Version JJO/5

EGT2
ENGINEERING TRIPOS PART IIA

Tuesday 30 April 2024 14.00 to 15.40

Module 3D3

STRUCTURAL MATERIALS AND DESIGN

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3D3 Structural Materials and Design data sheet (18 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) Explain the general Eurocode approach in designing steel structural elements for instability. In particular:

(i) Explain why the classical solutions, e.g. the Euler buckling load of columns, cannot directly be applied in design. [15%]

(ii) Explain the concept of 'slenderness' in general terms. [15%]

(iii) Discuss the analogies between the design of steel columns for flexural buckling, the design of steel beams for lateral-torsional buckling, and the design of steel beams for web crippling. [20%]

(b) Consider the tree-shaped column shown in Fig. 1. The column is fully fixed at the base (point A). Segment AB consists of a circular hollow section with an outside diameter of 355.6 mm and a thickness of 12.5 mm (Class 1). Segments BC and BD consist of a circular hollow section with an outside diameter of 193.7 mm and a thickness of 8.0 mm (Class 1). The tie member CD is a rectangular hollow section RHS 150×100×5.0. All members are cold-rolled and made of grade S355 steel.

(i) Determine the design capacity of the tie CD. The connections at C and D are identical. The connection at C is detailed in Fig. 2. M20 bolts are used. [20%]

(ii) A computer analysis assuming elastic material behaviour but accounting for second-order geometric effects determines that the column buckles at a load $P = 250$ kN. The buckled shape is shown in Fig. 1 as a dotted line. Determine the design capacity P_d of the column. [30%]

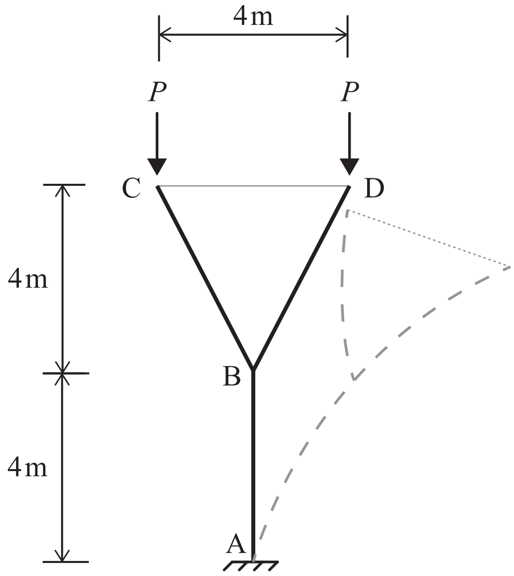

Fig. 1

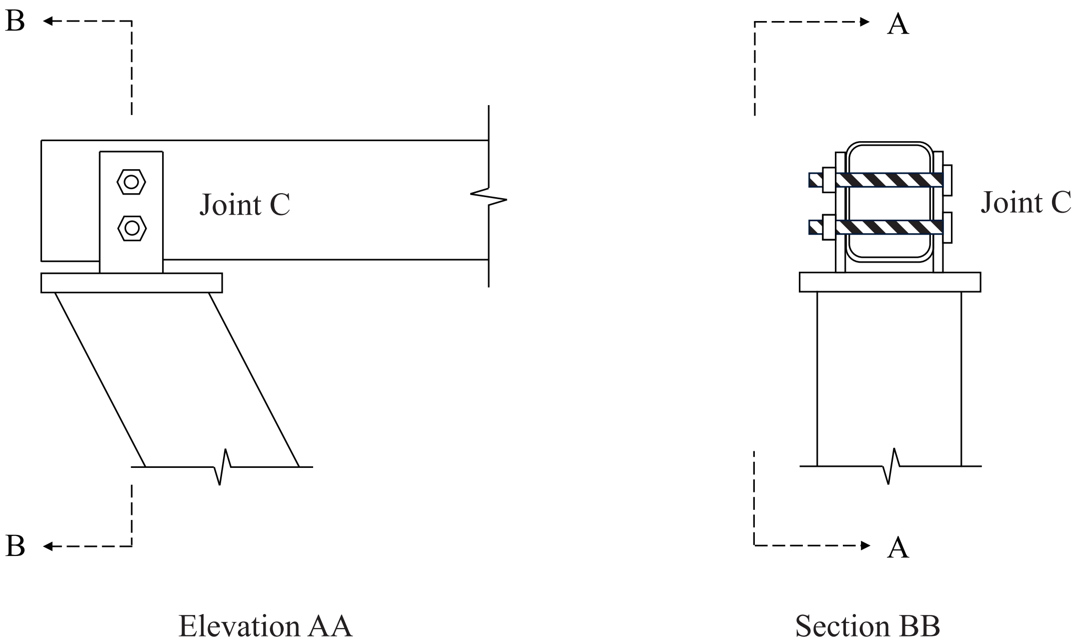

Fig. 2

(TURN OVER

2 (a) A reinforced concrete beam is to be added to the roof of an existing hospital building, to support a new helicopter landing pad, as shown in Fig. 3. The new beam is to be continuous over an existing reinforced concrete column AD and a masonry column BE before cantilevering a total distance of 8 m off the side of the roof. The helicopter and landing pad are represented as a single factored point load of 60 kN. The beam is to be designed in reinforced concrete. Concrete cover requirements for durability are 30 mm, the concrete is to have a characteristic compressive cube strength of 60 MPa and is reinforced with steel with a characteristic yield strength of 500 MPa. The material partial safety factor for concrete $\gamma_c = 1.50$ and for steel $\gamma_s = 1.15$.

(i) Stating your assumptions, determine suitable cross–section dimensions for the beam ABC. [5%]

(ii) Calculate the change in axial load in members AD and BE. [10%]

(iii) Draw the shear force and bending moment diagrams for the beam ABC. [10%]

(iv) Sketch a feasible deflected shape for the beam ABC. [10%]

(v) Design a longitudinal reinforcement layout at the most critical section for bending. [30%]

(vi) Without further calculations, suggest how the shape of the beam might be optimised along its length. [5%]

(b) The masonry column BE is 0.5 m wide, and of an unknown thickness, t. Assuming that the masonry wall carries no load prior to the helipad being installed, that the characteristic compressive strength of the masonry wall is 5 MPa, and the material partial safety factor for masonry is $\gamma_m = 3.0$, determine the thickness t required for the masonry wall. [30%]

