EGT2 ENGINEERING TRIPOS PART IIA

Tuesday 3 May 2022 2 to 3.40 pm

Module 3D3

STRUCTURAL MATERIALS AND DESIGN

Answer not more than three questions.

All questions carry the same number of marks.

The approximate percentage of marks allocated to each part of a question is indicated in the right margin.

Write your candidate number not your name on the cover sheet and at the top of each extra sheet.

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed Attachment: 3D3 Structural Materials and Design data sheet (18 pages). Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 An incomplete design for a steel framed building with concrete decking is shown in Fig. 1(a).

(a) With the aid of sketches where appropriate, devise a suitable structural scheme to provide overall stability to the structure. Describe the principal load paths for the complete structure for both horizontal and vertical loading. Explain any assumptions made. [40%]

(b) A cladding consultant has not yet been commissioned for the project, so you are asked to perform some initial calculations to get a sense of the glazing requirements. The glazing will be supported at the perimeter of the building by the slab edges, as shown in Fig. 1(b). You consider that a short-term uniformly distributed characteristic wind load of 1 kN m^{-2} is likely to govern and that self-weight of the glazing can be neglected. The partial safety factor for wind load is 1.5. Fully toughened glass is to be used, having $f_{rk} = 90$ N mm⁻² and $f_{gk} = 45$ N mm⁻²; material partial safety factors $\gamma_{mV} = 1.2$ and $\gamma_{mA} = 1.8$; the load duration factor $k_{mod} = 0.74$ and the stressed area factor $k_A = 1.0$.

(i) If single ply glazing is to be used, what minimum thickness of glass would be required for adequate flexural resistance? [30%]

(ii) For reasons of post-fracture safety, you decide that a 2-ply laminated glazing solution with a PVB interlayer is preferable to a single ply. Both layers are to be of equal thickness and the thickness of the PVB interlayer can be assumed to be negligible. Assuming that the PVB provides a degree of shear interaction of 20% in this short-term loading case, determine the required thickness of the laminated glass for adequate flexural resistance. [30%]

Fig. 1

2 A concrete slab is to span simply supported between parallel masonry walls as shown in Fig. 2.

Concrete cover requirements for durability are 25 mm and a fire resistance rating of 1 hour is required. The concrete is to have a characteristic compressive cube strength of 50 N mm⁻² and a compressive cylinder strength of 40 N mm⁻². Steel reinforcement is to have a characteristic yield strength 500 N mm⁻². The characteristic compressive strength of the masonry is 5 N mm⁻². The material partial safety factor for concrete is 1.50, for steel is 1.15 and for the masonry is 3. The partial safety factor for transient load is 1.50 and for permanent load is 1.35.

The concrete slab is to be designed to support a uniformly distributed transient load of 2.5 kN m^{-2} .

(b) The masonry walls supporting the slab are 102.5 mm thick and 2.4 m tall. Perform suitable checks to determine whether the masonry walls are adequate to resist the loads from the slab above. $[40\%]$

Fig. 2

Version RMF/6

3 A simply supported timber beam is to be designed to resist a uniformly distributed long-term load of 10 kN m^{-1} that includes self-weight, in addition to a single long-term point load of 30 kN from a heavy piece of equipment whose position is not yet known but may be placed anywhere along the beam except within 0.6 m of the supports. The loads given are design loads inclusive of partial safety factors. The structural arrangement and loading is summarized in Fig. 3(a).

The timber is grade C16 softwood. The beam can be considered as service class 1. The beam is fully laterally restrained along its length. The material partial safety factor for solid timber is 1.3. The size effect factor k_h can be taken as 1.0 and the load sharing factor k_{ls} can be taken as 1.0.

(a) The beam is initially designed by the engineer as a simple rectangular profile along the full length of the beam, having a height of 600 mm and breadth of 250 mm.

(b) The architect has requested that the beam end detail be changed for aesthetic reasons to include a rectangular notch as shown in Fig. 3(b).

