3D1 & 3D2 Geotechnical Engineering Databook (20 pages)

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) A rigid smooth plate of 3.0 m height interacts on its right side with a sand mass with friction angle $\varphi' = 30°$ and unit weight $\gamma = 20$ kN m^{-3} (Fig. 1). Force F equilibrates the resultant of the soil pressure acting on the plate and d is the distance from the ground surface to its application point. Calculate the range of possible values for F and d for the following two situations:

(i) no surcharge load applied at the ground surface; [10%]

(ii) uniform surcharge of 15 kPa applied at the surface. [10%]

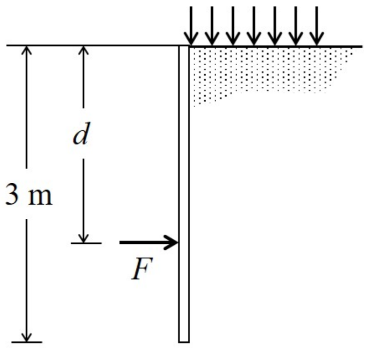

Fig. 1

(b) Figure 2 shows two sand masses in direct contact with two vertical, rigid, smooth plates conneceted by hinged struts spaced evenly 1.0 m apart, and provides some physical and mechanical characteristics of the soils, both of which are submerged. Assume that there is no friction at the base of the plates. It is possible to introduce small length variations in the struts by means of a mechanical system and also to measure the strut force, F. This has been measured as $F = 59$ kN, and it has been found that this value remains constant for (positive or negative) variations of strut length.

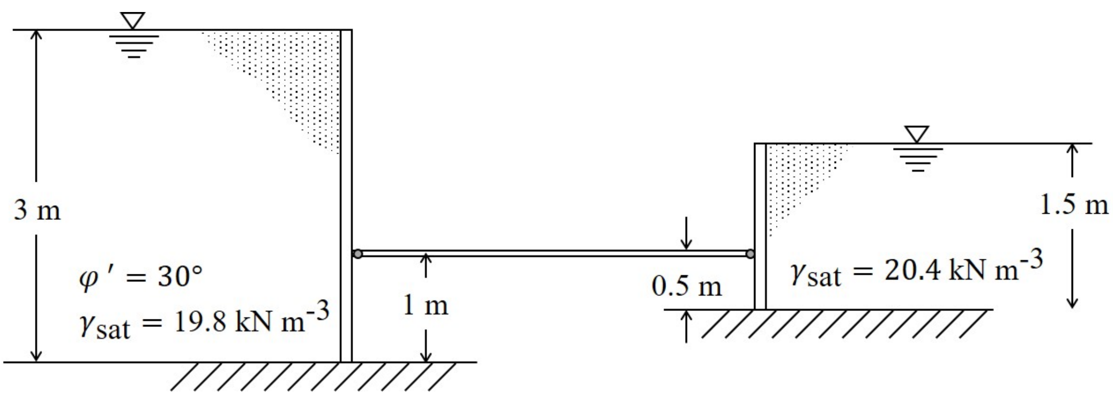

Fig. 2

(i) Determine the friction angle of the soil on the right side of Fig. 2. In what stress state is the soil on the left and right side of Fig. 2? Justify your answer. [20%]

(ii) Draw the Mohr circles representing the total and effective stress state at a point in the soil adjacent to each plate at 1.0 m depth. [20%]

(c) A 4 m deep excavation in sand is supported by a cantilevered sheet pile wall, as shown in Fig. 3. The water table is at dredge level and a uniform surcharge load of 10 kPa is applied at ground surface on the retained side. The unit weight of the sand in dry and saturated conditions are 15.3 $\mathrm{kN\,m^{-3}}$ and 19.7 $\mathrm{kN\,m^{-3}}$, respectively. The friction angle of the sand is $\varphi' = 33°$. It is assumed that the wall moves sufficiently to mobilise full active pressure on the retained side, while the mobilised lateral earth pressure coefficient K_P^* on the excavation side is considered to be a constant value.

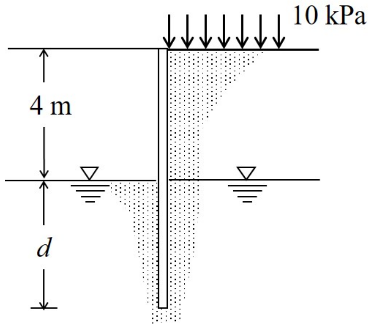

Fig. 3

(i) Assuming that the mobilised friction at the interface between the sheet pile wall and the sand on the passive side is $\delta = \varphi'/3$, compute the active and passive lateral earth pressure coefficients using Rankine's and Lancellotta's static solutions, respectively:

$$K_{\mathrm{A}} = \frac{1 - \sin\varphi'}{1 + \sin\varphi'}$$

$$K_{\mathrm{P}} = \frac{\cos\delta}{1 - \sin\varphi'}\left[\cos\delta + \sqrt{(\sin\varphi')^2 - (\sin\delta)^2}\right] e^{2\Theta\tan\varphi'}$$

where:

$$2\Theta = \sin^{-1}\frac{\sin\delta}{\sin\varphi'} + \delta$$

[10%]

(ii) Compute the depth of embedment, d, required to obtain $K_{\mathrm{P}}/K_P^* = 1.7$, and the maximum bending moment in the wall for this embedment depth. [30%]

2 (a) What are the main challenges for excavations and engineered slopes in soils whose specific volume lies below the critical state line drawn on a $v - \ln p'$ plot? Discuss the relative dangers incurred in drained and undrained conditions. [15%]

(b) Figure 4 shows a slope in quartz fine sand forming a layer about 6 m deep on top of a parallel sloping bedrock with an inclination of 30°. The fine sand has a critical state friction angle of 36° and a void ratio of 0.78. The maximum and minimum void ratios for the fine sand are 0.95 and 0.6, respectively. The specific gravity of the fine sand is 2.7. The slope has shown signs of instability over periods of heavy rainfall. Groundwater monitoring over a 12 month period showed that the phreatic surface in the sand ran parallel to the bedrock and at an elevation of 4 m and 2 m above the bedrock in the wet season and in the dry season, respectively.

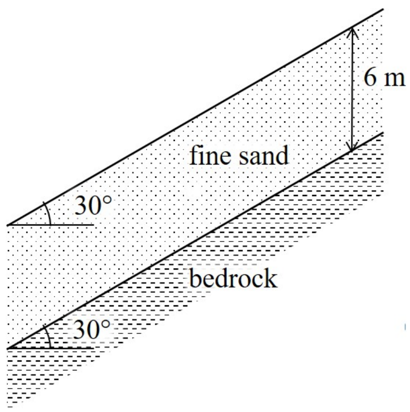

Fig. 4

(i) Calculate the dry and saturated unit weights of the fine sand layer. [10%]

(ii) Use infinite slope analysis, and other carefully justified assumptions, to determine whether the slope would fail in the dry season or the following wet season. Assume that the corresponding groundwater conditions from the previous monitoring exercise are exactly duplicated. [30%]

(iii) What is the maximum elevation that the phreatic surface can reach above the bedrock before the slope will definitely fail? [30%]

(iv) Given that a highway passes at the toe of the slope, stabilisation works are being considered. One of the proposed solution consists in reducing the thickness of the sand layer to 4 m, with the assumption that the maximum elevation of the water table will coincide with the soil surface after excavation. Is this a good idea? Justify your answer. [15%]

3 A cylindrical oil storage tank with a diameter of 10 m and a height of 15 m is to be founded on the surface of the subsoil shown in Fig. 5, with the hydrostatic water table at a depth of 2 m, as indicated. The dry and saturated unit weight of the sand are 18 kN m^{-3} and 21 kN m^{-3} respectively. The clay has a saturated unit weight of 18 kN m^{-3}, and a critical state friction angle of 25°. At a depth of 5 m, the clay is overconsolidated with a coefficient of earth pressure at rest $K_0 = 1$ and an undrained shear strength $s_u = 25$ kPa. Before filling the tank with oil, it is to be proof-tested by filling it with water. A piezometer is installed at point A beneath the centre of the tank to measure the pore pressure. From elastic theory, the total stress changes at point A due to a circular uniform surface stress σ_s applied rapidly (under undrained conditions in the clay) are $\Delta\sigma_v = 0.70\sigma_s$ and $\Delta\sigma_h = 0.23\sigma_s$ where $\Delta\sigma_v$ and $\Delta\sigma_h$ are the increases in total vertical and horizontal stress respectively.

