

1 (a)

NC to 400 kPa

$$v = N - \lambda \ln \sigma' = \bar{P} + \lambda - \kappa - \lambda \ln \sigma'$$

$$= 2.759 + 0.161 - 0.062 - 0.161 \ln 400$$

$$= 1.893$$

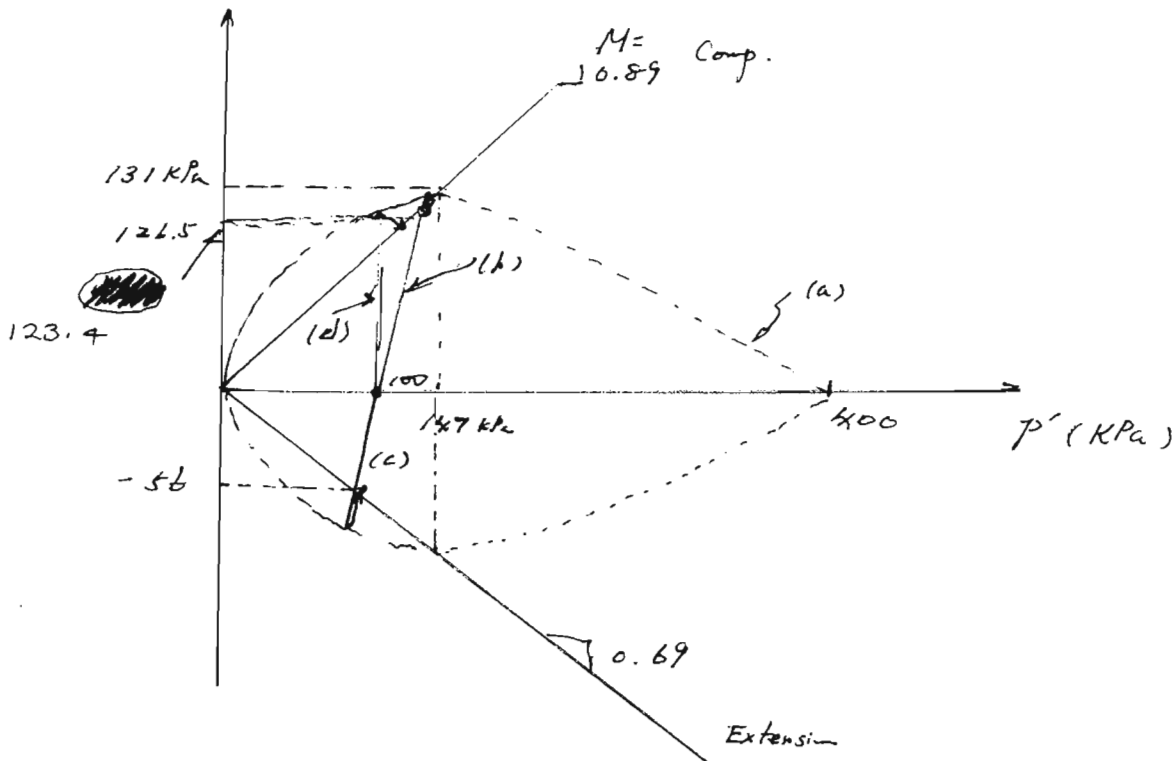
Swelled back to 100 kPa

$$v = 1.893 + \kappa \ln \left(\frac{400}{100} \right) = 1.893 + 0.062 \ln \left(\frac{400}{100} \right)$$

$$= 1.979$$

$$s = \sigma_1 - \sigma_3 \text{ (kPa)}$$

$$= 1.979$$



(b)