Fig. 3

4 A steel overhead crane girder is 16 m long and simply supported. At both ends the warping displacements are unrestrained but twisting about the longitudinal axis of the girder is prevented. The girder has been welded together from individual grade S355 steel plates with the following dimensions (see Fig. 4a): $b_f = 300$ mm; $t_f = 40$ mm; $h_w = 800$ mm; $t_w = 10$ mm.

Both flanges are identical. The girder is laterally unsupported along its length. The moving crane load can be represented by a point load. The mass of the crane itself (trolley without load) is 750 kg. Note that the support system consists of two identical girders, as shown in Fig. 4(b).

Material partial safety factors are $\gamma_{M0} = 1$ for the resistance of the cross section, $\gamma_{M1} = 1$ for the resistance of the member to buckling and $\gamma_{M2} = 1.25$ for the resistance of the cross section in tension to fracture. The partial safety factor for transient load is 1.50 and for permanent load is 1.35.

(a) Based on the ULS for bending, determine the safe hoisting load (ignore any dynamic effects). The torsional properties of the girder can be calculated as follows: [50%]

$$
I_t = \frac{2b_f t_f^3}{3} + \frac{h_w t_w^3}{3}
$$

$$
I_w = C_w = \frac{t_f}{24} (h_w + t_f)^2 b_f^3
$$

(b) Approximate the lateral-torsional buckling capacity of the girder by equating lateraltorsional buckling to column buckling of the compressed flange about its major axis. Discuss the reason(s) for any discrepancy with the result found in part (a). [25%]

(c) Determine whether any shear stiffening of the girder is required. If so, propose an appropriate stiffener spacing. [25%]

(a) Cross-sectional dimensions (b) Crane girders with trolley

Fig. 4

END OF PAPER

Version RMF/6

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University of Cambridge Department of Engineering

Engineering Tripos Part IIA

Module 3D3 Structural Materials & Design

Datasheets

Michaelmas 2021

THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

Example: $\Phi(3.57) = .9^38215 = 0.9998215.$

Steel Data Sheet

(EN 1993-1-1)

Table 3.1: Nominal values of yield strength f_y and ultimate tensile strength f_u for
hot rolled structural steel

Standard and	Nominal thickness of the element t [mm]					
		$t < 40$ mm	$40 \text{ mm} < t \leq 80 \text{ mm}$			
steel grade	f_v [N/mm ²]	f_u [N/mm ²]	f_v [N/mm ²]	f_u [N/mm ²]		
EN 10025-2						
	235	360	215	360		
S 235 S 275 S 355	275	430	255	410		
	355	AC_2 490 AC_2	335	470		
S 450	440	550	410	550		

Tension members

Yielding of the gross cross-section *A_g*: Fracture of the net cross-section *A_n*:

$$
N_{pl, Rd} = \frac{A_g f_y}{\gamma_{M0}}
$$

$$
N_{u, Rd} = \frac{0.9 A_n f_u}{\gamma_{M2}}
$$

Staggered bolt holes:

$$
A_n = A_g - n_b d_0 t + \sum_{stagger ggers} \frac{s_p^2 t}{4s_g}
$$

d⁰ = bolt hole diameter

 n_b = number of bolt lines across the member

Reduction factor for shear lag in eccentrically connected angles:

Column buckling

BS EN 1993-1-1:2005 EN 1993-1-1:2005 (E)

