i)

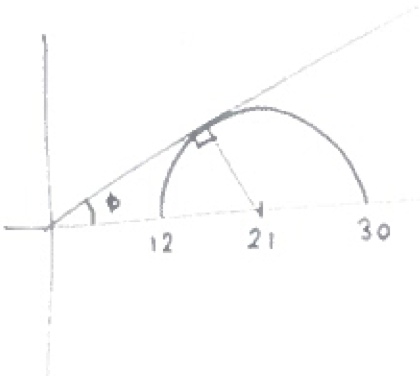
$\sigma_V = 80\ kPa$

$\sigma_V' = 30\ kPa$

$\sigma_H' = 12\ kPa$

$\sigma_H = 62\ kPa$

ii)



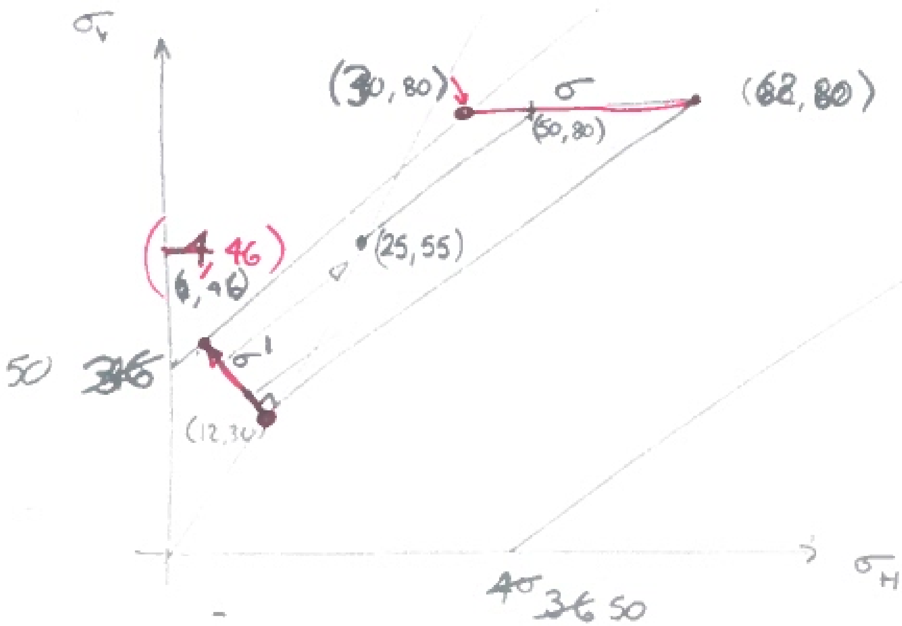
$\frac{9.285}{21.66}$

23.2

$\sin\phi = \frac{9}{21}$

$\phi = 25.4^{\circ}$

iii)