$$\frac{\Delta s}{\Delta p} = \frac{\Delta \sigma_1}{\Delta \sigma_1 / 3} = 3$$

$$M = \frac{s}{p'} = \frac{\Delta s}{100 + \Delta s / 3} = 0.89$$

$$0.89 (100 + \Delta s / 3) = \Delta s$$

$$s = \Delta s = \frac{100 \times 0.89}{(1 - 0.89/3)} = 126.5 \text{ kPa}$$

$$p' = 126.5 / 0.89 = 142.2 \text{ kPa}$$

$$\dots = 1.961$$

Question 2

(a) $\sigma_v = 20 \times 20 = 400 \text{ kPa}$

$u = 18 \times 10 = 180 \text{ kPa}$

$\therefore \sigma_v' = 400 - 180 = 220 \text{ kPa}$

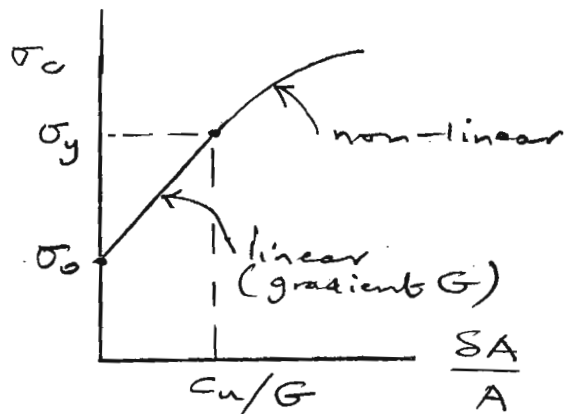
$K_0 = \frac{\sigma_h'}{\sigma_v'} \Rightarrow \sigma_h' = 1.5 \times 220 = 330 \text{ kPa}$

$\sigma_o = \sigma_{ho} = \sigma_h' + u = 330 + 180 = \underline{510 \text{ kPa}} \quad [15\%]$

(b) yield first occurs at cavity pressure $\sigma_c = \sigma_y$

when $\sigma_y = \sigma_o + c_u$

i.e. at $\sigma_y = 510 + 150 = \underline{660 \text{ kPa}}$



$$\frac{\delta A}{A} = \frac{c_u}{G} = \frac{150}{50 \times 10^3} = 3 \times 10^{-3}$$

$$\frac{\delta A}{A} \approx 2 \epsilon_c \text{ for small strains.}$$

$$\epsilon_c = \text{cavity strain} = \frac{\rho_c}{r_{co}}$$

$$\rho_c = \text{radial displacement}$$

$$r_{co} = \text{cavity radius}$$

$\therefore \rho_c = \epsilon_c \cdot r_{co} = \frac{1}{2} \times 3 \times 10^{-3} \times 40 = \underline{0.06 \text{ mm}}$

[30%]

(c) $\delta \sigma_c = \sigma_c - \sigma_o = c_u \left[1 + \ln \frac{G}{c_u} + \ln \frac{\delta A}{A} \right]$ DATA BOOK

limit pressure when $\frac{\delta A}{A} \rightarrow 1$, $\ln \frac{\delta A}{A} \rightarrow 0$

$$\begin{aligned} \text{i.e. } \sigma_c &= \sigma_o + c_u \left[1 + \ln \frac{G}{c_u} \right] \\ &= 510 + 150 \left[1 + \ln \frac{50 \times 10^3}{150} \right] \\ &= 510 + 1021 = \underline{1531 \text{ kPa}} \end{aligned}$$

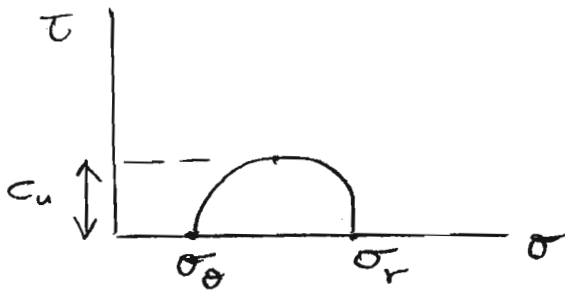
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(note $\sigma_o = \sigma_{ho}$)

$$(d) \quad \sigma_r = \sigma_{ho} + G \delta A / \pi r^2$$

$$\sigma_\theta = \sigma_{ho} - G \delta A / \pi r^2$$

$$\text{In plastic zone } \sigma_r - \sigma_\theta = 2c_w$$



r_c = radius of cavity

At edge of plastic zone $r = r_p$

$$\therefore \sigma_r - \sigma_\theta = 2G \delta A / \pi r_p^2 = 2c_w$$

$$\therefore r_p^2 = G \delta A / \pi c_w$$

$$A = \pi r_c^2$$

$$\therefore \frac{r_p}{r_c} = \left(\frac{G}{c_w} \cdot \frac{\delta A}{A} \right)^{0.5}$$