Table 6.2: Selection of buckling curve for a cross-section

Figure 6.4: Buckling curves

$6.3.1.2$ **Buckling curves**

For axial compression in members the value of χ for the appropriate non-dimensional slenderness $\bar{\lambda}$ (1) should be determined from the relevant buckling curve according to:

$$
\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \overline{\lambda}^2}} \quad \text{but } \chi \le 1,0
$$
\n
$$
\text{where } \Phi = 0,5\left[1 + \alpha\left(\overline{\lambda} - 0,2\right) + \overline{\lambda}^2\right]
$$
\n
$$
\overline{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}} \quad \text{for Class 1, 2 and 3 cross-sections}
$$
\n
$$
\overline{\lambda} = \sqrt{\frac{A_{\text{eff}}f_y}{N}} \quad \text{for Class 4 cross-sections}
$$
\n(6.49)

$$
\begin{array}{c}\n\mathbf{v} & \text{if } \mathbf{v} \\
\alpha & \text{if } \mathbf{v} \\
\mathbf{v} & \text{if } \mathbf{v}\n\end{array}
$$

is the elastic critical force for the relevant buckling mode based on the gross cross sectional N_{cr} properties.

The imperfection factor α corresponding to the appropriate buckling curve should be obtained from (2) Table 6.1 and Table 6.2.

Table 6.1: Imperfection factors for buckling curves

Buckling curve	ar			
Imperfection factor α			Δ	

Local buckling

$$
\sigma_{cr} = K \frac{\pi^2 E}{12(1 - v^2)} \left(\frac{t}{b}\right)^2
$$

where *b* is the width of the plate and *t* is its thickness.

For plates in uniform longitudinal compression:

- $K = 4$ for internal elements.
- $K = 0.43$ for outstand elements.

Beams

Elastic lateral-torsional buckling moment of a beam with doubly symmetric cross-section:

$$
M_{cr,0} = \frac{\pi^2 EI_z}{L_{cr}^2} \left[\frac{I_w}{I_z} + \frac{L_{cr}^2 GI_T}{\pi^2 EI_z} \right]^{0.5}
$$

where:

 I_T = torsional constant

 I_w = warping constant $(= d^2 I_{yy}/4$ for I-beams, with *d* the distance between the centerlines of the flanges)

 I_z = second moment of area about the minor axis

 $G =$ shear modulus

 L_{cr} = unrestrained length for lateral-torsional buckling

In the case of non-uniform bending:

$$
M_{cr} = C_1 M_{cr,0}
$$

(EN 1993-1-1)

6.3.2.2 Lateral torsional buckling curves - General case

(1) Unless otherwise specified, see 6.3.2.3, for bending members of constant cross-section, the value of χ _{LT} for the appropriate non-dimensional slenderness $\overline{\lambda}_{LT}$, should be determined from:

$$
\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \overline{\lambda}_{LT}^2}} \text{ but } \chi_{LT} \le 1,0
$$
\n(6.56)

where $\Phi_{LT} = 0.5 \left[1 + \alpha_{LT} \left(\overline{\lambda}_{LT} - 0.2\right) + \overline{\lambda}_{LT}^2\right]$

 α _{LT} is an imperfection factor

$$
\overline{\lambda}_{\rm LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}
$$

 M_{cr} is the elastic critical moment for lateral-torsional buckling

M_{cr} is based on gross cross sectional properties and takes into account the loading conditions, the real (2) moment distribution and the lateral restraints.

NOTE The imperfection factor α _{LT} corresponding to the appropriate buckling curve may be obtained from the National Annex. The recommended values α _{LT} are given in Table 6.3.

Table 6.3: Recommended values for imperfection factors for lateral torsional buckling curves

The recommendations for buckling curves are given in Table 6.4.

Cross-section	Limits	Buckling curve
Rolled I-sections	$h/b \leq 2$	
	h/b > 2	
Welded I-sections	h/h < 2	
	h/b > 2	
Other cross-sections		