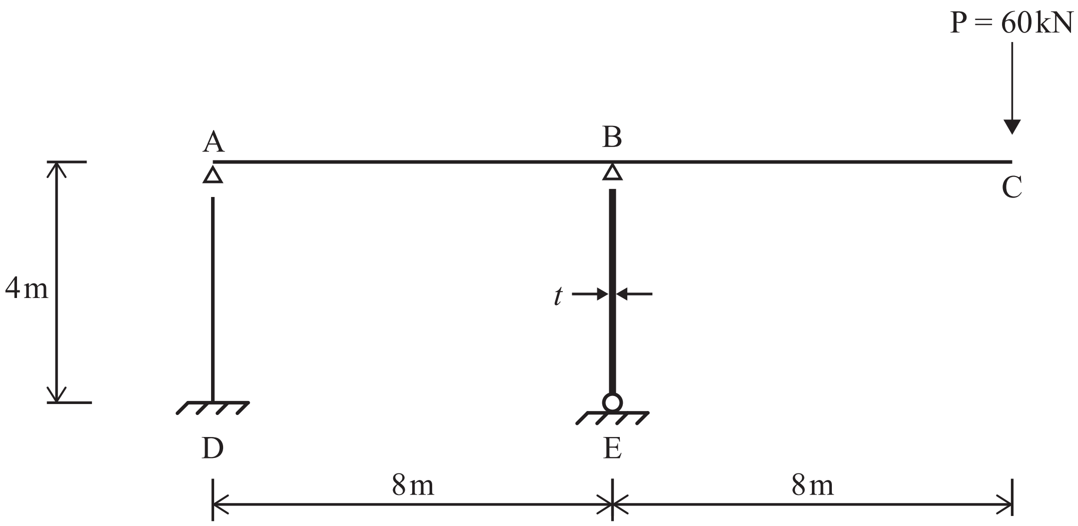

Fig. 3

(TURN OVER

3 An incomplete design of a multi-storey office building is shown in Fig. 4. The pin jointed structural frame is to be in S355 steel with 200 mm thick reinforced concrete flat slabs and a reinforced concrete core. The building is clad in glass from floors 1 to 5. At ground level, public access must be provided across the site, which means that no structural elements may be placed at ground floor level, apart from the core walls. The building has five floors of office space and a roof above. The unfactored vertical load on each office floor is 2 kPa. There is no vertical load on the roof level. The horizontal wind load is 1.5 kPa, and is uniform over the height of the building.

(a) With the aid of sketches, devise a suitable structural scheme for the building, and describe the vertical and horizontal load paths for the whole structure. Describe how your design meets the site constraints. [30%]

(b) Assuming that the steel beams in the floor are fully restrained, determine the lightest possible UB section for the primary floor beams supporting the office areas, when the SLS deflection limit is span/180 and the ULS load combination is (1.35 $\times$ dead load) + (1.5 $\times$ live load). [30%]

(c) Calculate the maximum moment, shear force, and axial force on the concrete core when the building is unoccupied. You may assume a load factor of 1.2 on the horizontal wind load. [30%]

(d) Suggest one way in which the design of the building might be changed or improved to reduce its carbon footprint. [10%]

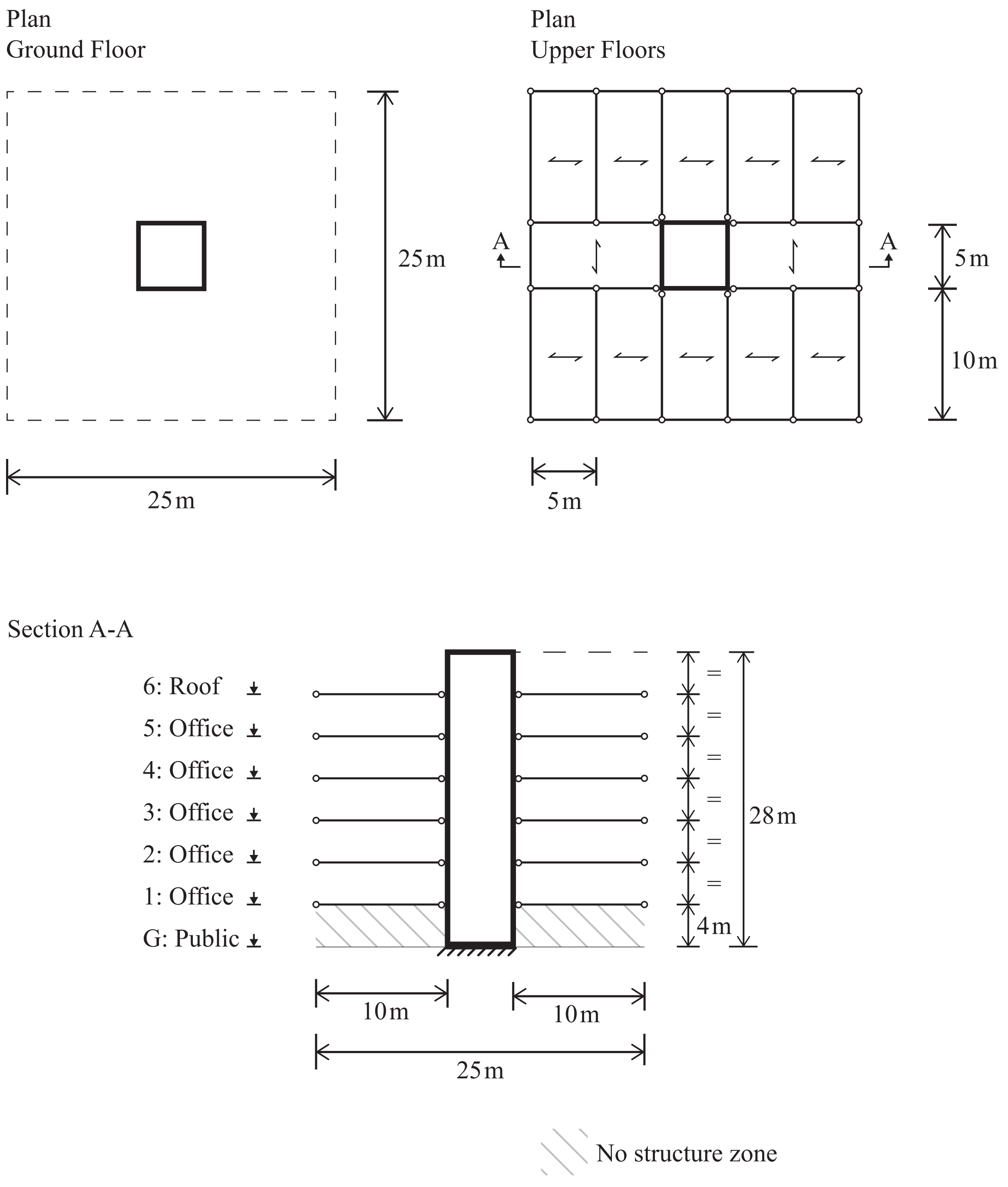

Fig. 4

(TURN OVER

4 (a) A 1 m wide, 4 m span footbridge is to be supported on a pair of simply supported timber beams that each span 4 m. The unfactored imposed load consists of a uniformly distributed load of 4 kPa, and a point load of 4 kN applied at mid-span. The unfactored dead plus superimposed dead load (which includes self-weight, the deck, and balustrades) is 0.75 kPa. The beams are to be made in C24 timber with breadth of 100 mm. You may assume Service Class 2, crack factor k_{cr} = 0.67, depth factor k_h = 1.0, and the load sharing factor k_{ls} = 1.0. The material partial safety factor for solid timber is 1.3. The bridge is to have a service life of 50 years. ULS load factors are 1.35 on dead load and 1.5 on live load.