[20%]

$$(e) \quad \sigma_c = 1.4 \times \sigma_y = 1.4 \times 660 = 924 \text{ kPa}$$

$$\sigma_c - \sigma_0 = c_w \left[1 + \ln \frac{G}{c_w} + \ln \frac{\delta A}{A} \right] \quad \text{DATA BOOK}$$

$$\therefore \frac{924 - 510}{150} = 1 + 5.81 + \ln \frac{\delta A}{A}$$

$$\therefore \ln \frac{\delta A}{A} = -4.05 \Rightarrow \frac{\delta A}{A} = 0.017$$

$$\therefore \frac{r_p}{r_c} = \left(\frac{50 \times 10^3}{150} \times 0.017 \right)^{0.5} = 2.38$$

$$r_c \approx r_{c0} = 40 \text{ mm (for small strains)}$$

$$\therefore r_p = 2.38 \times 40 = \underline{95 \text{ mm}}$$

[20%]

Question #3

(a) Before embankment construction, stresses at A:

$$\sigma_v = 20 \times 10 = 200 \text{ kN/m}^2$$

$$u_0 = 10 \times 9 = 90 \text{ kN/m}^2$$

$$\therefore \sigma_v' = \sigma_v - u_0 = 200 - 90 = 110 \text{ kN/m}^2$$

$$K_0 = 1.0$$

$$\therefore \sigma_h' = \sigma_v' = 110 \text{ kN/m}^2$$

$$\therefore \sigma_h = 110 + 90 = 200 \text{ kN/m}^2$$

$$t = \frac{1}{2} (\sigma_v - \sigma_h) = 0$$

$$s' = \frac{1}{2} (\sigma_v' + \sigma_h') = 110 \text{ kN/m}^2$$

$$s = \frac{1}{2} (\sigma_v + \sigma_h) = 200 \text{ kN/m}^2$$

\therefore Effective stress (t, s') at A' $(0, 110)$

Total stress (t, s) at A $(0, 200)$

During embankment raising, $\Delta\sigma_h = 0.25 \Delta\sigma_v$

$$\Delta t = \frac{1}{2} (\Delta\sigma_v - \Delta\sigma_h) = \frac{1}{2} \Delta\sigma_v \left(1 - \frac{1}{4}\right) = \frac{3}{8} \Delta\sigma_v$$

$$\Delta s = \frac{1}{2} (\Delta\sigma_v + \Delta\sigma_h) = \frac{1}{2} \Delta\sigma_v \left(1 + \frac{1}{4}\right) = \frac{5}{8} \Delta\sigma_v$$

$$\therefore \frac{\Delta t}{\Delta s} = \frac{\frac{3}{8} \Delta\sigma_v}{\frac{5}{8} \Delta\sigma_v} = \frac{3}{5} \quad \text{this is slope of total stress path, TSP}$$

Until yield, effective stress path remains vertical (because behaviour elastic inside yield surface)

Yield first occurs at embankment height H,

when $\Delta\sigma_v = 50 \text{ kN/m}^2$ at ~~point~~ A

$$\therefore \Delta\sigma_h = 0.25 \times 50 = 12.5 \text{ kN/m}^2$$

$$\therefore \Delta t = \frac{1}{2} (50 - 12.5) = 18.75 \text{ kN/m}^2$$

$$\Delta s = \frac{5}{3} \times 18.75 = 31.25 \text{ kN/m}^2$$

soil element

∴ point C is total stress state (18.75, 231.25) ⁽²⁾
for embankment height H,

Effective stress path remains vertical, then
point C' is effective stress state (18.75, 110).

When embankment height is 0.5H,
total stress at point B for which
 $t = \frac{1}{2} \times 18.75 = 9.4 \text{ kN/m}^2$
 $s = 200 + \frac{1}{2} \times 31.25 = 215.6 \text{ kN/m}^2$

effective stress at point B' for which
 $t = 9.4 \text{ kN/m}^2$ (as for TSP)
 $s' = 110 \text{ kN/m}^2$ (remains unchanged).