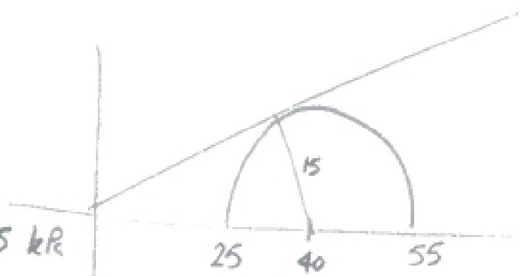


$\sigma_H = \underline{30\ kPa}$

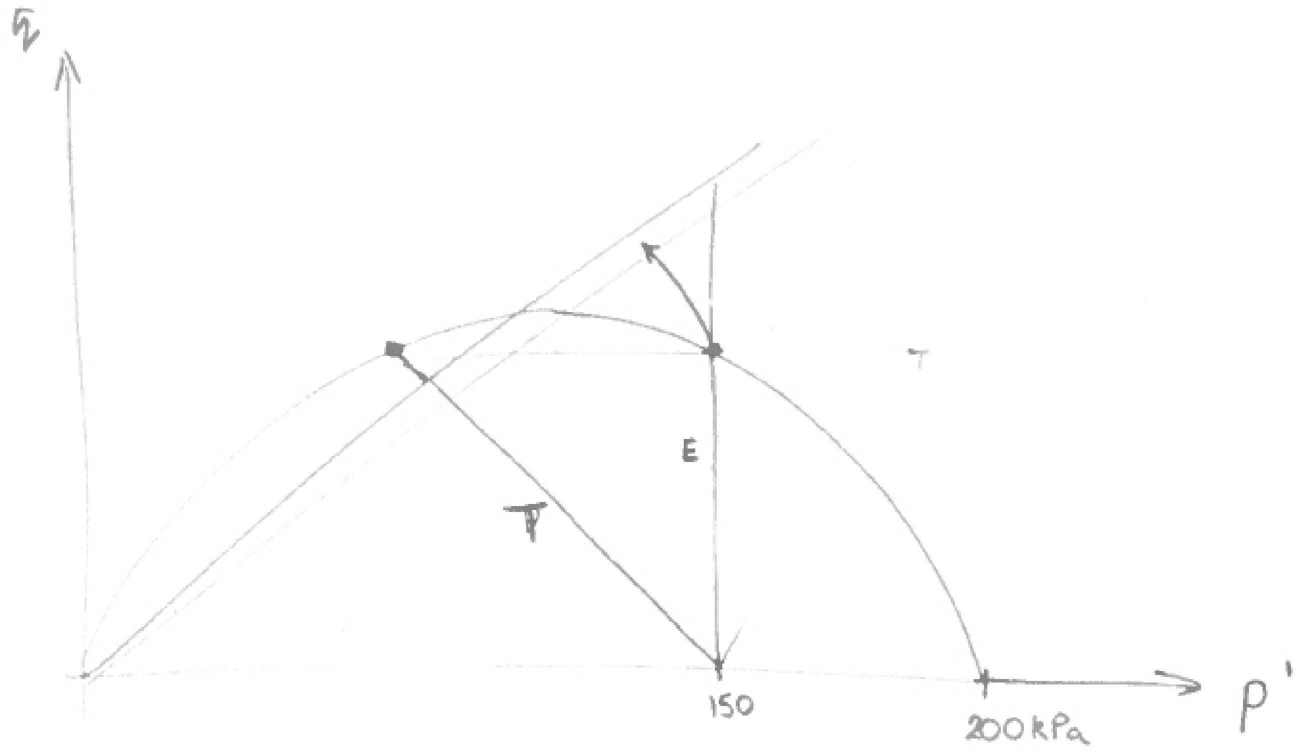
$u = \underline{\underline{34\ kPa}}$

iv) $\sigma_H = 50\ kPa$, $\sigma_V = 80\ kPa$ $u = 25\ kPa$

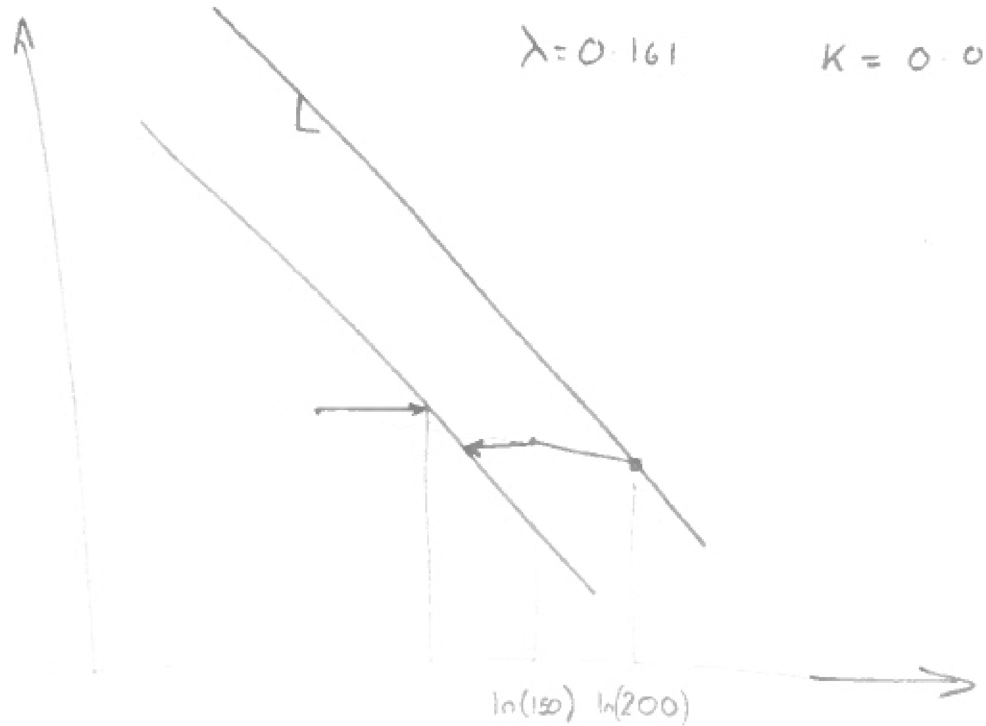
$\sigma_H' = 25\ kPa$ $\sigma_V' = 55\ kPa$