(i) Calculate the ULS loading on each beam. [5%]

(ii) Draw ULS shear force and bending moment diagrams for the beam. [10%]

(iii) Determine the depth of the beam required to carry ULS shear and bending, at suitable critical locations along its length. [30%]

(iv) What other parameters should be considered in sizing the beams? [10%]

(b) The designer wishes to express the pinned end connections of the bridge as shown in Fig 5. The connection between steel plate and timber is to be made with bolts of diameter $d = 12$ mm. Each bolt has a characteristic strength of 5 kN perpendicular to the grain and 7 kN parallel to the grain. The loaded end distance may be taken as $8d$, while all other minimum bolt spacings are $4d$.

(i) Calculate the design strength per bolt. [10%]

(ii) Design a suitable arrangement of bolts in the connection. You may assume the final beam design has a total depth of 300 mm, as shown in Fig. 5. As a first approximation, the eccentricity moment can be carried as a vertical push/pull. [35%]

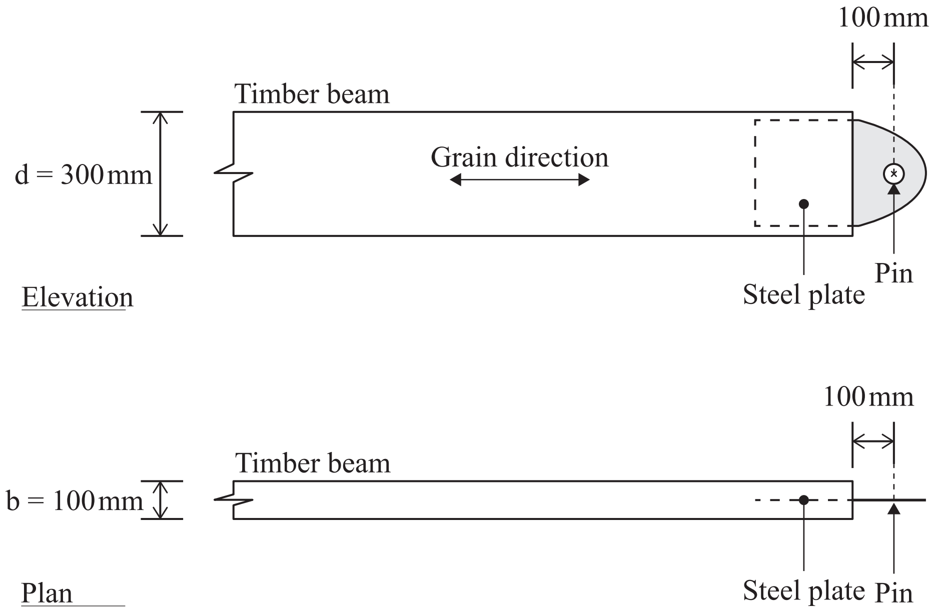

Fig. 5

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University of Cambridge
Department of Engineering

Engineering Tripos Part IIA

Module 3D3
Structural Materials & Design

Datasheets

Michaelmas 2023

THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$\Phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u} e^{-\frac{x^2}{2}} dx \quad \text{FOR} \quad 0{\cdot}00 \leq u \leq 4{\cdot}99.$$

u	·00	·01	·02	·03	·04	·05	·06	·07	·08	·09
·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7703	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
1·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
1·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·90147
1·3	·90320	·90490	·90658	·90824	·90988	·91149	·91309	·91466	·91621	·91774
1·4	·91924	·92073	·92220	·92364	·92507	·92647	·92785	·92922	·93056	·93189
1·5	·93319	·93448	·93574	·93699	·93822	·93943	·94062	·94179	·94295	·94408
1·6	·94520	·94630	·94738	·94845	·94950	·95053	·95154	·95254	·95352	·95449
1·7	·95543	·95637	·95728	·95818	·95907	·95994	·96080	·96164	·96246	·96327
1·8	·96407	·96485	·96562	·96638	·96712	·96784	·96856	·96926	·96995	·97062
1·9	·97128	·97193	·97257	·97320	·97381	·97441	·97500	·97558	·97615	·97670
2·0	·97725	·97778	·97831	·97882	·97932	·97982	·98030	·98077	·98124	·98169
2·1	·98214	·98257	·98300	·98341	·98382	·98422	·98461	·98500	·98537	·98574
2·2	·98610	·98645	·98679	·98713	·98745	·98778	·98809	·98840	·98870	·98899
2·3	·98928	·98956	·98983	·9^{2}0097	·9^{2}0358	·9^{2}0613	·9^{2}0863	·9^{2}1106	·9^{2}1344	·9^{2}1576
2·4	·9^{2}1802	·9^{2}2024	·9^{2}2240	·9^{2}2451	·9^{2}2656	·9^{2}2857	·9^{2}3053	·9^{2}3244	·9^{2}3431	·9^{2}3613
2·5	·9^{2}3790	·9^{2}3963	·9^{2}4132	·9^{2}4297	·9^{2}4457	·9^{2}4614	·9^{2}4766	·9^{2}4915	·9^{2}5060	·9^{2}5201
2·6	·9^{2}5339	·9^{2}5473	·9^{2}5604	·9^{2}5731	·9^{2}5855	·9^{2}5975	·9^{2}6093	·9^{2}6207	·9^{2}6319	·9^{2}6427
2·7	·9^{2}6533	·9^{2}6636	·9^{2}6736	·9^{2}6833	·9^{2}6928	·9^{2}7020	·9^{2}7110	·9^{2}7197	·9^{2}7282	·9^{2}7365
2·8	·9^{2}7445	·9^{2}7523	·9^{2}7599	·9^{2}7673	·9^{2}7744	·9^{2}7814	·9^{2}7882	·9^{2}7948	·9^{2}8012	·9^{2}8074
2·9	·9^{2}8134	·9^{2}8193	·9^{2}8250	·9^{2}8305	·9^{2}8359	·9^{2}8411	·9^{2}8462	·9^{2}8511	·9^{2}8559	·9^{2}8605
3·0	·9^{2}8650	·9^{2}8694	·9^{2}8736	·9^{2}8777	·9^{2}8817	·9^{2}8856	·9^{2}8893	·9^{2}8930	·9^{2}8965	·9^{2}8999
3·1	·9^{3}0324	·9^{3}0646	·9^{3}0957	·9^{3}1260	·9^{3}1553	·9^{3}1836	·9^{3}2112	·9^{3}2378	·9^{3}2636	·9^{3}2886
3·2	·9^{3}3129	·9^{3}3363	·9^{3}3590	·9^{3}3810	·9^{3}4024	·9^{3}4230	·9^{3}4429	·9^{3}4623	·9^{3}4810	·9^{3}4991
3·3	·9^{3}5166	·9^{3}5335	·9^{3}5499	·9^{3}5658	·9^{3}5811	·9^{3}5959	·9^{3}6103	·9^{3}6242	·9^{3}6376	·9^{3}6505
3·4	·9^{3}6631	·9^{3}6752	·9^{3}6869	·9^{3}6982	·9^{3}7091	·9^{3}7197	·9^{3}7299	·9^{3}7398	·9^{3}7493	·9^{3}7585
3·5	·9^{3}7674	·9^{3}7759	·9^{3}7842	·9^{3}7922	·9^{3}7999	·9^{3}8074	·9^{3}8146	·9^{3}8215	·9^{3}8282	·9^{3}8347
3·6	·9^{3}8409	·9^{3}8469	·9^{3}8527	·9^{3}8583	·9^{3}8637	·9^{3}8689	·9^{3}8739	·9^{3}8787	·9^{3}8834	·9^{3}8879
3·7	·9^{3}8922	·9^{3}8964	·9^{4}0039	·9^{4}0426	·9^{4}0799	·9^{4}1158	·9^{4}1504	·9^{4}1838	·9^{4}2159	·9^{4}2468
3·8	·9^{4}2765	·9^{4}3052	·9^{4}3327	·9^{4}3593	·9^{4}3848	·9^{4}4094	·9^{4}4331	·9^{4}4558	·9^{4}4777	·9^{4}4988
3·9	·9^{4}5190	·9^{4}5385	·9^{4}5573	·9^{4}5753	·9^{4}5926	·9^{4}6092	·9^{4}6253	·9^{4}6406	·9^{4}6554	·9^{4}6696
4·0	·9^{4}6833	·9^{4}6964	·9^{4}7090	·9^{4}7211	·9^{4}7327	·9^{4}7439	·9^{4}7546	·9^{4}7649	·9^{4}7748	·9^{4}7843
4·1	·9^{4}7934	·9^{4}8022	·9^{4}8106	·9^{4}8186	·9^{4}8263	·9^{4}8338	·9^{4}8409	·9^{4}8477	·9^{4}8542	·9^{4}8605
4·2	·9^{4}8665	·9^{4}8723	·9^{4}8778	·9^{4}8832	·9^{4}8882	·9^{4}8931	·9^{4}8978	·9^{5}0226	·9^{5}0655	·9^{5}1066
4·3	·9^{5}1460	·9^{5}1837	·9^{5}2199	·9^{5}2545	·9^{5}2876	·9^{5}3193	·9^{5}3497	·9^{5}3788	·9^{5}4066	·9^{5}4332
4·4	·9^{5}4587	·9^{5}4831	·9^{5}5065	·9^{5}5288	·9^{5}5502	·9^{5}5706	·9^{5}5902	·9^{5}6089	·9^{5}6268	·9^{5}6439
4·5	·9^{5}6602	·9^{5}6759	·9^{5}6908	·9^{5}7051	·9^{5}7187	·9^{5}7318	·9^{5}7442	·9^{5}7561	·9^{5}7675	·9^{5}7784
4·6	·9^{5}7888	·9^{5}7987	·9^{5}8081	·9^{5}8172	·9^{5}8258	·9^{5}8340	·9^{5}8419	·9^{5}8494	·9^{5}8566	·9^{5}8634
4·7	·9^{5}8699	·9^{5}8761	·9^{5}8821	·9^{5}8877	·9^{5}8931	·9^{5}8983	·9^{6}0320	·9^{6}0789	·9^{6}1235	·9^{6}1661
4·8	·9^{6}2067	·9^{6}2453	·9^{6}2822	·9^{6}3173	·9^{6}3508	·9^{6}3827	·9^{6}4131	·9^{6}4420	·9^{6}4696	·9^{6}4958
4·9	·9^{6}5208	·9^{6}5446	·9^{6}5673	·9^{6}5889	·9^{6}6094	·9^{6}6289	·9^{6}6475	·9^{6}6652	·9^{6}6821	·9^{6}6981