∴ (i) pore pressure for embankment height 0.5H,
 $= s - s'$ (for points B, B')
 $= 215.6 - 110 = \underline{105.6 \text{ kN/m}^2}$

(ii) pore pressure for embankment height H,
 $= s - s'$ (for points C, C')
 $= 231.25 - 110 = \underline{121.25 \text{ kN/m}^2}$

~~40%~~ [40%]

(b) Embankment to height H₂

$$\Delta\sigma_v = 80 \text{ kN/m}^2$$

$$\Delta t = \frac{3}{8} \Delta\sigma_v \quad (\text{from before})$$

$$= 30 \text{ kN/m}^2$$

Critical state Line reached at point D'

$$s' = \frac{t}{\sin 25^\circ} = \frac{30}{\sin 25^\circ} = \frac{30}{0.423} = 70.9 \text{ kN/m}^2$$

③

) point D (TSP)

$$\frac{\Delta \sigma}{\Delta s} = \frac{3}{5} \quad (\text{from before})$$

$$\therefore \text{for } \Delta \sigma = 30 \text{ kN/m}^2$$

$$\Delta s = \frac{5}{3} \times 30 = 50 \text{ kN/m}^2$$

\therefore point D is at (30, 250)

$$\text{hence pore pressure} = s - s' \quad (\text{for points D, D'})$$

$$= 250 - 70.9 = \underline{179.1 \text{ kN/m}^2} \quad [40\%]$$

(c) Original pore pressure $u_0 = 90 \text{ kN/m}^2$

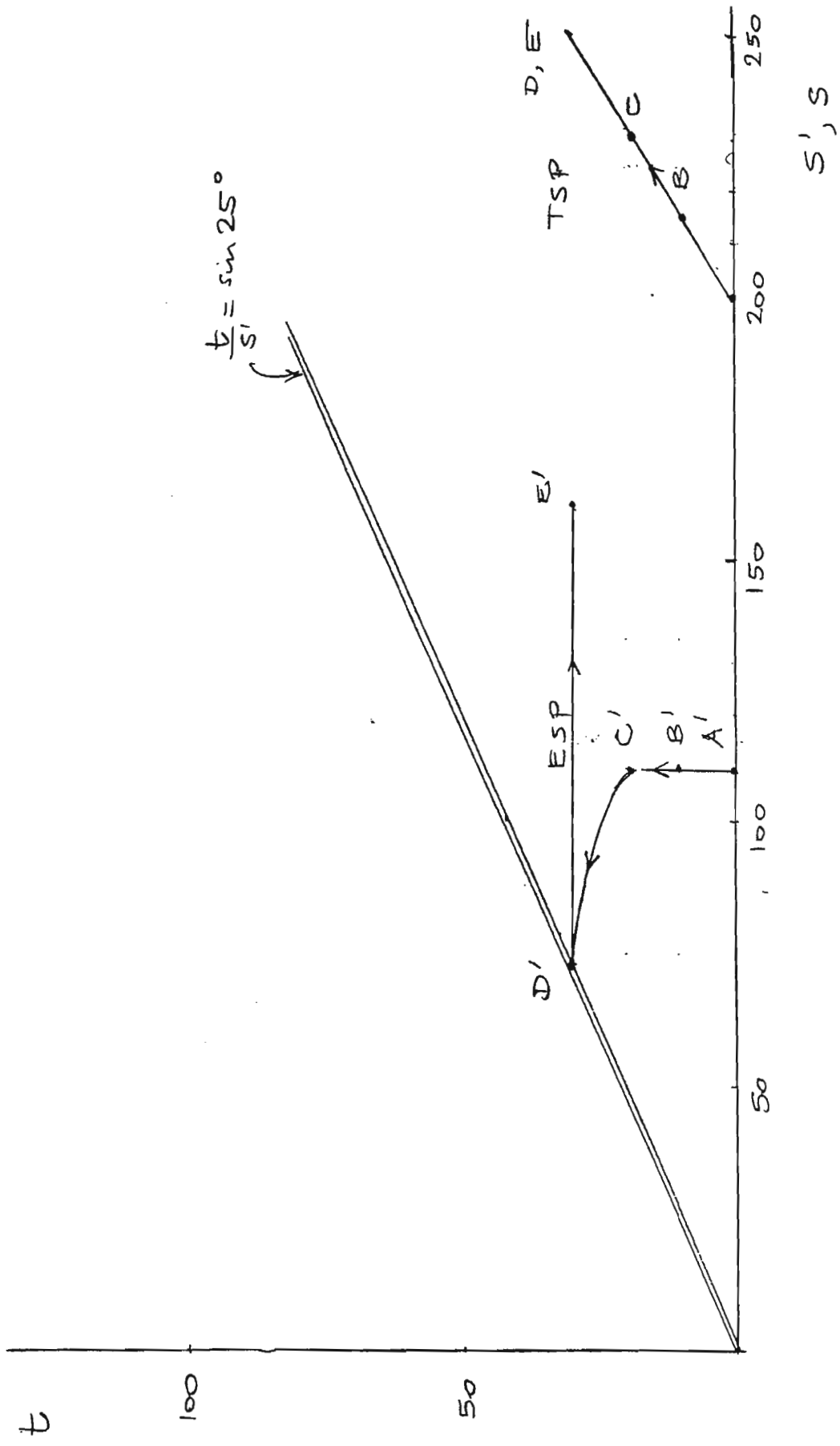
\therefore change in pore pressure as effective stress

$$\text{path moves from D' to E'} = 179.1 - 90$$

$$= \underline{89.1 \text{ kN/m}^2}$$

Total stress path does not change.