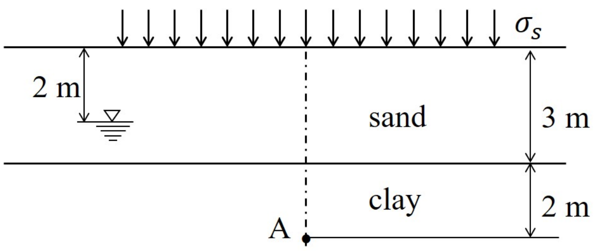

Fig. 5

(a) Calculate the initial total and effective vertical stress in the ground at point A before the proof test begins, ignoring the weight of the tank. Plot the effective and total stress paths in $q - p'$ and $q - p$ space as the tank is rapidly filled with water. [35%]

(b) At point A the clay just yields when the height of the water in the tank is $H_1 = 8.5$ m. Calculate the corresponding pore pressure measured at A. [15%]

(c) The height of the water is increased to H_2 when the clay at point A first attains its undrained strength. Find H_2 and the corresponding pore pressure measured at A. For simplicity, assume that the elastic theory given above for the total stress changes remains applicable after yield occurs. Sketch the total and effective stress paths. [30%]

(d) Comment on what will happen if the water height is maintained at H_2 for a long period, showing the likely effective stress path assuming that the total stresses remain unchanged. What is the relevance of this to the eventual filling of the tank with oil of specific gravity 0.8? [20%]

4 Triaxial compression tests are performed on reconstituted samples of Kaolin clay. Use the parameters given in the Data Book for Kaolin and the Cam Clay model to answer the following questions.

(a) One sample is consolidated isotropically to 200 kPa and then sheared to failure in drained conditions. For the following two tests, calculate the drained strength of the sample and the volumetric strain at failure, and compute the final size of the Cam Clay yield surface p'_c at failure. Plot the corresponding stress paths in $q - p'$ space and $v - p'$ space.

(i) Drained triaxial compression test, in which the axial stress is increased and the radial stress is kept constant [25%]

(ii) Drained triaxial extension test, in which the radial stress is increased and the axial stress is kept constant [25%]

(b) Another sample is consolidated isotropically to 600 kPa, and then unloaded drained to 200 kPa. It is then subjected to undrained triaxial compression, in which the axial stress is increased and the radial stress is kept constant.

(i) Estimate the undrained shear strength and the excess pore water pressure at failure [20%]

(ii) Calculate the yield stress at which the soil starts to exhibit plastic behaviour [15%]

(iii) To what isotropic pressure does the clay need to be normally consolidated in order to give the same undrained shear strength computed in part (b) (i)? [15%]

END OF PAPER

Engineering Tripos Part IIA

3D1 & 3D2
Geotechnical Engineering

Data Book 2019-2020

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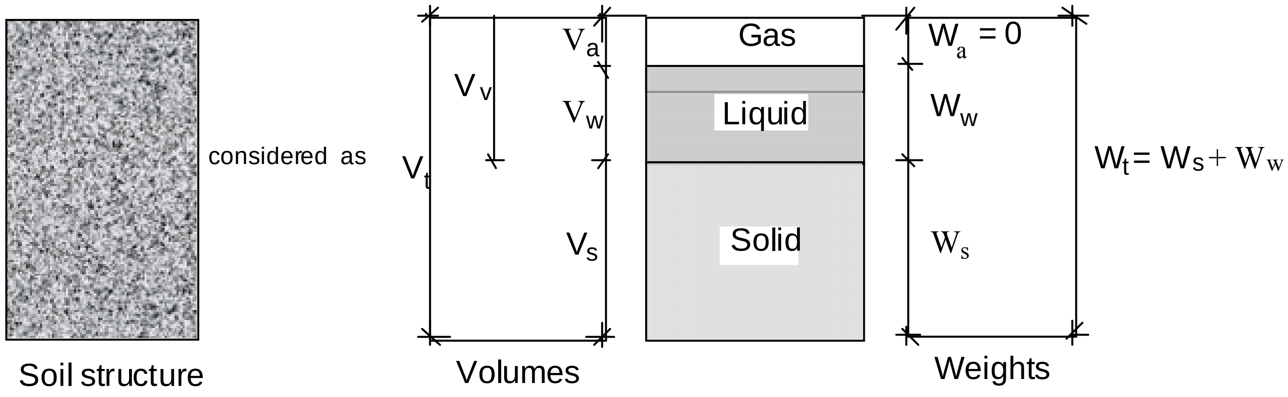
General definitions



Specific gravity of solid	G_s
Voids ratio	$e = V_v / V_s$
Specific volume	$v = V_t / V_s = 1 + e$
Porosity	$n = V_v / V_t = e/(1 + e)$
Water content	$w = (W_w / W_s)$
Degree of saturation	$S_r = V_w / V_v = (w\,G_s/e)$
Unit weight of water	$\gamma_w = 9.81\ \text{kN/m}^3$
Unit weight of soil	$\gamma = W_t / V_t = \left(\dfrac{G_s + S_r e}{1 + e}\right) \gamma_w$
Buoyant saturated unit weight	$\gamma' = \gamma - \gamma_w = \left(\dfrac{G_s - 1}{1 + e}\right) \gamma_w$
Unit weight of dry solids	$\gamma_d = W_s / V_t = \left(\dfrac{G_s}{1 + e}\right) \gamma_w$
Air volume ratio	$A = V_a/V_t = \left(\dfrac{e\,(1 - S_r)}{1+e}\right)$

Soil classification (BS1377)

Liquid limit w_L

Plastic Limit w_P

Plasticity Index $I_P = w_L - w_P$

Liquidity Index $I_L = \dfrac{w - w_P}{w_L - w_P}$

Activity $= \dfrac{\text{Plasticity Index}}{\text{Percentage of particles finer than 2 μm}}$

Sensitivity $= \dfrac{\text{Unconfined compressive strength of an undisturbed specimen}}{\text{Unconfined compressive strength of a remoulded specimen}}$ (at the same water content)

Classification of particle sizes:–

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

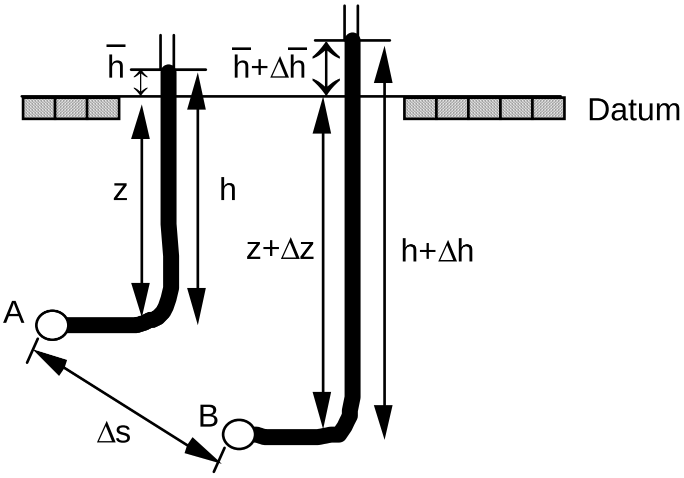
D equivalent diameter of soil particle

D_{10}, D_{60} etc. particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

C_U uniformity coefficient D_{60} / D_{10}

Seepage

Flow potential:
(piezometric level)



Total gauge pore water pressure at A: $u = \gamma_w h = \gamma_w(\bar{h} + z)$

B: $u + \Delta u = \gamma_w(h + \Delta h) = \gamma_w(\bar{h} + z + \Delta\bar{h} + \Delta z)$

Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w(\bar{h} + \Delta\bar{h})$

Hydraulic gradient A → B $\quad i = -\dfrac{\Delta\bar{h}}{\Delta s}$

Hydraulic gradient (3D) $\quad i = -\nabla\bar{h}$

Darcy's law $\quad V = ki$

V = superficial seepage velocity

k = coefficient of permeability

Typical permeabilities:

$D_{10} > 10$ mm	:	non-laminar flow
10 mm $> D_{10} >$ 1μm	:	$k \cong 0.01\,(D_{10}$ in mm$)^2$ m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

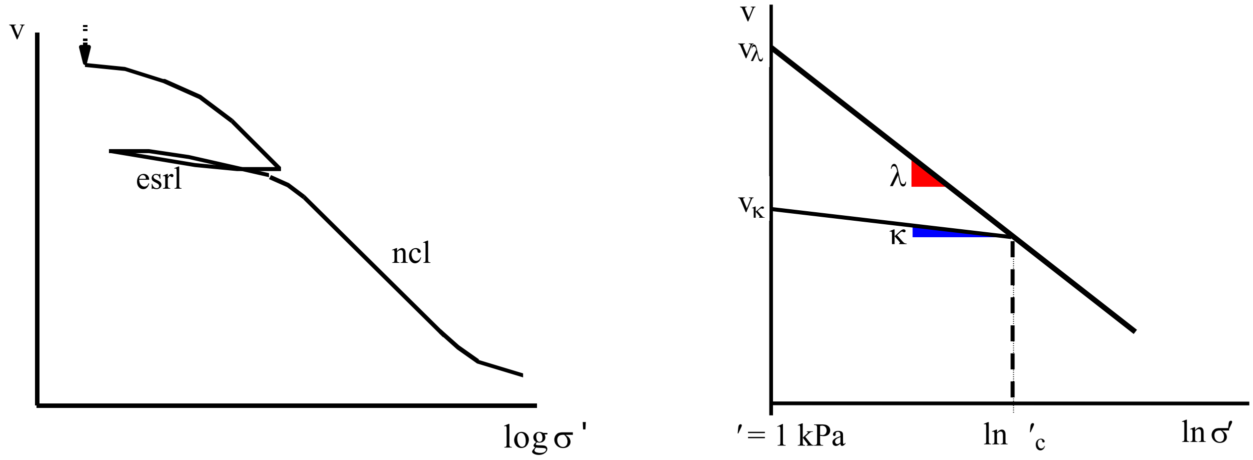
Saturated capillary zone

$h_c = \dfrac{4T}{\gamma_w d}$: capillary rise in tube diameter d, for surface tension T

$h_c \approx \dfrac{3\times10^{-5}}{D_{10}}$ m : for water at 10°C; note air entry suction is $u_c = -\gamma_w h_c$

One-Dimensional Compression

• Fitting data



Plastic compression stress σ'_c is taken as the larger of the initial aggregate crushing stress and the historic maximum effective vertical stress. Clay muds are taken to begin with $\sigma'_c \approx 1$ kPa.

Plastic compression (normal compression line, ncl): $v = v_\lambda - \lambda \ln \sigma'$ for $\sigma' = \sigma'_c$

Elastic swelling and recompression line (esrl): $v = v_c + \kappa (\ln \sigma'_c - \ln \sigma'_v)$

$= v_\kappa - \kappa \ln \sigma'_v$ for $\sigma' < \sigma'_c$

Equivalent parameters for $\log_{10}$ stress scale:

Terzaghi's compression index $C_c = \lambda \log_{10} e$

Terzaghi's swelling index $C_s = \kappa \log_{10} e$

• Deriving confined soil stiffnesses

Secant 1D compression modulus $E_o = (\Delta\sigma' / \Delta\varepsilon)_o$

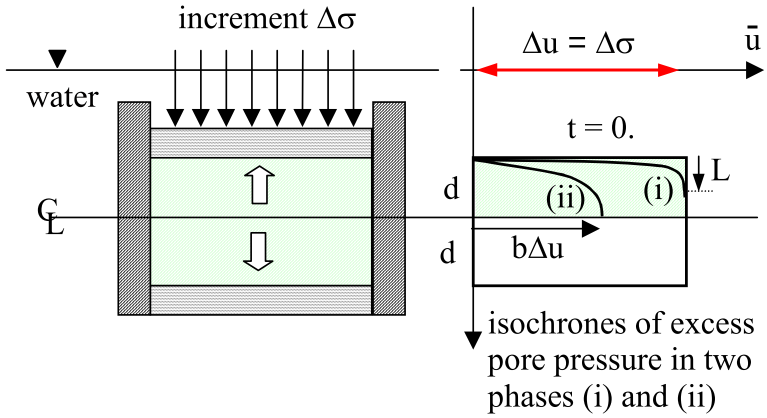
Tangent 1D plastic compression modulus $E_o = v\,\sigma' / \lambda$

Tangent 1D elastic compression modulus $E_o = v\,\sigma' / \kappa$

One-Dimensional Consolidation

Settlement	ρ	$= \int m_v (\Delta u - \bar{u})\, dz$	$= \int (\Delta u - \bar{u}) / E_o\, dz$
Coefficient of consolidation	c_v	$= \dfrac{k}{m_v \gamma_w}$	$= \dfrac{kE_o}{\gamma_w}$
Dimensionless time factor	T_v	$= \dfrac{c_v t}{d^2}$	
Relative settlement	R_v	$= \dfrac{\rho}{\rho_{ult}}$	

• Solutions for initially rectangular distribution of excess pore pressure



Approximate solution by parabolic isochrones:

Phase (i) $\quad L^2 = 12\, c_v t$

$$R_v = \sqrt{\frac{4T_v}{3}} \qquad \text{for } T_v < {}^1/_{12}$$

Phase (ii) $\quad b = \exp(\tfrac{1}{4} - 3T_v)$

$$R_v = [1 - {}^2/_3 \exp(\tfrac{1}{4} - 3T_v)] \qquad \text{for } T_v > {}^1/_{12}$$

Solution by Fourier Series:

T_v	0	0.01	0.02	0.04	0.08	0.15	0.20	0.30	0.40	0.50	0.60	0.80	1.00
R_v	0	0.12	0.17	0.23	0.32	0.45	0.51	0.62	0.70	0.77	0.82	0.89	0.94

Stress and strain components

• **Principle of effective stress (saturated soil)**

total stress σ = effective stress σ' + pore water pressure u

• **Principal components of stress and strain**

sign convention	compression positive
total stress	$\sigma_1, \sigma_2, \sigma_3$
effective stress	$\sigma_1', \sigma_2', \sigma_3'$
strain	$\varepsilon_1, \varepsilon_2, \varepsilon_3$

• **Simple Shear Apparatus (SSA)** ($\varepsilon_2 = 0$; other principal directions unknown)

The only stresses that are readily available are the shear stress τ and normal stress σ applied to the top platen. The pore pressure u can be controlled and measured, so the normal effective stress σ' can be found. Drainage can be permitted or prevented. The shear strain γ and normal strain ε are measured with respect to the top platen, which is a plane of zero extension. Zero extension planes are often identified with slip surfaces.

work increment per unit volume $\delta W = \tau\,\delta\gamma + \sigma'\delta\varepsilon$

• **Biaxial Apparatus - Plane Strain (BA-PS)** ($\varepsilon_2 = 0$; rectangular edges along principal axes)

Intermediate principal effective stress σ_2', in zero strain direction, is frequently unknown so that all conditions are related to components in the 1-3 plane.

mean total stress	$s = (\sigma_1 + \sigma_3)/2$
mean effective stress	$s' = (\sigma_1' + \sigma_3')/2 = s - u$
shear stress	$t = (\sigma_1' - \sigma_3')/2 = (\sigma_1 - \sigma_3)/2$
volumetric strain	$\varepsilon_v = \varepsilon_1 + \varepsilon_3$
shear strain	$\varepsilon_\gamma = \varepsilon_1 - \varepsilon_3$
work increment per unit volume	$\delta W = \sigma_1'\delta\varepsilon_1 + \sigma_3'\delta\varepsilon_3$
	$\delta W = s'\delta\varepsilon_v + t\delta\varepsilon_\gamma$

providing that principal axes of strain increment and of stress coincide.