Example: $\Phi(3{\cdot}57) = {\cdot}9^{3}8215 = 0{\cdot}9998215.$

Steel Data Sheet

(EN 1993-1-1)

Table 3.1: Nominal values of yield strength f_y and ultimate tensile strength f_u for hot rolled structural steel

Standard and steel grade	Nominal thickness of the element t [mm]			
	$t \leq 40$ mm		40 mm < $t \leq 80$ mm	
	f_y [N/mm^2]	f_u [N/mm^2]	f_y [N/mm^2]	f_u [N/mm^2]
EN 10025-2				
S 235	235	360	215	360
S 275	275	430	255	410
S 355	355	AC2 490 AC2	335	470
S 450	440	550	410	550

Tension members

Yielding of the gross cross-section A_g:

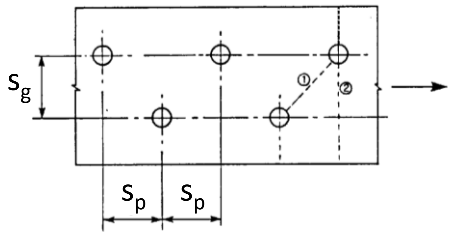
$$N_{pl,Rd} = \frac{A_g f_y}{\gamma_{M0}}$$

Fracture of the net cross-section A_n:

$$N_{u,Rd} = \frac{0.9 A_n f_u}{\gamma_{M2}}$$

Staggered bolt holes:

$$A_n = A_g - n_b d_0 t + \sum_{staggers} \frac{s_p^2 t}{4 s_g}$$



d_0 = bolt hole diameter
n_b = number of bolt lines across the member

Bolt size	**12**	**14**	**16**	**18**	**20**	**22**	**24**	**27 to 36**
Clearance (mm)	1	1	2	2	2	2	2	3

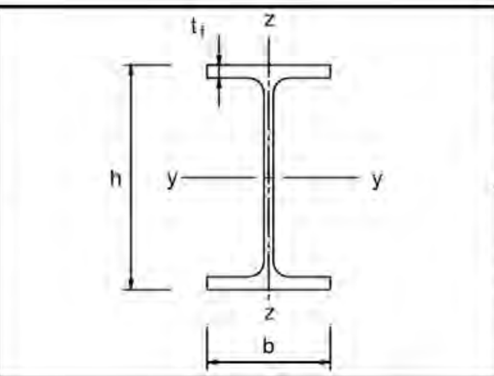
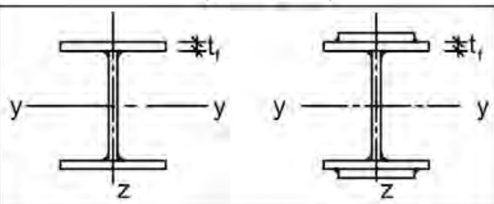
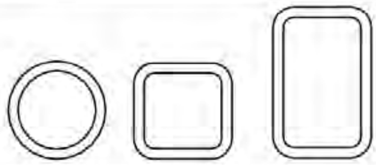
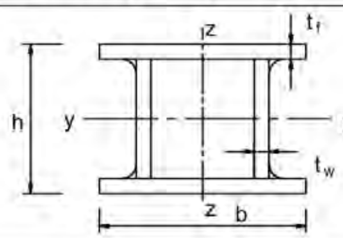
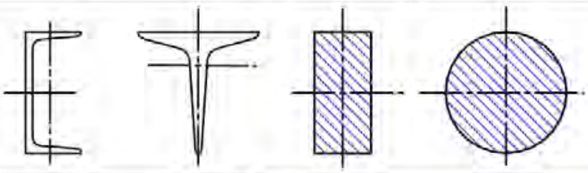
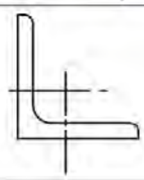
Reduction factor for shear lag in eccentrically connected angles:

Pitch	p_1	$\leq 2{,}5\ d_o$	$\geq 5{,}0\ d_o$
2 bolts	β_2	0,4	0,7
3 bolts or more	β_3	0,5	0,7

Column buckling

BS EN 1993-1-1:2005
EN 1993-1-1:2005 (E)

Table 6.2: Selection of buckling curve for a cross-section

Cross section		Limits		Buckling about axis	Buckling curve S 235 S 275 S 355 S 420	Buckling curve S 460
Rolled sections		$h/b > 1,2$	$t_f \leq 40$ mm	y – y z – z	a b	a_0 a_0
		$h/b > 1,2$	40 mm < $t_f \leq 100$	y – y z – z	b c	a a
		$h/b \leq 1,2$	$t_f \leq 100$ mm	y – y z – z	b c	a a
		$h/b \leq 1,2$	$t_f > 100$ mm	y – y z – z	d d	c c
Welded I-sections			$t_f \leq 40$ mm	y – y z – z	b c	b c
			$t_f > 40$ mm	y – y z – z	c d	c d
Hollow sections			hot finished	any	a	a_0
			cold formed	any	c	c
Welded box sections			generally (except as below)	any	b	b
			thick welds: $a > 0,5t_f$ $b/t_f < 30$ $h/t_w < 30$	any	c	c
U-, T- and solid sections				any	c	c
L-sections				any	b	b

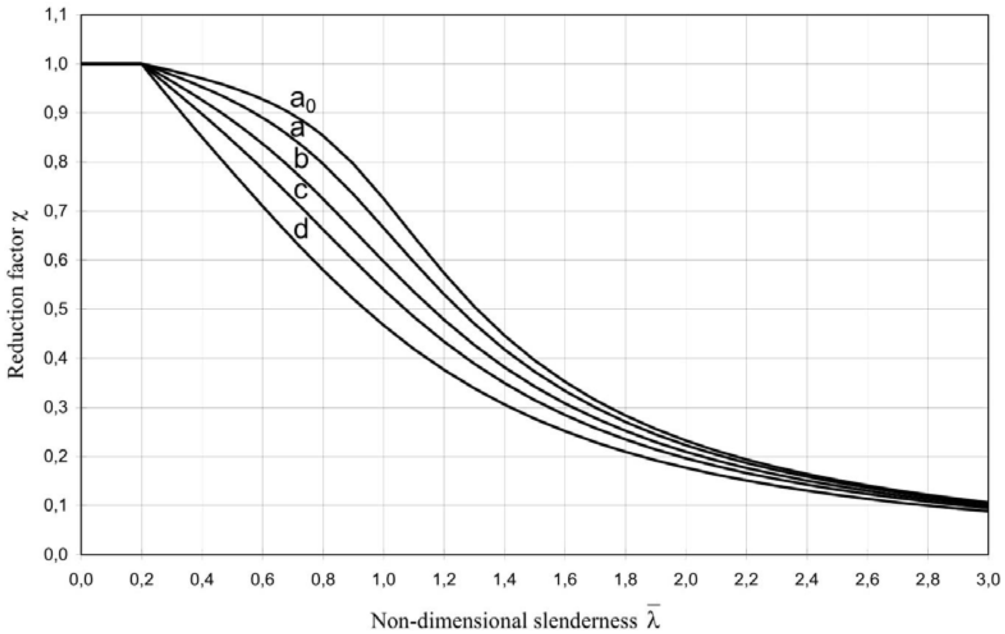


Figure 6.4: Buckling curves

6.3.1.2 Buckling curves

(1) For axial compression in members the value of χ for the appropriate non-dimensional slenderness $\bar{\lambda}$ should be determined from the relevant buckling curve according to:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} \quad \text{but } \chi \leq 1{,}0 \tag{6.49}$$

where $\Phi = 0{,}5\left[1 + \alpha\left(\bar{\lambda} - 0{,}2\right) + \bar{\lambda}^2\right]$

$$\bar{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}} \quad \text{for Class 1, 2 and 3 cross-sections}$$

$$\bar{\lambda} = \sqrt{\frac{A_{eff} f_y}{N_{cr}}} \quad \text{for Class 4 cross-sections}$$

α is an imperfection factor

N_{cr} is the elastic critical force for the relevant buckling mode based on the gross cross sectional properties.

(2) The imperfection factor α corresponding to the appropriate buckling curve should be obtained from Table 6.1 and Table 6.2.

Table 6.1: Imperfection factors for buckling curves

Buckling curve	a_0	a	b	c	d
Imperfection factor α	0,13	0,21	0,34	0,49	0,76

Local buckling

$$\sigma_{cr} = K \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$$

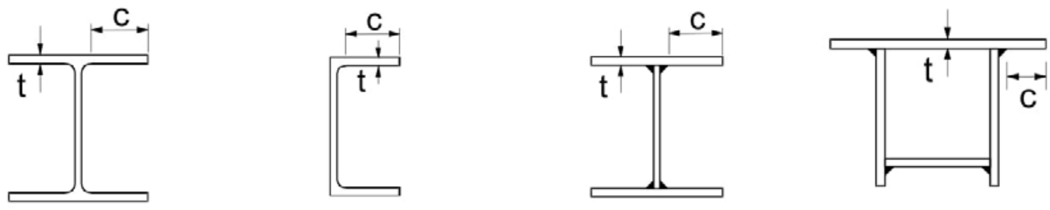
where *b* is the width of the plate and *t* is its thickness.

For plates in uniform longitudinal compression:

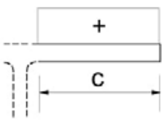
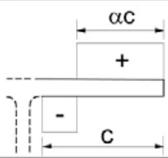
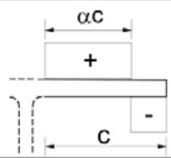
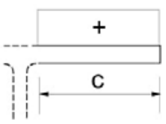
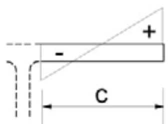
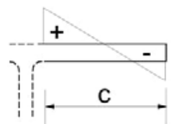
$K = 4$ for internal elements.

$K = 0.43$ for outstand elements.