$\sin\phi = 15/40$ $\phi = \underline{\underline{22^{\circ}}}$



$\lambda = 0.161$ $\kappa = 0.062$ $\Gamma = 2.759$



a) $V_i = \overbrace{2.759 - 0.161 \ln(200)}^{0.853}$

$+ 0.161 - 0.062 + \underbrace{0.062 \ln\left(\frac{200}{150}\right)}_{0.018} = 2.023$ ~~1.924~~

b) Undrained $p' = $ const to yield

@ Yield $p' = 150 \quad p'_c = 200$

$$\frac{q}{p'} = M \ln \frac{p'_c}{p'_o} = 0.89 \ln\left(\frac{200}{150}\right)$$

$$q = \underline{\underline{38.4 \text{ kPa}}}$$

~~$\sigma_v = 150 - 38.4$~~ $- 38.4$

~~$\sigma_H = 150 - 38.4$~~

$$p = 150 - 2/3(38.4)$$

$$= \underline{\underline{124.4 \text{ kPa}}}$$

$\sigma_v = 150$

$\sigma_H = 150 - 38.4$

$$u = -\underline{\underline{25.6 \text{ kPa}}}$$

@ Failure Const vol on c.s.

$$2.759 - 0.161 \ln(p') = 2.023 \ \cancel{1.924}$$

$p' = \cancel{178.8 \text{ kPa}}$ $\underline{\underline{96.7 \text{ kPa}}}$

$q = Mp' = \underline{\underline{86 \text{ kPa}}}$

$u = \underline{\underline{-4 \text{ kPa}}}$

$\sigma_v = 150$

$\sigma_H = 150 - 86$

$= 64$

$p = \frac{278}{3} = \underline{\underline{92.7}}$

c) Drained

$$p' = 150 - \tfrac{2}{3}\Delta\sigma$$
$$q = \Delta\sigma$$

@ yield

$$\frac{q}{Mp'} = M \ln\left(\frac{p'_c}{p'}\right)$$

$$\frac{\Delta\sigma}{150 - \tfrac{2}{3}\Delta\sigma} = 0.89 \ln\left(\frac{200}{150 - \tfrac{2}{3}\Delta\sigma}\right)$$

		LHS	RHS
Try $p' = (150 - \tfrac{2}{3}\Delta\sigma)$ ($\Delta\sigma = 75$)	= 100	0.75	0.617
$\Delta\sigma = 90$	90	1	0.711
$\Delta\sigma = 60$	110	0.545	0.532
$\Delta\sigma = 58.5$	111	0.527	0.524
57	112	0.509	0.516

$$\Delta\sigma \sim \underline{\underline{58.5 \text{ kPa}}} = q$$

$$v = 2.023 + 0.062 \ln\left(\frac{150}{111}\right) = 2.042$$

$$\epsilon_v = \frac{0.01867}{2.023} = \underline{\underline{0.92\%}}$$ expansion

@ failure $q = Mp'$

$$\Delta\sigma = 0.89\left(150 - \tfrac{2}{3}\Delta\sigma\right)$$

$$\Delta\sigma = \frac{133.5}{1.593} = \underline{\underline{83.8 \text{ kPa}}}$$ $p' = 94.1$ kPa

$v = 2.027$

$$\epsilon_v = \frac{0.004}{2.023} = \cancel{1.4\%}\ -0.19\%$$

d) @ failure

$v = 2.042$

$$\Gamma - \lambda \ln p' = 2.042$$

$$p' = 85.9 \text{ kPa}$$

$$q = Mp' = \underline{\underline{76.5 \text{ kPa}}}$$

3D2 - Geotechnical Engineering II, 23-24 Exam Paper

Problem 3

Part (a): From GE Databook: $\tan\phi_{mob} = \dfrac{\tan\beta}{\left(1 - \dfrac{\gamma_w z_w}{\gamma z}\right)}$

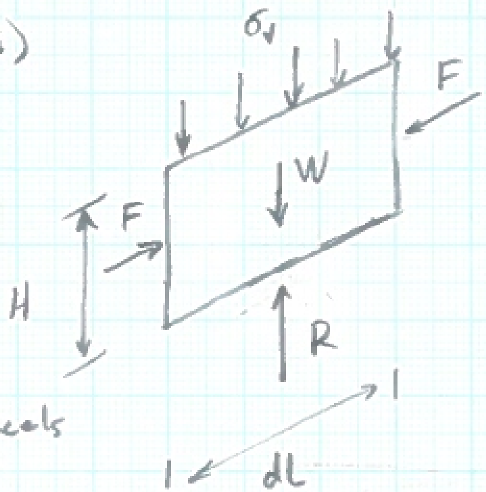
$z_w = 0 \Rightarrow \tan\phi_{mob} = \tan\beta$

$$SF = \frac{\tan\phi}{\tan\beta} = \frac{\tan 35°}{\tan 25°} = 1.502$$

$$\boxed{SF = 1.5}$$

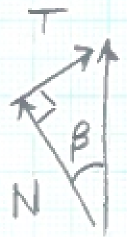
Part (b): Consider stress within sand layer

Strategy (i)



(force F cancels itself)

$R = W + \sigma_v dL$ ... σ_v equivalent to increasing unit weight



(#)

$$\tan\phi_{mob} = \frac{\tau}{\sigma} = \frac{(T/dL)}{(N/dL)} = \frac{T}{N}$$

From force polygon (#):