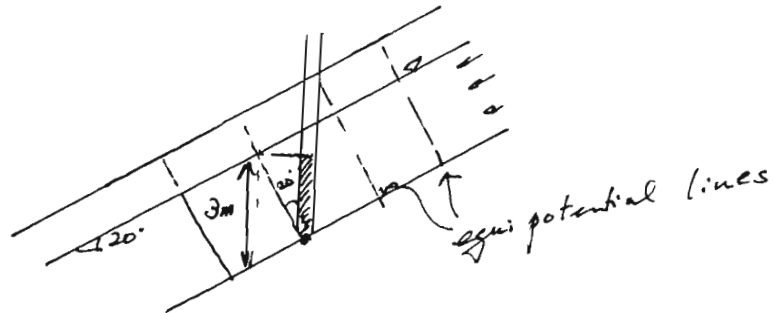
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4 (a)

$$\begin{aligned} \gamma &= \left(\frac{G_s + e}{1 + e} \right) \gamma_w \\ &= \left(\frac{2.65 + 0.7}{1 + 0.7} \right) 9.8 \\ &= \underline{19.31 \text{ kN/m}^3} \end{aligned}$$

(b)



$$\begin{aligned} u &= 3 \times 9.8 \times \cos 20^\circ \times \cos 20^\circ \\ &= \underline{26.0 \text{ kPa}} \end{aligned}$$

(c)

$$\begin{aligned} \sigma'_i &= (1 \text{ m} \times 18 + 3 \text{ m} \times 19.3) \cos^2 20^\circ \\ &= 75.9 \times \cos^2 20^\circ \\ &= \underline{67.0 \text{ kPa}} \\ \tau &= 75.9 \times \cos 20^\circ \times \sin 20^\circ \\ &= \underline{24.4 \text{ kPa}} \end{aligned}$$

$$\begin{aligned} \phi'_{mbb} &= \tan^{-1} \left(\frac{24.4}{41.0} \right) \\ &= \underline{30.8^\circ} \end{aligned}$$

$$\sigma' = 67.0 - 26.0 = \underline{41.0 \text{ kPa}}$$

(d)

$$Z_D = \frac{0.9 - 0.7}{0.9 - 0.5} = 0.5$$

$$Z_c = \ln \left(\frac{20.000}{41.0} \right) = 6.19$$

$$Z_R = 0.5 \times 6.19 - 1 = 2.10$$

$$\phi_{peak} = \phi_{crit} + s Z_R = 36^\circ + 5 \times 2.10 = 46.5^\circ$$

$$\phi_{crit} = 36^\circ$$

$$F.S' \text{ for peak} = \frac{\tan 46.5^\circ}{\tan 30.8^\circ} = \underline{1.76}$$

$$F.S \text{ for critical} = \frac{\tan 36^\circ}{\tan 30.8^\circ} = \underline{1.22}$$

(e) At the top interface,

$$T = 1 \times 18 \times \cos^2 20^\circ = 15.9 \text{ kPa}$$

$$Z = 1 \times 18 \times \cos 20^\circ \sin 20^\circ = 5.8 \text{ kPa}$$

$$\tan 26^\circ = \frac{5.8}{(15.9 - u)} = 0.488$$

$$u = \frac{15.9 \times 0.488 - 5.8}{0.488} = 4.01 \text{ kPa}$$

At the sandstone interface

$$\tan 36^\circ = \frac{24.4}{(67 - u)} = 0.727$$

$$u = \frac{67 \times 0.727 - 24.4}{0.727} = 33.4 \text{ kPa}$$

At the top $0 \rightarrow 4.01 \text{ kPa}$

At the bottom $26.0 \rightarrow 33.4 \text{ kPa}$

probably fail at the top interface first.