Table 6.4: Recommended values for lateral torsional buckling curves for crosssections using equation (6.56)

$$
M_{b, Rd} = \chi_{LT} W_y \frac{f_y}{\gamma_{M1}}
$$

Interaction between moment and shear in the cross-section:

$$
f_{yr} = (1 - \rho)f_y \qquad \rho = \left(\frac{2V_{Ed}}{V_{pl, Rd}} - 1\right)^2 \qquad \text{(for } V_{Ed} > 0.5_{\text{Vpl, Rd}}\text{)}
$$

$$
M_{y,V,Rd} = \left[W_{pl,y} - \frac{\rho A_w^2}{4t_w}\right] \frac{f_y}{\gamma_{M0}} \le M_{y,c,Rd}
$$

$$
\leq M_{y,c,Rd} \qquad \qquad \text{where } A_w = h_w t_w
$$

Shear

$$
V_{pl, Rd} = A_v \frac{(f_y/\sqrt{3})}{\gamma_{M0}}
$$

$$
A_v = A - 2bt_f + (t_w + 2r)t_f \qquad \text{but} \quad \geq h_w t_w
$$

where:

- $b = \text{flange width}$
- t_f = flange thickness
- t_w = web thickness
- h_w = web height
- *r* = transition radius between web and flange

Shear buckling:

$$
\tau_{cr} = K \frac{\pi^2 E}{12(1 - v^2)} \left(\frac{t}{b}\right)^2
$$

$$
K = 5.34 + \frac{4}{(a/b)^2} \quad \text{if } a > b
$$

$$
K = 5.34 + \frac{4}{(b/a)^2} \quad \text{if } b > a
$$

Shear buckling needs to be checked if: $\frac{h_w}{h_w}$ $\frac{n_w}{t_w} \geq 72\varepsilon$

where h_w is the web height, t_w is the web thickness and $\varepsilon = \sqrt{235/f_y}$ (with f_y in MPa).

$$
V_{b, Rd} = \chi_w \frac{\left(f_y / \sqrt{3}\right) h_w t_w}{\gamma_{M1}} \qquad \qquad \lambda_w = 0.76 \sqrt{\frac{f_y}{\tau_{cr}}}
$$

Table 5.1: Contribution from the web χ_w to shear buckling resistance

	Rigid end post	Non-rigid end post		
$\lambda_{\rm w}$ < 0.83/ η				
$0.83/\eta \le \lambda_{\rm w} < 1.08$	$0.83/\lambda_{\rm w}$	$0.83/\lambda_{\rm w}$		
$\lambda_{\rm w} \ge 1.08$	$1,37/(0,7+\lambda_{\rm w})$	$0.83/\lambda_{\rm w}$		

Web crippling:

$$
\bar{\lambda}_F = \sqrt{\frac{F_y}{F_{cr}}} = \sqrt{\frac{l_y t_w f_{yw}}{F_{cr}}} \qquad \qquad F_{cr} = 0.9 k_F E \frac{t_w^3}{h_w}
$$
\n
$$
\chi_F = \frac{0.5}{\bar{\lambda}_F} \le 1.0 \qquad \qquad F_{Rd} = \chi_F \frac{l_y t_w f_{yw}}{\gamma_{M1}}
$$

Deflections:

÷

			C14	C16	C18	C ₂₂	C ₂₄	C27	C40
$f_{m,k}$	bending	MPa	14	16	18	22	24	27	40
$f_{t,0,k}$	tens	MPa	8	10	11	13	14	16	24
$f_{t,90,k}$	tens	MPa	0.3	0.3	0.3	0.3	0.4	0.4	0.4
$f_{c,0,k}$	comp	MPa	16	17	18	20	21	22	26
$f_{c,90,k}$	$comp \perp$	MPa	4.3	4.6	4.8	5.1	5.3	5.6	6.3
$f_{v,k}$	shear	MPa	1.7	1.8	2.0	2.4	2.5	2.8	3.8
$E_{0,mean}$	tens mod	GPa	$\overline{7}$	8	9	10	11	12	14
$E_{0,05}$	tens mod	GPa	4.7	5.4	6	6.7	7.4	8	9.4
E 90, mean	tens mod	GPa	0.23	0.27	0.3	0.33	0.37	0.4	0.47
G_{mean}	shear mod	GPa	0.44	0.50	0.56	0.63	0.69	0.75	0.88
$\rho_{\scriptscriptstyle{k}}$	density	kg/m^3	290	310	320	340	350	370	420
ρ_{mean}	density	kg/m ³	350	370	380	410	420	450	500