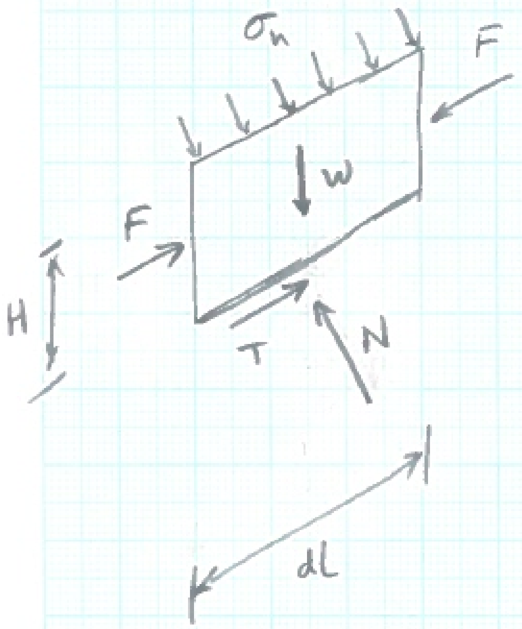
$\dfrac{T}{N} = \tan\beta$ $\quad\therefore\quad$ $\tan\phi_{mob} = \tan\beta$

or $SF = \dfrac{\tan\phi}{\tan\beta}$

Safety factor is same as if $\sigma_v = 0$, so no, strategy (i) will not help stabilise the slope.

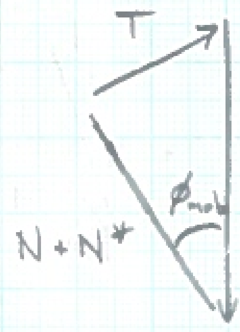
Adding σ_v is equivalent to increasing weight of material (unit weight), which is irrelevant.

Strategy (ii)



$N^* = \sigma_n \, dL$ ... normal force due to σ_n

Force polygon



$$\tan \phi_{mob} = \frac{\tau}{\sigma} = \frac{(T/dL)}{(N+N^*)/dL} = \frac{T}{N+N^*}$$

Normal force N, shear force T, and weight W correspond to $\sigma_n = 0$

Hence, mobilised friction angle ϕ_{mob} is <u>less</u> with $\sigma_n > 0$.

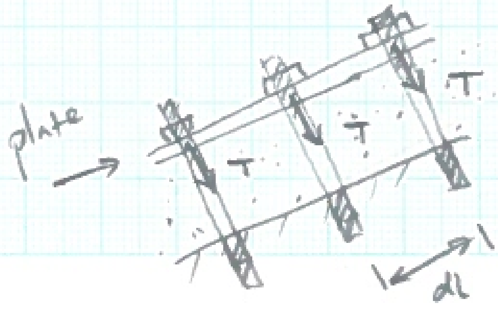
Strategy (ii) <u>will</u> help stabilise the slope by increasing normal stress

Could compute SF for $\sigma_n > 0$ that will be larger than with $\sigma_n = 0$

<u>Part (c)</u> :

Vertical stress σ_v could be applied by adding a layer of material above the slope : $\sigma_v = \gamma_{mat} H_{mat}$, where γ_{mat} is unit weight of the added material and H_{mat} is its height.

Normal stress σ_n could be applied by installing pretensioned anchors, mounted to the rock. Other strategies can also be envisioned.



$$\sigma_n \sim \frac{T}{dL}$$

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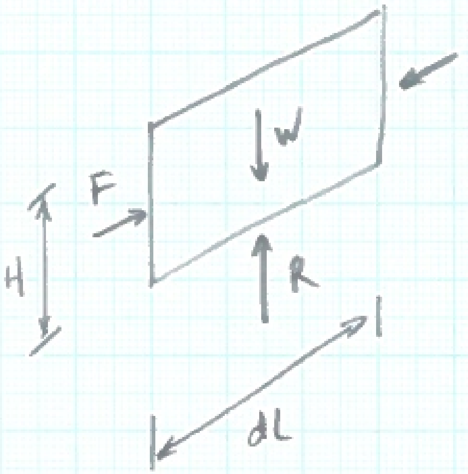
Part (d):



$$W = \gamma_s H \, dL \cos\beta = R$$

$$\sigma = \frac{R\cos\beta}{dL} \quad \text{... normal stress at depth } H$$

$$\tau = \frac{R\sin\beta}{dL} \quad \text{... shear stress — " —}$$

$$\tau = \frac{\gamma_s H \cancel{dL} \cos\beta \sin\beta}{\cancel{dL}}$$

$$SF = \frac{S_u}{\tau} = \frac{S_u}{\gamma_s H \cos\beta \sin\beta}$$

$$\boxed{SF = \frac{S_u}{\gamma_s H \cos\beta \sin\beta}}$$

For slope to fail, $SF = 1$:

$$H = \frac{S_u}{\gamma_s \cos\beta \sin\beta}$$

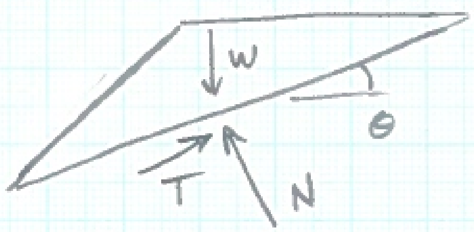
$$= \frac{15 \text{ kPa}}{(18 \text{ kN/m}^3) \cos 40° \sin 40°} = 1.69 \text{ m}$$