Outstand flanges

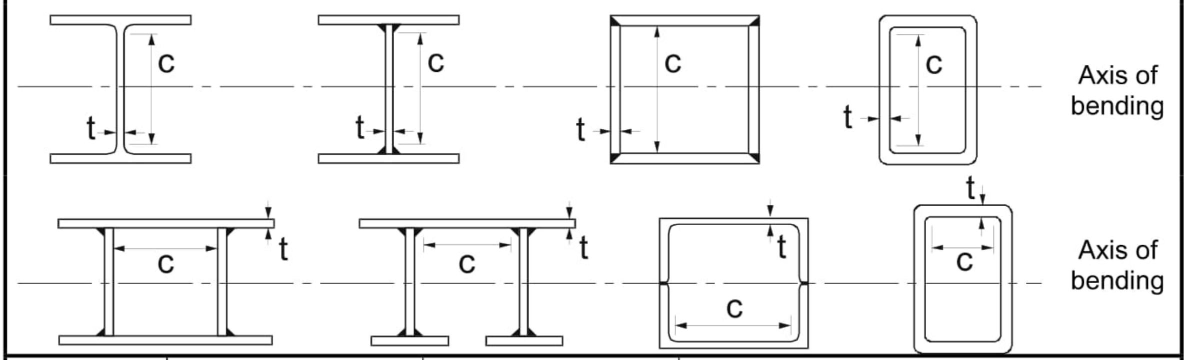


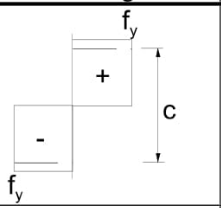
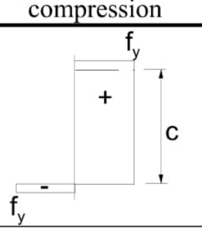
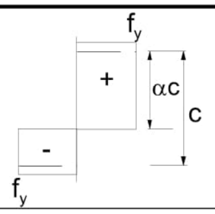
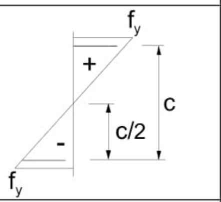
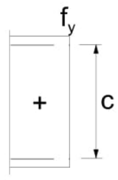
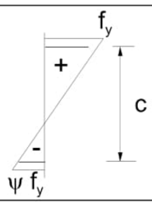
Rolled sections   Welded sections

Class	Part subject to compression	Part subject to bending and compression: Tip in compression	Part subject to bending and compression: Tip in tension
Stress distribution in parts (compression positive)			
1	$c/t \leq 9\varepsilon$	$c/t \leq \frac{9\varepsilon}{\alpha}$	$c/t \leq \frac{9\varepsilon}{\alpha\sqrt{\alpha}}$
2	$c/t \leq 10\varepsilon$	$c/t \leq \frac{10\varepsilon}{\alpha}$	$c/t \leq \frac{10\varepsilon}{\alpha\sqrt{\alpha}}$
Stress distribution in parts (compression positive)			
3	$c/t \leq 14\varepsilon$	$c/t \leq 21\varepsilon\sqrt{k_\sigma}$ For k_σ see EN 1993-1-5	

$\varepsilon = \sqrt{235/f_y}$						
	f_y	235	275	355	420	460
	ε	1,00	0,92	0,81	0,75	0,71

Internal compression parts



Class	Part subject to bending	Part subject to compression	Part subject to bending and compression
Stress distribution in parts (compression positive)			
1	$c/t \le 72\varepsilon$	$c/t \le 33\varepsilon$	when $\alpha > 0{,}5$: $c/t \le \dfrac{396\varepsilon}{13\alpha - 1}$ when $\alpha \le 0{,}5$: $c/t \le \dfrac{36\varepsilon}{\alpha}$
2	$c/t \le 83\varepsilon$	$c/t \le 38\varepsilon$	when $\alpha > 0{,}5$: $c/t \le \dfrac{456\varepsilon}{13\alpha - 1}$ when $\alpha \le 0{,}5$: $c/t \le \dfrac{41{,}5\varepsilon}{\alpha}$
Stress distribution in parts (compression positive)			
3	$c/t \le 124\varepsilon$	$c/t \le 42\varepsilon$	when $\psi > -1$: $c/t \le \dfrac{42\varepsilon}{0{,}67 + 0{,}33\psi}$ when $\psi \le -1^{*)}$: $c/t \le 62\varepsilon(1-\psi)\sqrt{(-\psi)}$

$\varepsilon = \sqrt{235/f_y}$	f_y	235	275	355	420	460
	ε	1,00	0,92	0,81	0,75	0,71

*) $\psi \le -1$ applies where either the compression stress $\sigma \le f_y$ or the tensile strain $\varepsilon_y > f_y/E$

Beams

Elastic lateral-torsional buckling moment of a beam with doubly symmetric cross-section:

$$M_{cr,0} = \frac{\pi^2 EI_z}{L_{cr}^2}\left[\frac{I_w}{I_z} + \frac{L_{cr}^2 GI_T}{\pi^2 EI_z}\right]^{0.5}$$

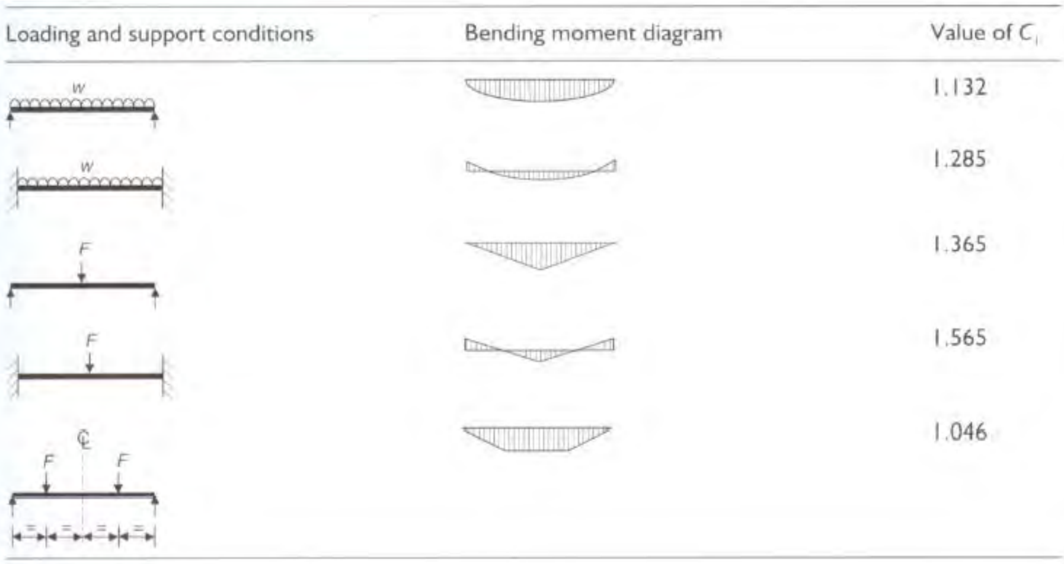
where:

I_T = torsional constant
I_w = warping constant (= $d^2I_{yy}/4$ for I-beams, with d the distance between the centerlines of the flanges)
I_z = second moment of area about the minor axis
G = shear modulus
L_{cr} = unrestrained length for lateral-torsional buckling

In the case of non-uniform bending:

$$M_{cr} = C_1\, M_{cr,0}$$

Loading and support conditions	Bending moment diagram	Value of C_1
	$\psi = +1$	1.000
	$\psi = +0.75$	1.141
	$\psi = +0.5$	1.323
	$\psi = +0.25$	1.563
	$\psi = 0$	1.879
	$\psi = -0.25$	2.281
	$\psi = -0.5$	2.704
	$\psi = -0.75$	2.729
	$\psi = -1$	2.752