• **Triaxial Apparatus – Axial Symmetry (TA-AS)** (cylindrical element with radial symmetry)

total axial stress	σ_a	=	$\sigma'_a + u$
total radial stress	σ_r	=	$\sigma'_r + u$
total mean normal stress	p	=	$(\sigma_a + 2\sigma_r)/3$
effective mean normal stress	p'	=	$(\sigma'_a + 2\sigma'_r)/3 = p - u$
deviatoric stress	q	=	$\sigma'_a - \sigma'_r = \sigma_a - \sigma_r$
stress ratio	η	=	q/p'
axial strain	ε_a		
radial strain	ε_r		
volumetric strain	ε_v	=	$\varepsilon_a + 2\varepsilon_r$
triaxial shear strain	ε_s	=	$\frac{2}{3}(\varepsilon_a - \varepsilon_r)$
work increment per unit volume	δW	=	$\sigma_a'\delta\varepsilon_a + 2\sigma_r'\delta\varepsilon_r$
	δW	=	$p'\delta\varepsilon_v + q\delta\varepsilon_s$

Types of triaxial test include:

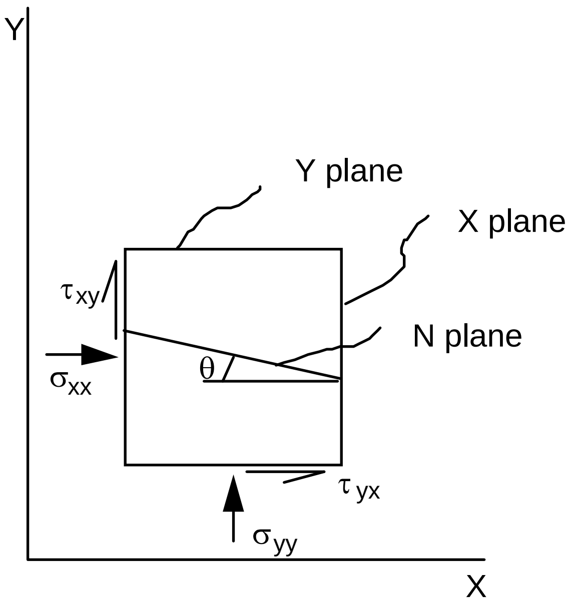
isotropic compression in which p' increases at zero q

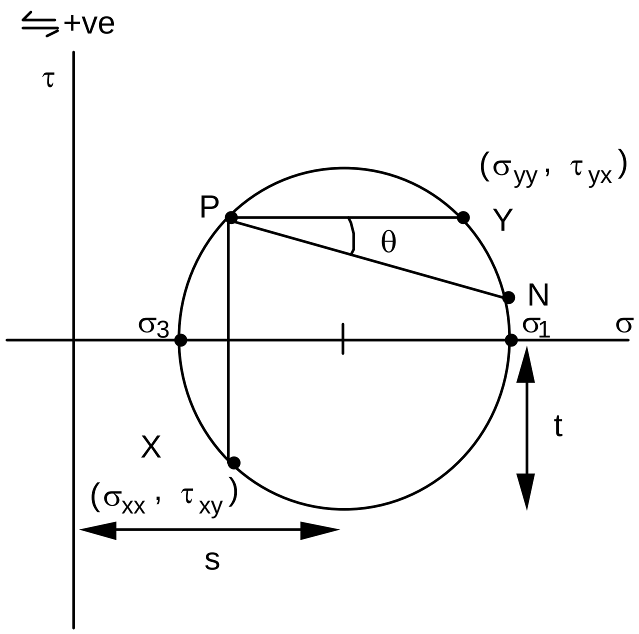
triaxial compression in which q increases *either* by increasing σ_a *or* by reducing σ_r

triaxial extension in which q reduces *either* by reducing σ_a *or* by increasing σ_r

• **Mohr's circle of stress (1–3 plane)**

Sign of convention: compression, and counter-clockwise shear, positive





Poles of planes P : the components of stress on the N plane are given by the intersection N of the Mohr circle with the line PN through P parallel to the plane.

Elastic stiffness relations

These relations apply to tangent stiffnesses of over-consolidated soil, with a state point on some swelling and recompression line (κ-line), and remote from gross plastic yielding.

One-dimensional compression (axial stress and strain increments $d\sigma'$, $d\varepsilon$)

compressibility $m_v = d\varepsilon / d\sigma'$

constrained modulus $E_o = 1 / m_v$

Physically fundamental parameters

shear modulus $G' = dt / d\varepsilon_\gamma$

bulk modulus $K' = dp' / d\varepsilon_v$

Parameters which can be used for constant-volume deformations

undrained shear modulus $G_u = G'$

undrained bulk modulus $K_u = \infty$ (neglecting compressibility of water)

Alternative convenient parameters

Young's moduli E' (effective), E_u (undrained)

Poisson's ratios ν' (effective), $\nu_u = 0.5$ (undrained)

Typical value of Poisson’s ratio for small changes of stress: $\nu' = 0.2$

Relationships:

$$G = \frac{E}{2\,(1+\nu)}$$

$$K = \frac{E}{3\,(1-2\nu)}$$

$$E_o = \frac{E\,(1-\nu)}{(1+\nu)\,(1-2\nu)}$$

Cam Clay

• Interchangeable parameters for stress combinations at yield, and plastic strain increments

System	Effective normal stress	Plastic normal strain	Effective shear stress	Plastic shear strain	Critical stress ratio	Plastic normal stress	Critical normal stress
General	σ^*	ε^*	τ^*	γ^*	μ^*_{crit}	σ^*_c	σ^*_{crit}
SSA	σ'	ε	τ	γ	$\tan\phi_{crit}$	σ'_c	σ'_{crit}
BA-PS	s'	ε_v	t	ε_γ	$\sin\phi_{crit}$	s'_c	s'_{crit}
TA-AS	p'	ε_v	q	ε_s	M	p'_c	p'_{crit}

• General equations of plastic work

Plastic work and dissipation $\quad \sigma^* \delta\varepsilon^* + \tau^* \delta\gamma^* = \mu^*_{crit} \sigma^* \delta\gamma^*$

Plastic flow rule – normality $\quad \dfrac{d\tau^*}{d\sigma^*} \cdot \dfrac{d\gamma^*}{d\varepsilon^*} = -1$

• General yield surface

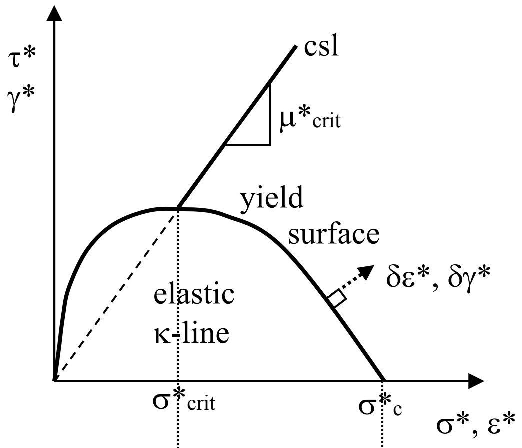
$$\frac{\tau^*}{\sigma^*} = \mu^* = \mu^*_{crit} \,.\, \ln\left[\frac{\sigma_c^*}{\sigma^*}\right]$$

• Parameter values which fit soil data

	London Clay	**Weald Clay**	**Kaolin**	**Dog's Bay Sand**	**Ham River Sand**
λ^*	0.161	0.093	0.26	0.334	0.163
κ^*	0.062	0.035	0.05	0.009	0.015
Γ^* at 1 kPa	2.759	2.060	3.767	4.360	3.026
$\sigma^*_{c,\ virgin}$ kPa	1	1	1	Loose 500	Loose 2500
				Dense 1500	Dense 15000
ϕ_{crit}	23°	24°	26°	39°	32°
M_{comp}	0.89	0.95	1.02	1.60	1.29
M_{extn}	0.69	0.72	0.76	1.04	0.90
w_L	0.78	0.43	0.74	--------------	--------------
w_P	0.26	0.18	0.42	--------------	--------------
G_s	2.75	2.75	2.61	2.75	2.65

Note: 1) parameters λ^*, κ^*, Γ^*, σ^*_c should depend to a small extent on the deformation mode, e.g. SSA, BA-PS, TA-AS, etc. This may be neglected unless further information is given.
2) Sand which is loose, or loaded cyclically, compacts more than Cam Clay allows.