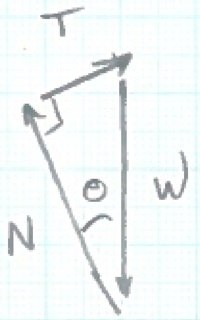
$$\boxed{H = 1.7 \text{ m}}$$

failure height for given properties

Part (e):

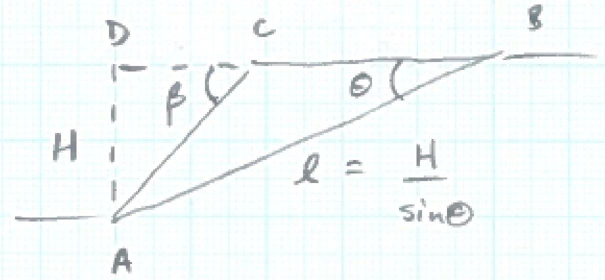
Consider equilibrium of wedge of soil above failure plane





$T = W\sin\theta$ (*)

Weight W:



$$\text{Area } ABD = \frac{1}{2} H \underbrace{\left(\frac{H}{\tan\theta} \right)}_{\text{length } DB} = \frac{H^2}{2\tan\theta}$$

$$\text{Area } ACD = \frac{1}{2} H \underbrace{\left(\frac{H}{\tan\beta} \right)}_{\text{length } DC} = \frac{H^2}{2\tan\beta}$$

$$\text{Area } ABC = \text{Area } ABD - \text{Area } ACD = \frac{H^2}{2}\left(\frac{1}{\tan\theta} - \frac{1}{\tan\beta} \right)$$

$$W = \frac{1}{2}\gamma_s H^2 \left(\frac{1}{\tan\theta} - \frac{1}{\tan\beta} \right)$$

Now, use Eq. (*) from force polygon and compute <u>average</u> shear stress τ:

$$T = W\sin\theta$$

$$\tau = \frac{T}{\ell} = \frac{T}{(H/\sin\theta)} = \frac{W\sin^2\theta}{H}$$

Combine with expression for W above:

$$\tau = \frac{1}{2}\gamma_s H \left(\frac{\sin^2\theta}{\tan\theta} - \frac{\sin^2\theta}{\tan\beta} \right)$$

$$\frac{\sin^2\theta}{\tan\theta} = \frac{\sin^2\theta}{(\sin\theta/\cos\theta)} = \sin\theta\cos\theta$$

$$\tau = \frac{1}{2}\gamma_s H \left(\sin\theta\cos\theta - \frac{\sin^2\theta}{\tan\beta} \right)$$

JPH

$$SF = \frac{S_u}{\tau} = \frac{S_u}{\left[\frac{1}{2}\gamma_s H\left(\sin\theta\cos\theta - \frac{\sin^2\theta}{\tan\beta}\right)\right]}$$

$$= 2\,\frac{S_u}{\gamma_s H}\,\frac{\tan\beta}{\sin\theta(\cos\theta\tan\beta - \sin\theta)}$$

Alternatively, after some work using trigonometric identies,

$$SF = 2\,\frac{S_u}{\gamma H}\,\frac{\sin\beta}{\sin(\beta-\theta)\sin\theta}$$

$$\boxed{\begin{aligned} SF &= 2\,\frac{S_u}{\gamma_s H}\,\frac{\tan\beta}{\sin\theta(\cos\theta\tan\beta - \sin\theta)} \\ &= 2\,\frac{S_u}{\gamma_s H}\,\frac{\sin\beta}{\sin(\beta-\theta)\sin\theta} \end{aligned}}$$

To determine value of θ for which failure is most likely, compute <u>minimum SF</u> :

$$\frac{\partial}{\partial\theta}(SF) = 0 \quad \leftarrow \text{calculate derivative and solve for } \theta$$

This approach may <u>not</u> be realistic for various reasons. The assumed mode of failure is simple, and another failure mode (e.g. rotational failure) may be closer to reality. Furthermore, the safety factor for other failure modes may be lower, implying this approach is potentially <u>unsafe</u>. Averaging shear stress over the failure plane may be unsafe, since non-uniform stress is possible and may produce progressive failure much earlier than this calculation would predict.