3D3 – Structural Materials and Design – Timber Datasheet

Table 11.2 Selected strength classes - characteristic values according to EN 338 [11.3]-**Coniferous Species and Poplar (Table 1)**

Table $3.1.7$ Values of k_{mod}

Selected Modification Factors for Service Class and Duration of Load [11.2]

[11.2] DD ENV 1995-1-1 :1994 Eurocode 5: Design of timber structures – Part 1.1 General rules and rules for buildings [11.3] BS EN 338:1995 Structural Timber – Strength classes

Flexure - Design bending strength $f_{m,d} = k_{mod} k_h k_{crit} k_{ls} f_{m,k} / \gamma_m$

Shear – Design shear stress $f_{v,d} = k_{mod} k_{ls} f_{v,k} / \gamma_m$

Bearing – Design bearing stress $f_{c,90,d} = k_{ls} k_{c,90} k_{mod} f_{c,90,k} / \gamma_m$ *Stability – Relative slenderness for bending* $\lambda_{rel,m} = \sqrt{f_{m,k}/\sigma_{m,crit}}$

"For beams with an initial lateral deviation from straightness within the limits defined in chapter 7, k_{crit} may be determined from $(5.2.2$ c-e)"

$$
\begin{array}{c|cc}\n & 1 & \text{for} & \lambda_{rel,m} \leq 0.75 & (5.2.2c) \\
\hline\n1.56.0.75 & 0.75 & 0.75 & (5.2.2c)\n\end{array}
$$

$$
k_{crit} = \begin{cases} 1.56 - 0.75\lambda_{rel,m} & \text{for } 0.75 < \lambda_{rel,m} \le 1.4\\ 1/3^2 & \text{for } 1.4 < \lambda_{rel,m} \end{cases} \tag{5.2.2d}
$$

$$
\left\{\n\begin{array}{c}\n1/\lambda_{\text{rel},m}^2\n\end{array}\n\right.\n\text{for}\n\quad\n1.4 < \lambda_{\text{rel},m}\n\tag{5.2.2e}
$$

Extract from [11.2] - *kcrit*

Joints

For bolts and for nails *with* predrilled holes, the characteristic embedding strength $f_{h,0,k}$ is: $f_{h,0,k} = 0.082(1 - 0.01d)\rho_k$ N/mm²

For bolts up to 30 mm diameter at an angle α to the grain:

for hardwood $k_{90} = 0.90 + 0.015d$ for softwood $k_{90} = 1.35 + 0.015d$ *k f* $f_{h,\alpha,k} = \frac{J_{h,0,k}}{2}$ $h, \alpha, k = \frac{k_{90} \sin^2 \alpha + \cos^2 \alpha}{k_{90} \sin^2 \alpha + \cos^2 \alpha}$ for hardwood k_{90} 90 , ,0 $k = \frac{k_{90} \sin^2 \alpha + k_{90} \sin^2 \alpha +$ = α , κ α $\sin^2 \alpha$ + $\cos^2 \alpha$

Design yield moment for round steel bolts: $M_{y,d} = (0.8 f_{u,k} d^3) / (6 \gamma_m)$ Design embedding strength e.g. for material 1: $f_{h,1,d} = (k_{mod,1} f_{h,1,k}) / \gamma_m$