• The yield surface in (σ*, τ*, v) space



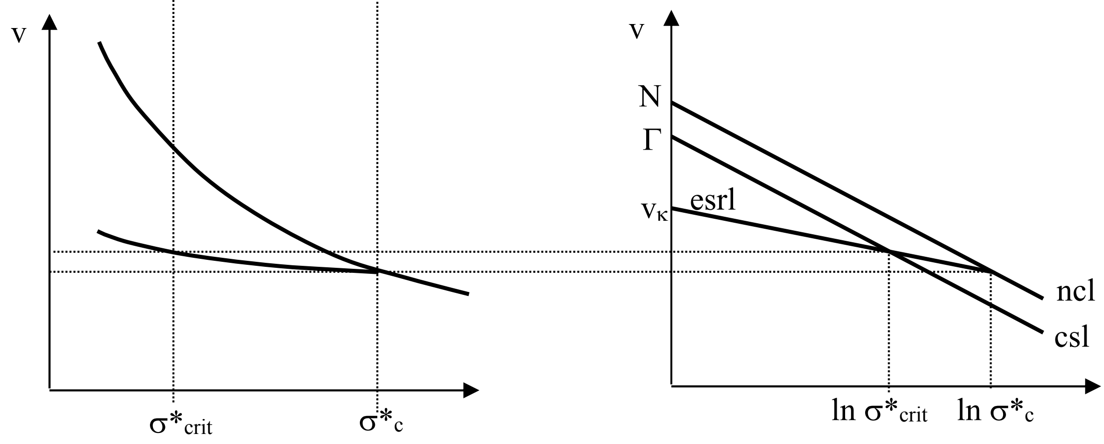
ncl: normal compression line

$$v = N - \lambda \ln \sigma^*$$

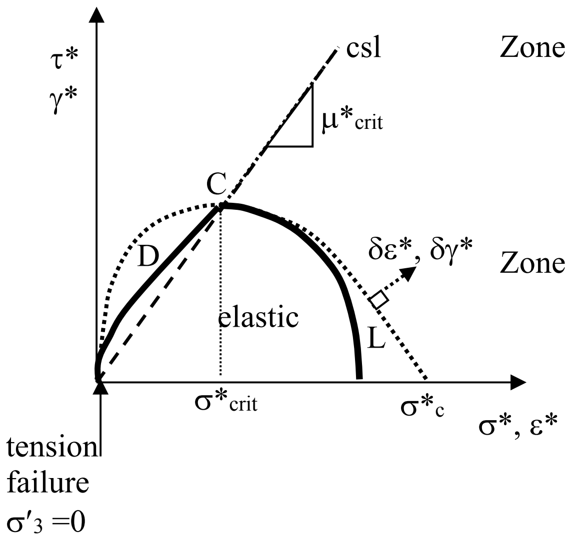
csl: critical state line

$$v = \Gamma - \lambda \ln \sigma^*$$

where $N = \Gamma + \lambda - \kappa$



• Regions of limiting soil behaviour



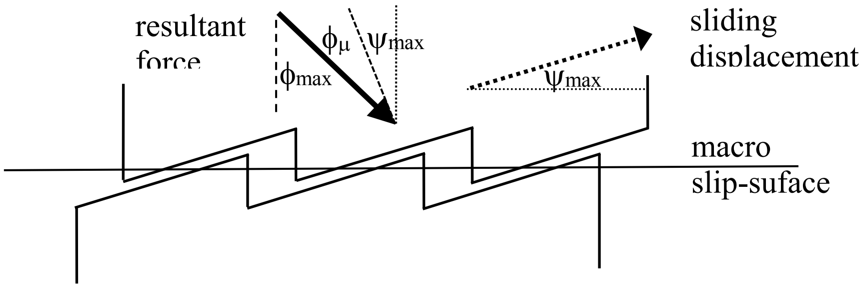
Variation of Cam Clay yield surface

Zone D: denser than critical, “dry”,
dilation or negative excess pore pressures,
Hvorslev strength envelope,
friction-dilatancy theory,
unstable shear rupture, progressive failure

Zone L: looser than critical, “wet”,
compaction or positive excess pore pressures,
Modified Cam Clay yield surface,
stable strain-hardening continuum

Strength of soil: friction and dilation

• Friction and dilatancy: the saw-blade model of direct shear

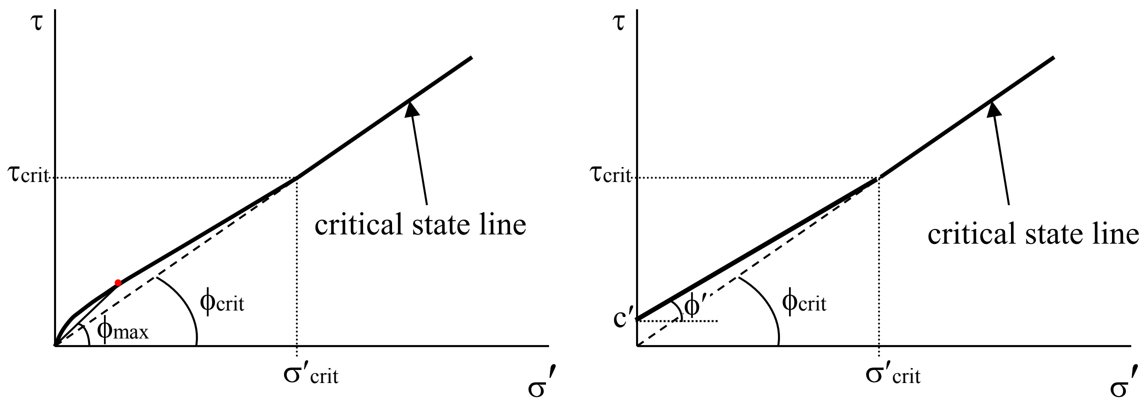


Intergranular angle of friction at sliding contacts ϕ_μ

Angle of dilation ψ_{max}

Angle of internal friction $\phi_{max} = \phi_\mu + \psi_{max}$

• Friction and dilatancy: secant and tangent strength parameters



Secant angle of internal friction

$\tau = \sigma' \tan \phi_{max}$
$\phi_{max} = \phi_{crit} + \Delta\phi$
$\Delta\phi = f(\sigma'_{crit}/\sigma')$

typical envelope fitting data:
power curve
$(\tau/\tau_{crit}) = (\sigma'/\sigma'_{crit})^\alpha$
with $\alpha \approx 0.85$

Tangent angle of shearing envelope

$\tau = c' + \sigma' \tan \phi'$
$c' = f(\sigma'_{crit})$

typical envelope:
straight line
$\tan \phi' = 0.85 \tan \phi_{crit}$
$c' = 0.15\, \tau_{crit}$

• Friction and dilation: data of sands

The inter-granular friction angle of quartz grains, $\phi_\mu \approx 26°$. Turbulent shearing at a critical state causes ϕ_{crit} to exceed this. The critical state angle of internal friction ϕ_{crit} is a function of the uniformity of particle sizes, their shape, and mineralogy, and is developed at large shear strains irrespective of initial conditions. Typical values of ϕ_{crit} ($\pm 2^o$) are:

well-graded, angular quartz or feldspar sands	40°
uniform sub-angular quartz sand	36°
uniform rounded quartz sand	32°

Relative density $\quad I_D = \dfrac{(e_{max} - e)}{(e_{max} - e_{min})}$ $\quad$ where:

e_{max} is the maximum void ratio achievable in quick-tilt test
e_{min} is the minimum void ratio achievable by vibratory compaction

Relative crushability $\quad I_C = \ln(\sigma_c / p')$ $\quad$ where:

σ_c is the aggregate crushing stress, taken to be a material constant, typical values being: 80 000 kPa for quartz silt, 20 000 kPa for quartz sand, 5 000 kPa for carbonate sand.

p' is the mean effective stress at failure which may be taken as approximately equal to the effective stress σ' normal to a shear plane.