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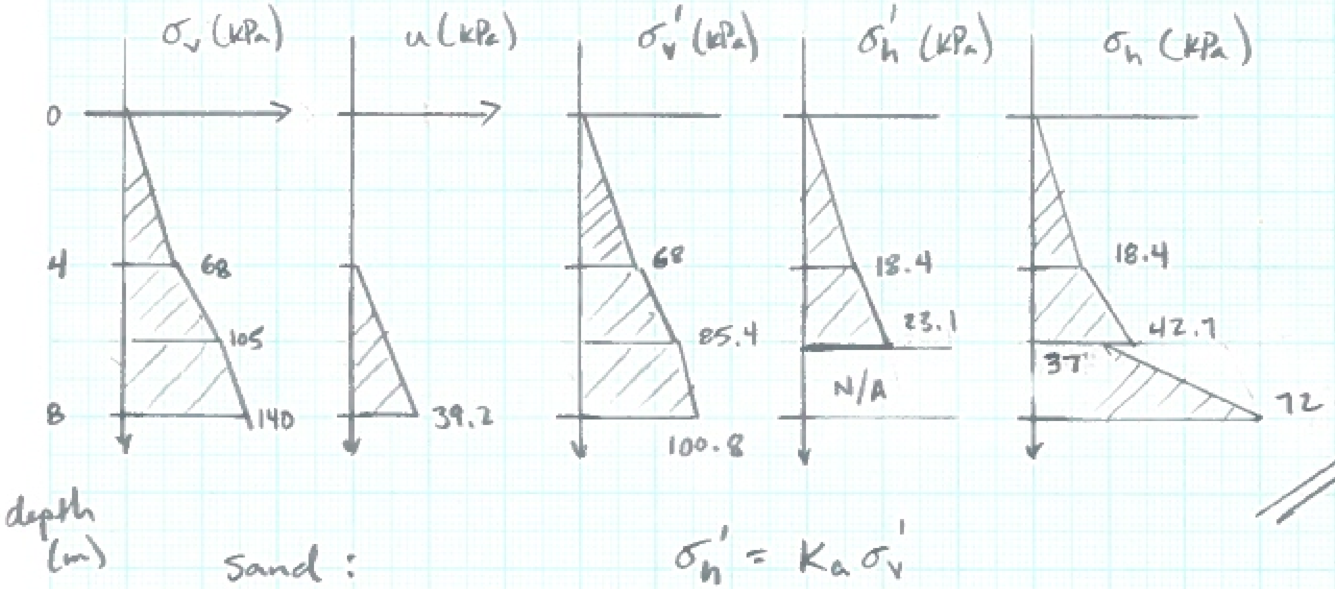
Problem 4

Part (a)

back of wall (active)

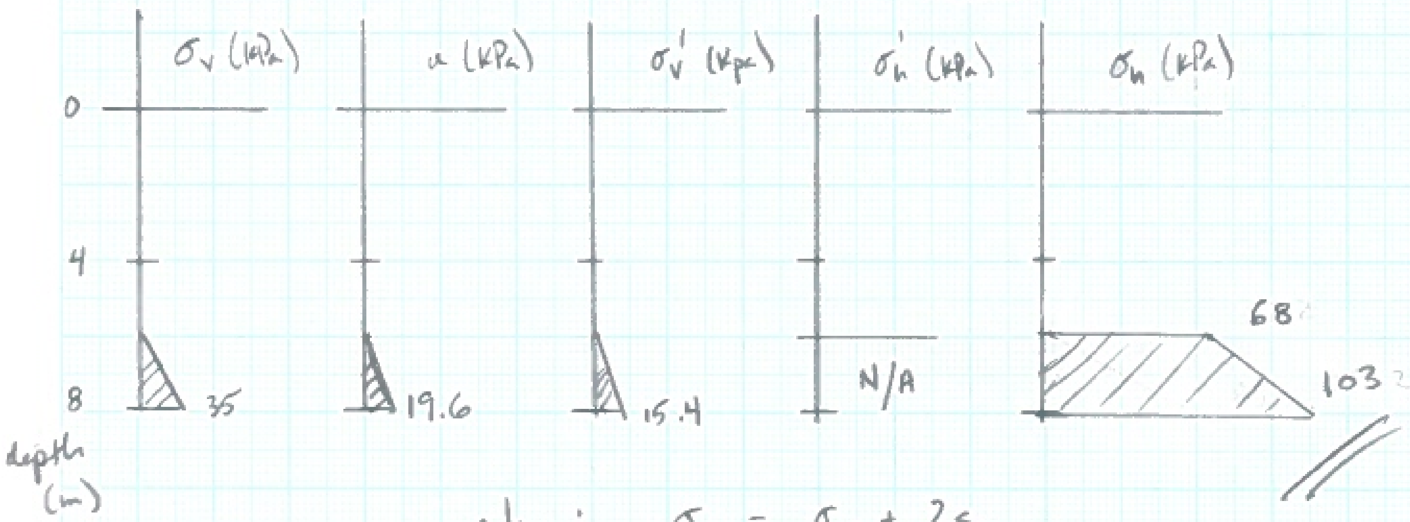


sand : $\sigma_h' = K_a \sigma_v'$

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 35^\circ}{1+\sin 35^\circ} = 0.271$$

clay : $\sigma_h = \sigma_v - 2s_u = \sigma_v - 68$ kPa

front of wall (passive)