Loading and support conditions	Bending moment diagram	Value of C_1
		1.132
		1.285
		1.365
		1.565
		1.046

(EN 1993-1-1)

6.3.2.2 Lateral torsional buckling curves – General case

(1) Unless otherwise specified, see 6.3.2.3, for bending members of constant cross-section, the value of χ_{LT} for the appropriate non-dimensional slenderness $\bar{\lambda}_{LT}$, should be determined from:

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \quad \text{but } \chi_{LT} \leq 1{,}0 \tag{6.56}$$

where $\Phi_{LT} = 0{,}5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0{,}2\right) + \bar{\lambda}_{LT}^2\right]$

α_{LT} is an imperfection factor

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$$

M_{cr} is the elastic critical moment for lateral-torsional buckling

(2) M_{cr} is based on gross cross sectional properties and takes into account the loading conditions, the real moment distribution and the lateral restraints.

NOTE The imperfection factor α_{LT} corresponding to the appropriate buckling curve may be obtained from the National Annex. The recommended values α_{LT} are given in Table 6.3.

Table 6.3: Recommended values for imperfection factors for lateral torsional buckling curves

Buckling curve	a	b	c	d
Imperfection factor α_{LT}	0,21	0,34	0,49	0,76

The recommendations for buckling curves are given in Table 6.4.

Table 6.4: Recommended values for lateral torsional buckling curves for cross-sections using equation (6.56)

Cross-section	Limits	Buckling curve
Rolled I-sections	h/b ≤ 2 h/b > 2	**a** **b**
Welded I-sections	h/b ≤ 2 h/b > 2	**c** **d**
Other cross-sections	-	**d**

$$M_{b,Rd} = \chi_{LT}\ W_y \frac{f_y}{\gamma_{M1}}$$

<u>Interaction between moment and shear in the cross-section</u>:

$$f_{yr} = (1-\rho) f_y \qquad \rho = \left(\frac{2V_{Ed}}{V_{pl,Rd}} - 1\right)^2 \qquad \text{(for } V_{Ed} > 0.5_{Vpl,Rd}\text{)}$$

$$M_{y,V,Rd} = \left[W_{pl,y} - \frac{\rho {A_W}^2}{4t_w}\right]\frac{f_y}{\gamma_{M0}} \leq M_{y,c,Rd} \qquad \text{where } A_w = h_w t_w$$

<u>Shear</u>

$$V_{pl,Rd} = A_v \frac{(f_y/\sqrt{3})}{\gamma_{M0}}$$

$$A_v = A - 2bt_f + (t_w + 2r)t_f \qquad \text{but} \quad \geq \ h_w t_w$$

where:

b = flange width
t_f = flange thickness
t_w = web thickness
h_w = web height
r = transition radius between web and flange

Shear buckling:

$$\tau_{cr} = K \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

$$K = 5.34 + \frac{4}{(a/b)^2} \qquad \text{if } \ a > b$$

$$K = 5.34 + \frac{4}{(b/a)^2} \qquad \text{if } \ b > a$$

Shear buckling needs to be checked if: $\frac{h_w}{t_w} \geq 72\varepsilon$

where h_w is the web height, t_w is the web thickness and $\varepsilon = \sqrt{235/f_y}$ (with f_y in MPa).

$$V_{b,Rd} = \chi_w \frac{(f_y/\sqrt{3})h_w t_w}{\gamma_{M1}} \qquad \lambda_w = 0.76\sqrt{\frac{f_y}{\tau_{cr}}}$$

Table 5.1: Contribution from the web χ_w to shear buckling resistance

	Rigid end post	Non-rigid end post
$\bar{\lambda}_w < 0{,}83/\eta$	η	η
$0{,}83/\eta \le \bar{\lambda}_w < 1{,}08$	$0{,}83/\bar{\lambda}_w$	$0{,}83/\bar{\lambda}_w$
$\bar{\lambda}_w \ge 1{,}08$	$1{,}37/(0{,}7+\bar{\lambda}_w)$	$0{,}83/\bar{\lambda}_w$

<u>Web crippling</u>:

$$\bar{\lambda}_F = \sqrt{\frac{F_y}{F_{cr}}} = \sqrt{\frac{l_y t_w f_{yw}}{F_{cr}}} \qquad F_{cr} = 0.9 k_F E \frac{t_w^3}{h_w}$$

$$\chi_F = \frac{0.5}{\bar{\lambda}_F} \le 1.0 \qquad F_{Rd} = \chi_F \frac{l_y t_w f_{yw}}{\gamma_{M1}}$$

IOF/ITF: $\ell_y = s_s + 2t_f\left(1+\sqrt{m_1+m_2}\right) \le a$

EOF: $\min \begin{cases} \ell_y = \ell_e + t_f\sqrt{\dfrac{m_1}{2} + \left(\dfrac{\ell_e}{t_f}\right)^2 + m_2} \\ \ell_y = \ell_e + t_f\sqrt{m_1+m_2} \end{cases}$

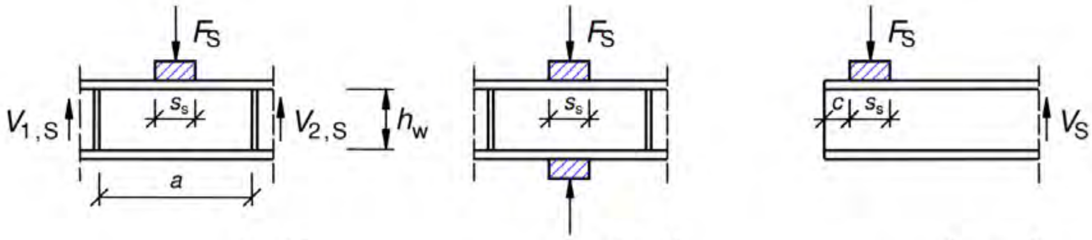
with: $\ell_e = \dfrac{k_F E t_w^2}{2 f_{yw} h_w} \le s_s + c$

$$m_1 = \frac{f_{yf} b_f}{f_{yw} t_w}$$

$$m_2 = 0{,}02\left(\frac{h_w}{t_f}\right)^2 \quad if\ \bar{\lambda}_F > 0{,}5$$

$$m_2 = 0 \quad if\ \bar{\lambda}_F \le 0{,}5$$

(IOF) (ITF) (EOF)



$$k_F = 6 + 2\left(\frac{h_w}{a}\right)^2 \qquad k_F = 3{,}5 + 2\left(\frac{h_w}{a}\right)^2 \qquad k_F = 2 + 6\left(\frac{s_s + c}{h_w}\right) \le 6$$

<u>Deflections:</u>

Vertical deflection	
Cantilevers	Length/180
Beams carrying plaster or other brittle finish	Span/360
Other beams (except purlins and sheeting rails)	Span/200
Purlins and sheeting rails	To suit the characteristics of particular cladding