Dilatancy contribution to the peak angle of internal friction is $\Delta\phi = (\phi_{max} - \phi_{crit}) = f(I_R)$

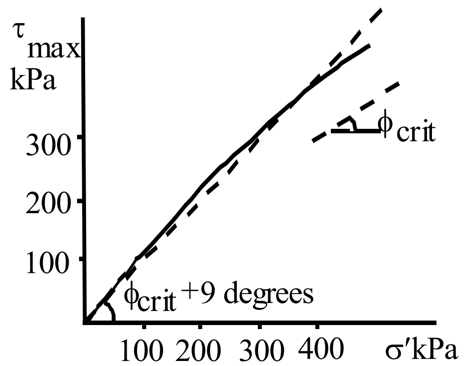
Relative dilatancy index $I_R = I_D I_C - 1$ $\quad$ where:

$I_R < 0$ indicates compaction, so that I_D increases and $I_R \to 0$ ultimately at a critical state
$I_R > 4$ to be limited to $I_R = 4$ unless corroborative dilatant strength data is available

The following empirical correlations are then available

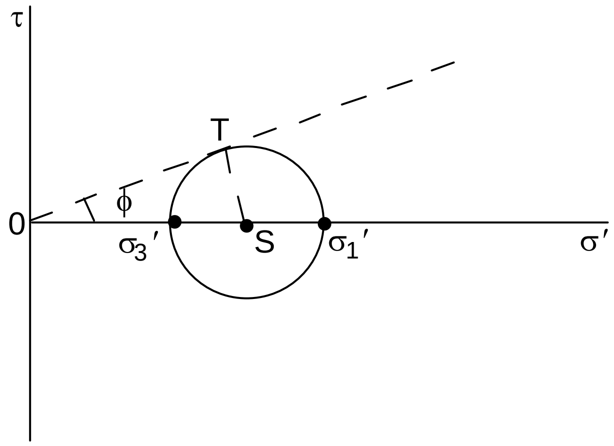
plane strain conditions	$(\phi_{max} - \phi_{crit})$	$= 0.8\ \psi_{max}$	$= 5\ I_R$ degrees
triaxial strain conditions	$(\phi_{max} - \phi_{crit})$	$= 3\ I_R$ degrees	
all conditions	$(-\delta\varepsilon_v / \delta\varepsilon_1)_{max}$	$= 0.3\ I_R$	

The resulting peak strength envelope for triaxial tests on a quartz sand at an initial relative density $I_D = 1$ is shown below for the limited stress range 10 - 400 kPa:



$\phi_{max} > \phi_{crit} + 9°$ for $I_D = 1$, $\sigma' = 400$ kPa

• Mobilised (secant) angle of shearing ϕ in the 1 – 3 plane



$$\sin\phi = TS/OS$$

$$= \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

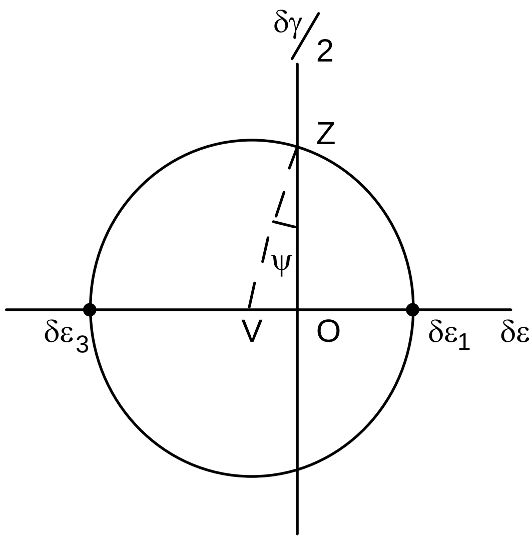
$$\left[\frac{\sigma_1'}{\sigma_3'}\right] = \frac{(1+\sin\phi)}{(1-\sin\phi)}$$

Angle of shearing resistance:

at peak strength ϕ_{max} at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state ϕ_{crit} after large shear strains

• Mobilised angle of dilation in plane strain ψ in the 1 – 3 plane



$$\sin\psi = VO/VZ$$

$$= -\frac{(\delta\varepsilon_1 + \delta\varepsilon_3)/2}{(\delta\varepsilon_1 - \delta\varepsilon_3)/2}$$

$$= -\frac{\delta\varepsilon_v}{\delta\varepsilon_\gamma}$$

$$\left[\frac{\delta\varepsilon_1}{\delta\varepsilon_3}\right] = -\frac{(1-\sin\psi)}{(1+\sin\psi)}$$

at peak strength $\psi = \psi_{max}$ at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state $\psi = 0$ since volume is constant

Plasticity: Cohesive material $\tau_{max} = c_u$ (or s_u)

- **Limiting stresses**

Tresca $|\sigma_1 - \sigma_3| = q_u = 2c_u$

von Mises $(\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2c_u^2$

where q_u is the undrained triaxial compression strength, and c_u is the undrained plane shear strength.

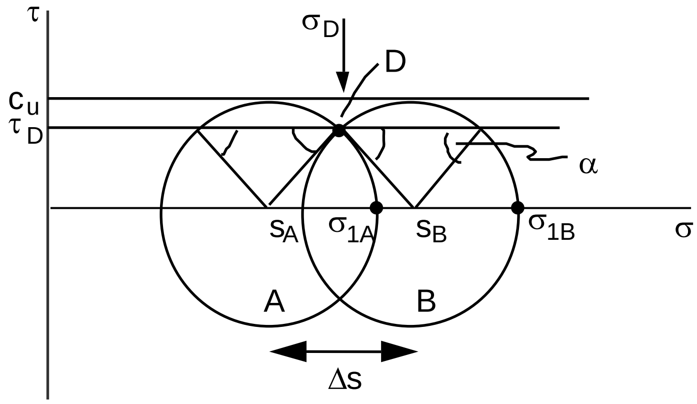
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = c_u \delta\varepsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength c_u, this becomes

$$D = A c_u x$$

- **Stress conditions across a discontinuity**



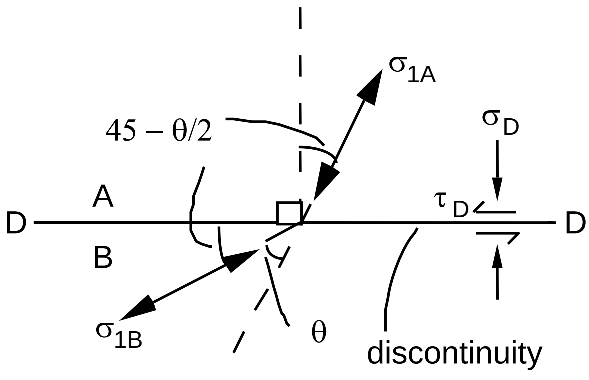
Rotation of major principal stress θ

$s_B - s_A = \Delta s = 2c_u \sin\theta$

$\sigma_{1B} - \sigma_{1A} = 2c_u \sin\theta$

In limit with $\theta \to 0$

$$ds = 2c_u\, d\theta$$



Useful example:

$\theta = 30^\circ$

$\sigma_{1B} - \sigma_{1A} = c_u$

$\tau_D / c_u = 0.87$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

Plasticity: Frictional material $(\tau/\sigma')_{max} = \tan\phi$

• Limiting stresses

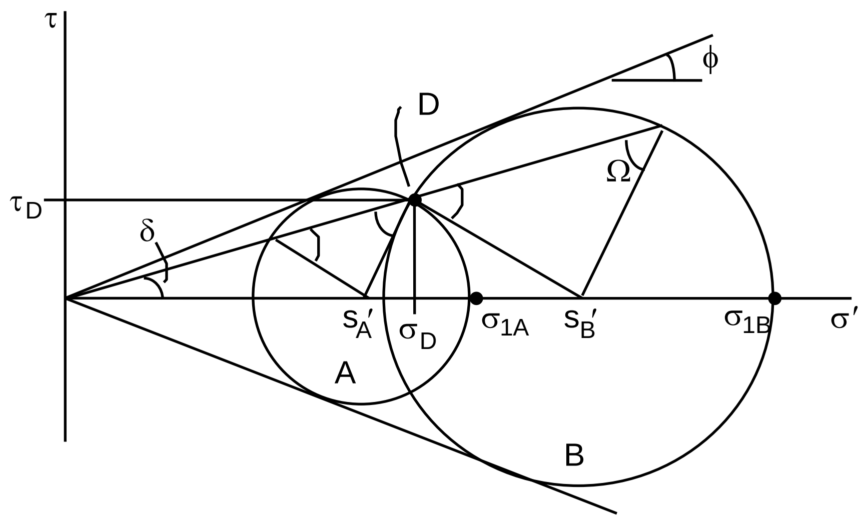
$$\sin\phi = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u_s)$$

where σ'_{1f} and σ'_{3f} are the major and minor principal effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principle total stresses at failure, and u_s is the steady state pore pressure.