Design load-carrying capacities for fasteners in single shear

$$
\int f_{h,1,d} t_1 d
$$
\n(6.2.1a)

$$
\begin{bmatrix} \frac{\partial n_{1,1,d} - 1}{\partial t} & \cdots & \cdots & \cdots & \cdots \\ f_{h,1,d} & t_2 & d\beta & & \cdots & \cdots & \cdots \end{bmatrix} (6.2.1b)
$$

$$
\frac{f_{h,1,d} t_1 d}{1+\beta} \left[\sqrt{\beta + 2\beta^2 \left[1 + \frac{t_2}{t_1} + \left(\frac{t_2}{t_1} \right)^2 \right] + \beta^3 \left(\frac{t_2}{t_1} \right)^2} - \beta \left(1 + \frac{t_2}{t_1} \right) \right] \tag{6.2.1c}
$$

$$
R_d = \min. \left\{ 1.1 \frac{f_{h,1,d} t_1 d}{2 + \beta} \left[\sqrt{2\beta (1 + \beta) + \frac{4\beta (2 + \beta) M_{y,d}}{f_{h,1,d} d t_1^2}} - \beta \right] \right\}
$$
(6.2.1d)

$$
\left| 1.1 \frac{f_{h,1,d} t_2 d}{1 + 2\beta} \right| \sqrt{2\beta^2 (1 + \beta) + \frac{4\beta (1 + 2\beta) M_{y,d}}{f_{h,1,d} d t_2^2}} - \beta \right| \tag{6.2.1e}
$$

$$
\left[1.1\sqrt{\frac{2\beta}{1+\beta}}\sqrt{2M_{y,d}f_{h,1,d}d}\right]
$$
\n(6.2.1f)

Extract from [11.2] – Timber-to-timber and panel-to-timber joints

3D3 – Oct 2010

3D3 – Structural Materials and Design – Masonry Datasheet

Bearing or crushing resistance per unit length

$$
P_b = \frac{f_k t}{\gamma_m}
$$

Buckling resistance per unit length

$$
P_b = \frac{\beta f_k t}{\gamma_m}
$$

*Graph for capacity reduction factor*β

Flexural resistance per unit length

m γ $M = \frac{f_{kx} Z}{f_{kx}}$

3D3 – Structural Materials and Design – Glass Datasheet

Explicit relationship between the flaw opening stress history and the initial flaw size:

$$
\int_0^{t_f} \sigma^n(t)dt \approx \frac{2}{(n-2)v_0 K_{IC}^{-n} \left(Y \sqrt{\pi}\right)^n a_i^{(n-2)/2}}
$$

Idealised v–K relationship:

2-parameter Weibull distribution:

$$
P_f = 1 - \exp\left[-kA\left(\sigma_f - f_{rk}\right)^m\right]
$$

Stressed surface area factor (uniform stress): Load duration factor (constant stress history):

$$
\frac{\sigma_f}{\sigma_{A0}} = \left(\frac{A_0}{A_f}\right)^{1/m} = k_A
$$

Laminated glass equivalent thickness for bending deflection:

$$
h_{eq,\delta} = \sqrt[3]{\left(1-\varpi\right)\sum_{i}h_i^3 + \varpi\left(\sum_{i}h_i\right)^3}
$$

Laminated glass equivalent thickness for bending stress:

$$
h_{eq,\,\sigma} = \sqrt{\frac{\left(h_{eq,\,\delta}\right)^3}{\left(h_i + 2\varpi h_{m,i}\right)}}
$$

$$
\frac{\sigma_f}{\sigma_{t0}} = \left(\frac{t_0}{t_f}\right)^{1/n} = k_{\text{mod}}
$$

3D3 – Oct 2013

G(t) of PVB and SGP interlayers:

Glass design strength:

$$
f_{gd} = \frac{k_{\text{mod}} k_A f_{gk}}{\gamma_{mA}} + \frac{f_{rk}}{\gamma_{mV}}
$$

Stress-history (load duration) interaction equation:

$$
\frac{\sigma_{1,S}}{f_{gd,S}} + \frac{\sigma_{1,M}}{f_{gd,M}} + \frac{\sigma_{1,L}}{f_{gd,L}} \le 1
$$