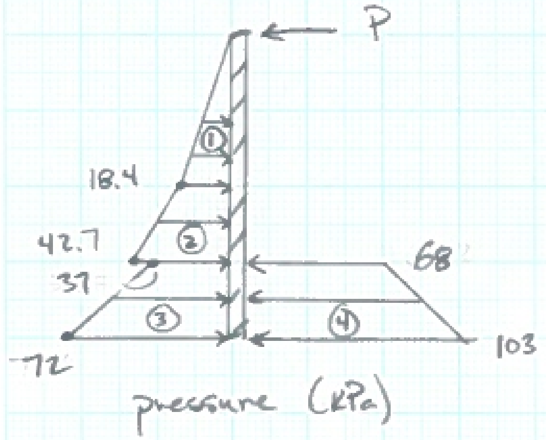


clay : $\sigma_h = \sigma_v + 2s_u$

$= \sigma_v + 68$ kPa

Part (b)

Free-body diagram for wall:



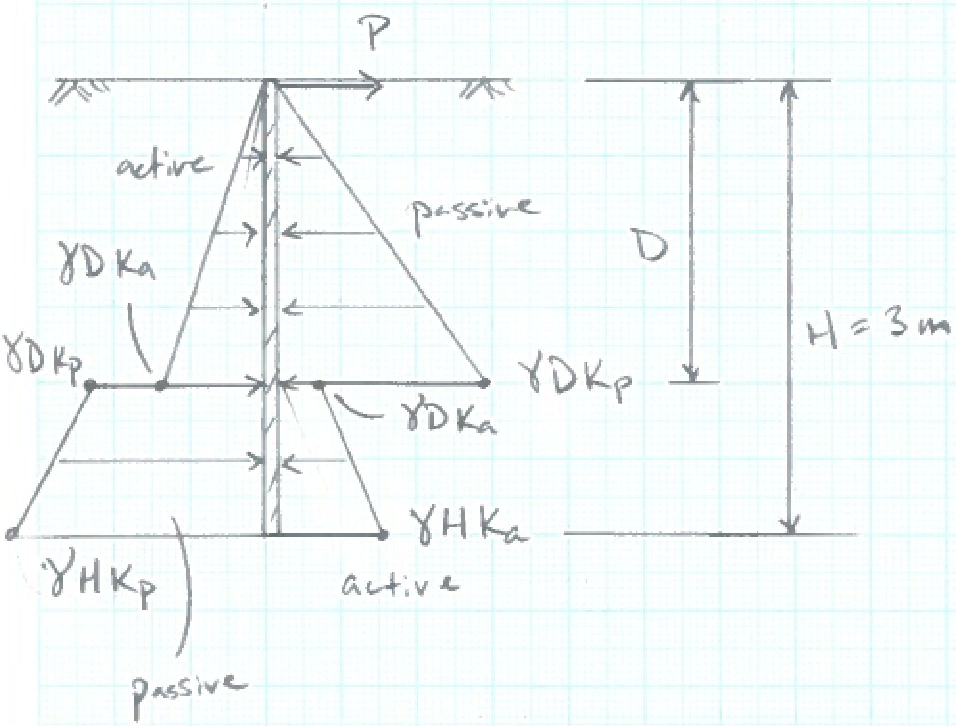
$$\Sigma_i F = \underbrace{F_1 + F_2 + F_3 + F_4}_{\text{forces for regions shown at left}} - P$$

$$= \frac{1}{2}(18.4\text{ kPa})(4\text{m})$$
$$+ \frac{1}{2}(42.7\text{ kPa} + 18.4\text{ kPa})(2\text{m})$$
$$+ \frac{1}{2}(72\text{ kPa} + 37\text{ kPa})(2\text{m})$$
$$- \frac{1}{2}(68\text{ kPa} + 103\text{ kPa})(2\text{m})$$
$$- P = 0$$

$$\Rightarrow \boxed{P = 35.9\text{ kN/m}}$$

Part (c)

Wall is loaded at top and therefore will rotate about unknown point at distance D below the surface:



$$\left.\begin{aligned} \Sigma_i F &= 0 \\ \Sigma_i M &= 0 \end{aligned}\right\}$$

use force and moment equilibrium (2 equations) to solve for the 2 unknowns: P and D

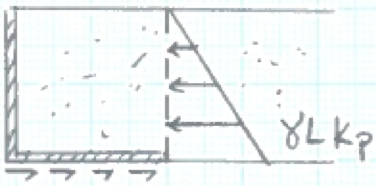
(From spreadsheet calculations, find

D = 2.4 m and P = 62 kN/m.)