Active pressure:
$\sigma'_v > \sigma'_h$
$\sigma'_1 = \sigma'_v$ (assuming principal stresses are horizontal and vertical)
$\sigma'_3 = \sigma'_h$
$K_a = (1 - \sin\phi)/(1 + \sin\phi)$

Passive pressure:
$\sigma'_h > \sigma'_v$
$\sigma'_1 = \sigma'_h$ (assuming principal stresses are horizontal and vertical)
$\sigma'_3 = \sigma'_v$
$K_p = (1 + \sin\phi)/(1 - \sin\phi) = 1/K_a$

• Stress conditions across a discontinuity



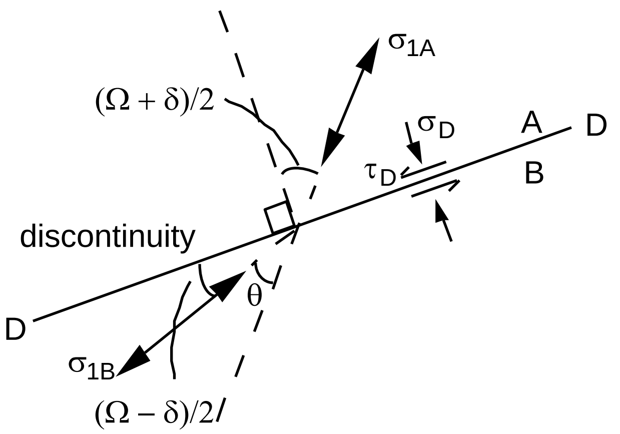
Rotation of major principal stress

$\theta = \pi/2 - \Omega$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

$\tan\delta = \tau_D / \sigma'_D$



$\sin\Omega = \sin\delta / \sin\phi$

$s'_B/s'_A = \sin(\Omega + \delta) / \sin(\Omega - \delta)$

In limit, $d\theta \rightarrow 0$ and $\delta \rightarrow \phi$

$ds' = 2s'.\, d\theta \tan\phi$

Integration gives $s'_B/s'_A = \exp(2\theta\tan\phi)$

Empirical earth pressure coefficients following one-dimensional strain

Coefficient of earth pressure in 1D plastic compression (normal compression)

$K_{o,nc} = 1 - \sin\phi_{crit}$

Coefficient of earth pressure during a 1D unloading-reloading cycle (overconsolidated soil)

$$K_o = K_{o,nc}\left[1 + \frac{(n-1)(n_{max}^{\alpha} - 1)}{(n_{max} - 1)}\right]$$

where n is current overconsolidation ratio (OCR) defined as $\sigma'_{v,max} / \sigma'_{v}$

n_{max} is maximum historic OCR defined as $\sigma'_{v,max} / \sigma'_{v,min}$

α is to be taken as $1.2 \sin\phi_{crit}$

Cylindrical cavity expansion

Expansion $\delta A = A - A_o$ caused by increase of pressure $\delta\sigma_c = \sigma_c - \sigma_o$

At radius r: small displacement $\rho = \dfrac{\delta A}{2\pi r}$

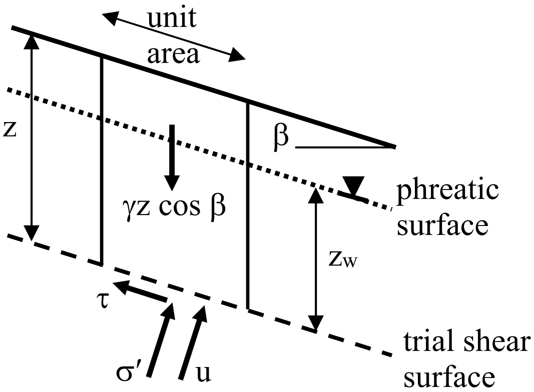
small shear strain $\gamma = \dfrac{2\rho}{r}$

Radial equilibrium: $r\dfrac{d\sigma r}{dr} + \sigma_r - \sigma_\theta = 0$

Elastic expansion (small strains) $\delta\sigma_c = G\dfrac{\delta A}{A}$

Undrained plastic-elastic expansion $\delta\sigma_c = c_u\left[1 + \ln\dfrac{G}{c_u} + \ln\dfrac{\delta A}{A}\right]$

Infinite slope analysis



$u = \gamma_w z_w \cos^2\beta$

$\sigma = \gamma z \cos^2\beta$

$\sigma' = (\gamma z - \gamma_w z_w)\cos^2\beta$

$\tau = \gamma z \cos\beta \sin\beta$

$$\tan\phi_{mob} = \frac{\tau}{\sigma'} = \frac{\tan\beta}{\left(1 - \dfrac{\gamma_w z_w}{\gamma z}\right)}$$

Shallow foundation design

Tresca soil, with undrained strength s_u

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

V_{ult} and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and γ (or γ') is the appropriate density of the overburden.

The exact bearing capacity factor N_c for a plane strain surface foundation (zero embedment) on uniform soil is:

$$N_c = 2 + \pi \qquad \text{(Prandtl, 1921)}$$

Shape correction factor:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_c = 1 + 0.2\, B / L$$

The exact solution for a rough circular foundation (D = B = L) is $q_f = 6.05 s_u$, hence $s_c = 1.18 \sim 1.2$.

Embedment correction factor:

A fit to Skempton's (1951) embedment correction factors, for an embedment of h, is:

$$d_c = 1 + 0.33 \tan^{-1}(h/B) \qquad \text{(or h/D for a circular foundation)}$$

Combined V-H loading

A curve fit to Green's lower bound plasticity solution for V-H loading is:

If $V/V_{ult} > 0.5$: $$\frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{H}{H_{ult}}} \quad \text{or} \quad \frac{H}{H_{ult}} = 1 - \left(2\frac{V}{V_{ult}} - 1\right)^2$$

If $V/V_{ult} < 0.5$: $H = H_{ult} = B s_u$

Combined V-H-M loading

With lift-off: combined Green-Meyerhof

Without lift-off: $$\left(\frac{V}{V_{ult}}\right)^2 + \left[\frac{M}{M_{ult}}\left(1 - 0.3\frac{H}{H_{ult}}\right)\right]^2 + \left|\left(\frac{H}{H_{ult}}\right)^3\right| - 1 = 0 \quad \text{(Taiebet \& Carter 2000)}$$

Frictional (Coulomb) soil, with friction angle ϕ

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation under drained loading (Coulomb soil) is:

$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + s_\gamma N_\gamma \frac{\gamma' B}{2}$$

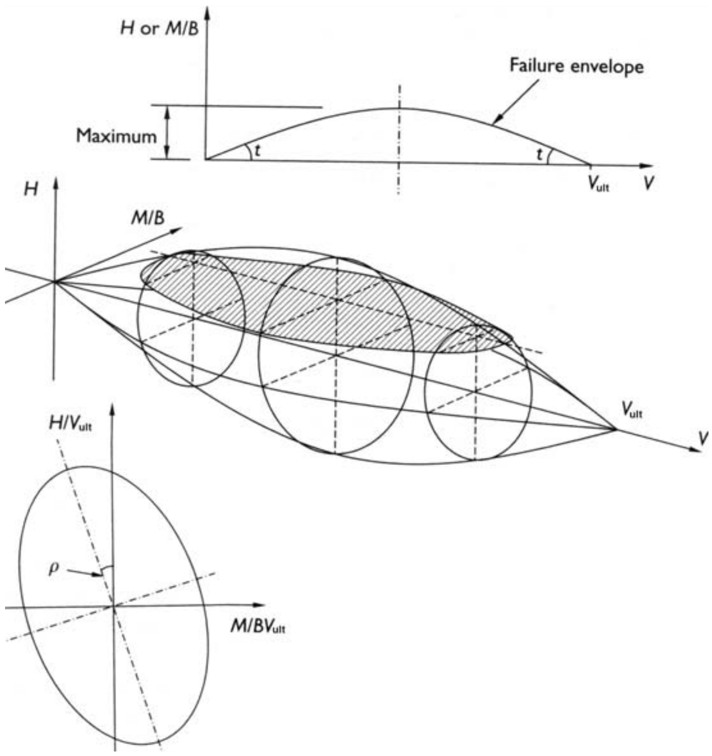
The bearing capacity factors N_q and N_γ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. σ'_{v0} is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for N_q is:

$N_q = \tan^2(\pi/4 + \phi/2)\, e^{(\pi \tan \phi)}$ (Prandtl 1921)

An empirical relationship to estimate N_γ from N_q is (Eurocode 7):

$N_\gamma = 2\,(N_q - 1) \tan \phi$



Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

Rough base: $N_\gamma = 0.1054\, e^{9.6\phi}$

Smooth base: $N_\gamma = 0.0663\, e^{9.3\phi}$

Shape correction factors:

For a rectangular footing of length L and breadth B (Eurocode 7):

$s_q = 1 + (B \sin \phi) / L$
$s_\gamma = 1 - 0.3\, B / L$

For circular footings take L = B.

Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.

Combined V-H-M loading

With lift-off- drained conditions - use Butterfield & Gottardi (1994) failure surface shown above

$$\left[\frac{H/V_{ult}}{t_h}\right]^2 + \left[\frac{M/BV_{ult}}{t_m}\right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m}\right] = \left[\frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)\right]^2$$

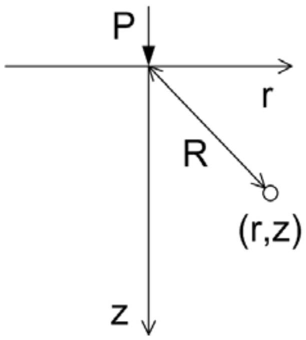
where $C = \tan\left(\frac{2\rho(t_h - t_m)(t_h + t_m)}{2 t_h t_m}\right)$ (Butterfield & Gottardi, 1994)

Typically, t_h~0.5, t_m~0.4 and ρ~15°. Note that t_h is the friction coefficient, H/V= $\tan\phi$, during sliding.

Settlement of Shallow Foundations

Elastic stress distributions below point, strip and circular loads

Point loading (Boussinesq solution)



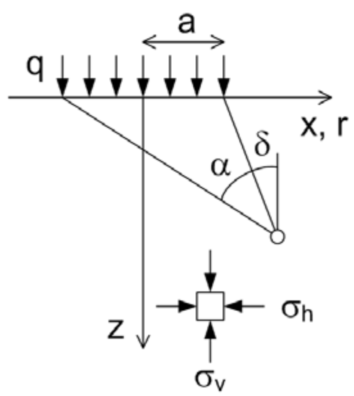
Vertical stress $\sigma_z = \dfrac{3Pz^3}{2\pi R^5}$

Radial stress $\sigma_r = \dfrac{P}{2\pi R^2}\left[\dfrac{3r^2 z}{R^3} - \dfrac{(1-2\nu)R}{R+z}\right]$

Tangential stress $\sigma_\theta = \dfrac{P(1-2\nu)}{2\pi R^2}\left[\dfrac{R}{R+z} - \dfrac{z}{R}\right]$

Shear stress $\tau_{rz} = \dfrac{3Prz^2}{2\pi R^5}$

Uniformly-loaded strip



Vertical stress $\sigma_v = \dfrac{q}{\pi}\left[\alpha + \sin\alpha\cos(\alpha+2\delta)\right]$

Horizontal stress $\sigma_h = \dfrac{q}{\pi}\left[\alpha - \sin\alpha\cos(\alpha+2\delta)\right]$

Shear stress $\tau_{vh} = \dfrac{q}{\pi}\sin\alpha\sin(\alpha+2\delta)$

Principal stresses $\sigma_1 = \dfrac{q}{\pi}(\alpha + \sin\alpha) \quad \sigma_3 = \dfrac{q}{\pi}(\alpha - \sin\alpha)$

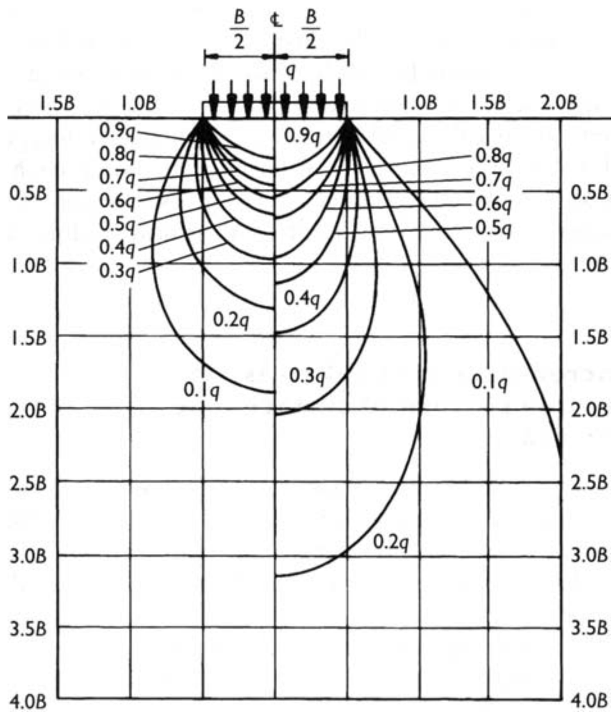
Uniformly-loaded circle radius *a* (on centerline, r=0)

Vertical stress

$$\sigma_v = q\left[1 - \left(\frac{1}{1+(a/z)^2}\right)^{\frac{3}{2}}\right]$$

Horizontal stress

$$\sigma_h = \frac{q}{2}\left[(1+2\nu) - \frac{2(1+\nu)z}{(a^2+z^2)^{1/2}} + \frac{z^3}{(a^2+z^2)^{3/2}}\right]$$



Contours of vertical stress below uniformly-loaded circular (left) and strip footings (right)

Elastic solutions for surface settlement
Isotropic, homogeneous, elastic half-space (semi-infinite)

Point load (Boussinesq solution)

Settlement, w, at distance s: $w(s) = \frac{1}{2\pi}\frac{(1-\nu)}{G}\frac{P}{s}$

Circular area (radius a), uniform soil

Uniform load: central settlement: $w_o = \frac{(1-\nu)}{G}qa$

edge settlement: $w_e = \frac{2}{\pi}\frac{(1-\nu)}{G}qa$

Rigid punch: (q_{avg}= V/πa^2) $w_r = \frac{\pi}{4}\frac{(1-\nu)}{G}q_{avg}a$

Rectangular area, uniform soil

Uniform load, corner settlement:

$$w_c = \frac{(1-\nu)}{G}\frac{qB}{2}I_{rect}$$

Where I_{rect} depends on the aspect ratio, L/B:

L/B	I_{rect}	L/B	I_{rect}	L/B	I_{rect}	L/B	I_{rect}
1	0.561	1.6	0.698	2.4	0.822	5	1.052
1.1	0.588	1.7	0.716	2.5	0.835	6	1.110
1.2	0.613	1.8	0.734	3	0.892	7	1.159
1.3	0.636	1.9	0.750	3.5	0.940	8	1.201
1.4	0.658	2	0.766	4	0.982	9	1.239
1.5	0.679	2.2	0.795	4.5	1.019	10	1.272

Rigid rectangle: $w_r = \frac{(1-\nu)}{G}\frac{q_{avg}\sqrt{BL}}{2}I_{rgd}$ where I_{rgd} varies from 0.9→0.7 for L/B = 1-10.

Note: $G = \frac{E}{2(1+\nu)}$ where ν= Poisson's ratio, E= Young's modulus.