Empirical stress concentration for bolted connections:

$$
K_t = 1.5 + 1.25 \left(\frac{H}{d} - 1\right) - 0.0675 \left(\frac{H}{d} - 1\right)^2
$$

where

$$
K_t = \frac{\sigma_{\text{max}}(H - d)t}{P}
$$

3D3 – Structural Materials and Design – Concrete Datasheet (pg 1 of 2)

highly stressed $\rho = 1.5\%$ and lightly stressed $\rho = 0.5\%$ (slabs are normally assumed to be lightly stressed) *Table 7.4N, NA.5 [1.2]

Table 1.2 Minimum member sizes and cover (to main reinforcement) for initial design of continuous members

Member	Fire resistance	Minimum dimension, mm		
		4 hours	2 hours	1 hour
Columns fully exposed	width	450	300	200
to fire				
Beams	width	240	200	200
	cover	70	50	45
Slabs with plain soffit	thickness	170	125	100
	cover	45	35	35

Extracts from Table 4.1 [1.1]

Fig 1.1 Interaction diagram from [1.3]

[1.1] Manual for the design of reinforced concrete building structures to EC2, IStructE, ICE, March 2000 - FM 507 [1.2] Eurocode 2: Design of concrete structures, EN 1992-1-1:2004, UK National Annex –NA to BS EN 1992-1-1:2004 [1.3] Structural design. Extracts from British Standards for Students of Structural design. PP7312:2002, BSi

Flexure

Under-reinforced – singly reinforced *s ys* $M_u = \frac{A_s f_y d(1 - 0.5x / d)}{u}$ γ $=\frac{A_s f_y d(1 - 0.5x / d)}{2}$ *bdf* $A_{s}f$ *d x* $S^{0.0}$ *cu* $c^A s J y$ $\gamma_s 0.6$ $=\frac{\gamma}{\sqrt{2}}$

if $x/d = 0.5$ $M_u = 0.225 f_{cu} b d^2 / \gamma_c$

Balanced section

$$
\rho_b = \frac{A_s}{bd} = \frac{\gamma_s 0.6 f_{cu}}{\gamma_c f_y} \cdot \frac{\varepsilon_{cu}}{\varepsilon_y + \varepsilon_{cu}}
$$

Shear

Without internal stirrups

 $V_{Rd,c} = \left[\frac{0.16}{\mu}k(100\rho_1 f_{ck})^{1/3}\right]b_w d \ge (0.035k^{3/2}f_{ck}^{1/2})b_w d$ *c* $f_{Rd,c} = \left[\frac{0.18}{\gamma_c} k (100 \rho_1 f_{ck})^{1/3}\right] b_w d \ge (0.035 k^{3/2} f_{ck}^{1/2})$ where: f_{ck} is the characteristic concrete compressive cylinder strength (MPa). $k = 1 + \sqrt{200/d} \le 2.0$ (*d* in mm) $\rho_1 = A_s/b_w d \leq 0.02$

With internal stirrups

- Concrete resistance $V_{Rd \text{ max}} = f_c \frac{1}{\text{max}} (b_w 0.9d) / (\cot \theta + \tan \theta)$ where: $f_{c,\text{max}} = 0.6(1 - f_{ck} / 250)f_{cd}$

- Shear stirrup resistance $V_{Rd,s} = A_{sw} f_v(0.9d)(\cot\theta)/(s\gamma_s)$

Columns – axial loading only

$$
\sigma_u = 0.6 \frac{f_{cu}}{\gamma_c} + \rho_c \frac{f_y}{\gamma_s}
$$

Standard steel diameters (in mm)

6, 8, 10, 12, 16, 20, 25, 32 and 40

3D3 Structural Materials and Design 2021-2022 – List of numerical answers

- 1) b) i) 13.9 mm
	- ii) 17.0 mm
- 2) a) i) approximately 290 mm
	- ii) 12 mm bars @200 mm c/c or similar
	- iii) no reinforcement required
	- b) walls are adequate
- 3) a) i) section not adequate for bending
	- ii) section adequate for shear
	- b i) 10.6 kN
- 4) a) 286 kN
	- b) 529 kNm
	- c) no shear stiffening required