JPH

Part (d)

First, assume wall will slide due to active earth pressure.
Configuration is symmetric, so consider one wall throughout.



$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 30^\circ}{1+\sin 30^\circ} = \frac{1}{3}$$

$\leftarrow$ assume rough wall so strength fully mobilised

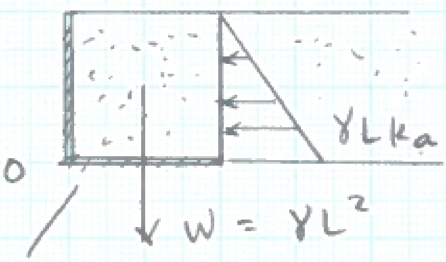
$$\Sigma_i F_{horizontal} = \frac{1}{2} K_a \gamma L^2 - S_u L = 0$$

$$= \left(\frac{1}{2} K_a \gamma L - S_u\right) L = 0$$

$$\Rightarrow \quad \frac{1}{2} K_a \gamma L - S_u = 0$$

$$L = \frac{2 S_u}{K_a \gamma} = \frac{2\,(80\text{ kPa})}{\left(\frac{1}{3}\right)(18\text{ kN/m}^3)} = 26.7\text{ m}$$

Second, assume wall will rotate at corner.



assume no tension on interface

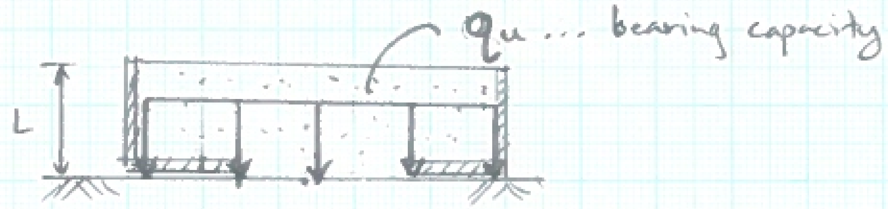
$$\Sigma M_o = \left(\frac{1}{2} K_p \gamma L^2\right)\left(\frac{1}{3} L\right) - (\gamma L^2)\left(\frac{1}{2} L\right) = 0$$

$$\left(\frac{1}{6} K_a - \frac{1}{2}\right)(\gamma L^3) = 0$$

$\Rightarrow$ stable for $K_a \le 3$

$\Rightarrow$ Wall is always stable against overturning for all values of L

Third, assume bearing capacity controls L.



limit: $\gamma L = q_u$

$q_u = S_u N_c = S_u(2+\pi)$

$$\gamma L = S_u(2+\pi)$$

$$L = \frac{S_u(2+\pi)}{\gamma}$$

$$= \frac{(80\text{ kPa})(2+\pi)}{(18\text{ kN/m}^3)}$$

$$= 22.9\text{ m}$$

Among all three modes, BC controls and $L = 22.9$ m.

bearing capacity

ENGINEERING TRIPOS PART IIA 2024

EXTRACTS FROM ASSESSOR'S REPORT, 3D2, GEOTECHNICAL ENGINEERING II

Q1 2D stress paths

attempted by 67% of students, average mark 10.6/20, maximum 19, minimum 3.

Not a very popular question on 2D stress paths. Most candidates could calculate the in-site stress state but in general the changes of stress with excavation were not particularly well handled.

Q2 Triaxial stress paths

attempted by 100% of students, average mark 13.6/20, maximum 20, minimum 3.

A popular question answered by all candidates which was well handled. The only thing that caused any real problem was the slightly unusual stress path, but the understanding of yield, failure and methods to calculate appropriate parameters were good.

Q3 Slope stability

attempted by 90% of students, average mark 12.8/20, maximum 19, minimum 7.

A popular question which was well handled overall. Some had difficulty recognising that the formula for Part (d) must be derived considering the Tresca yield criterion, and they instead attempted to use the formulas given in the data book. For Part (e), those who recognized the need to use global equilibrium of the wedge of soil above the failure plane tended to do well, but a significant number made fruitless attempts with other approaches (e.g., using formulas from the data book).

Q4 Retaining structures

attempted by 43% of students, average mark 10.0/20, maximum 18, minimum 2.

Fewer than half attempted this question, which differentiated quite clearly between those who fully understood the underlying concepts and those who did not. Those who mixed up active and passive states tended to do poorly throughout. For Part (c), many did not see that the point of rotation is unknown and must be determined. For Part (d), only a few understood that failure modes for sliding, rotation, and bearing must all be considered.