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			C14	C16	C18	C22	C24	C27	C40
$f_{m,k}$	bending	MPa	14	16	18	22	24	27	40
$f_{t,0,k}$	tens $\parallel$	MPa	8	10	11	13	14	16	24
$f_{t,90,k}$	tens $\perp$	MPa	0.3	0.3	0.3	0.3	0.4	0.4	0.4
$f_{c,0,k}$	comp $\parallel$	MPa	16	17	18	20	21	22	26
$f_{c,90,k}$	comp $\perp$	MPa	4.3	4.6	4.8	5.1	5.3	5.6	6.3
$f_{v,k}$	shear	MPa	1.7	1.8	2.0	2.4	2.5	2.8	3.8
$E_{0,mean}$	tens mod $\parallel$	GPa	7	8	9	10	11	12	14
$E_{0,05}$	tens mod	GPa	4.7	5.4	6	6.7	7.4	8	9.4
$E_{90,mean}$	tens mod $\perp$	GPa	0.23	0.27	0.3	0.33	0.37	0.4	0.47
G_{mean}	shear mod	GPa	0.44	0.50	0.56	0.63	0.69	0.75	0.88
ρ_k	density	kg/m^3	290	310	320	340	350	370	420
ρ_{mean}	density	kg/m^3	350	370	380	410	420	450	500

Table 11.2 Selected strength classes - characteristic values according to EN 338 [11.3]– Coniferous Species and Poplar (Table 1)

Table 3.1.7 Values of k_{mod}

Material/ load-duration class	Service class 1	2	3
Solid and glued laminated timber and plywood			
Permanent	0.60	0.60	0.50
Long-term	0.70	0.70	0.55
Medium-term	0.80	0.80	0.65
Short-term	0.90	0.90	0.70
Instantaneous	1.10	1.10	0.90

Selected Modification Factors for Service Class and Duration of Load [11.2]

[11.2] DD ENV 1995-1-1 :1994 Eurocode 5: Design of timber structures – Part 1.1 General rules and rules for buildings
[11.3] BS EN 338:1995 Structural Timber – Strength classes

Flexure - Design bending strength

$$f_{m,d} = k_{mod} k_h k_{crit} k_{ls} f_{m,k} / \gamma_m$$

Shear – Design shear stress

$$f_{v,d} = k_{mod} k_{ls} f_{v,k} / \gamma_m$$

Bearing – Design bearing stress

$$f_{c,90,d} = k_{ls} k_{c,90} k_{mod} f_{c,90,k} / \gamma_m$$

Stability – Relative slenderness for bending

$\lambda_{rel,m} = \sqrt{f_{m,k} / \sigma_{m,crit}}$

"For beams with an initial lateral deviation from straightness within the limits defined in chapter 7, k_{crit} may be determined from (5.2.2 c-e)"

$$k_{crit} = \begin{cases} 1 & \text{for} \quad \lambda_{rel,m} \le 0.75 & (5.2.2c) \\ 1.56 - 0.75\lambda_{rel,m} & \text{for} \quad 0.75 < \lambda_{rel,m} \le 1.4 & (5.2.2d) \\ 1/\lambda^2_{rel,m} & \text{for} \quad 1.4 < \lambda_{rel,m} & (5.2.2e) \end{cases}$$

Extract from [11.2] - k_{crit}

Joints

For bolts and for nails *with* predrilled holes, the characteristic embedding strength $f_{h,0,k}$ is:

$f_{h,0,k} = 0.082(1 - 0.01d)\rho_k$ N/mm^2

For bolts up to 30 mm diameter at an angle α to the grain:

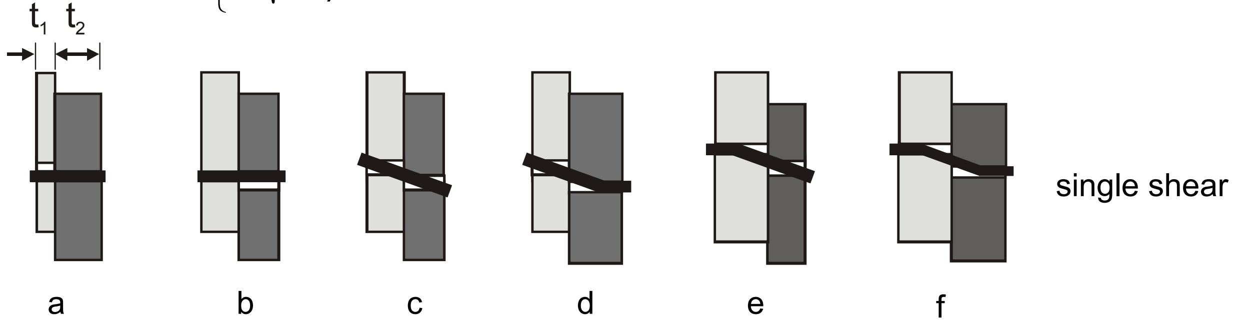
$$f_{h,\alpha,k} = \frac{f_{h,0,k}}{k_{90}\sin^2\alpha + \cos^2\alpha} \qquad \begin{array}{ll} \text{for softwood} & k_{90} = 1.35 + 0.015d \\ \text{for hardwood} & k_{90} = 0.90 + 0.015d \end{array}$$

Design yield moment for round steel bolts: $M_{y,d} = (0.8 f_{u,k} d^3)/(6\gamma_m)$

Design embedding strength e.g. for material 1: $f_{h,1,d} = (k_{mod,1} f_{h,1,k})/\gamma_m$

Design load-carrying capacities for fasteners in single shear

$$R_d = \min.\begin{cases} f_{h,1,d}\, t_1\, d & (6.2.1a) \\ f_{h,1,d}\, t_2\, d\beta & (6.2.1b) \\ \dfrac{f_{h,1,d}\, t_1 d}{1+\beta}\left[\sqrt{\beta + 2\beta^2\left[1 + \dfrac{t_2}{t_1} + \left(\dfrac{t_2}{t_1}\right)^2\right] + \beta^3\left(\dfrac{t_2}{t_1}\right)^2} - \beta\left(1 + \dfrac{t_2}{t_1}\right)\right] & (6.2.1c) \\ 1.1\dfrac{f_{h,1,d}\, t_1\, d}{2+\beta}\left[\sqrt{2\beta(1+\beta) + \dfrac{4\beta(2+\beta)M_{y,d}}{f_{h,1,d}\, d\, t_1^2}} - \beta\right] & (6.2.1d) \\ 1.1\dfrac{f_{h,1,d}\, t_2\, d}{1+2\beta}\left[\sqrt{2\beta^2(1+\beta) + \dfrac{4\beta(1+2\beta)M_{y,d}}{f_{h,1,d}\, d\, t_2^2}} - \beta\right] & (6.2.1e) \\ 1.1\sqrt{\dfrac{2\beta}{1+\beta}}\sqrt{2M_{y,d} f_{h,1,d}\, d} & (6.2.1f) \end{cases}$$



Extract from [11.2] – Timber to timber and panel to timber joints

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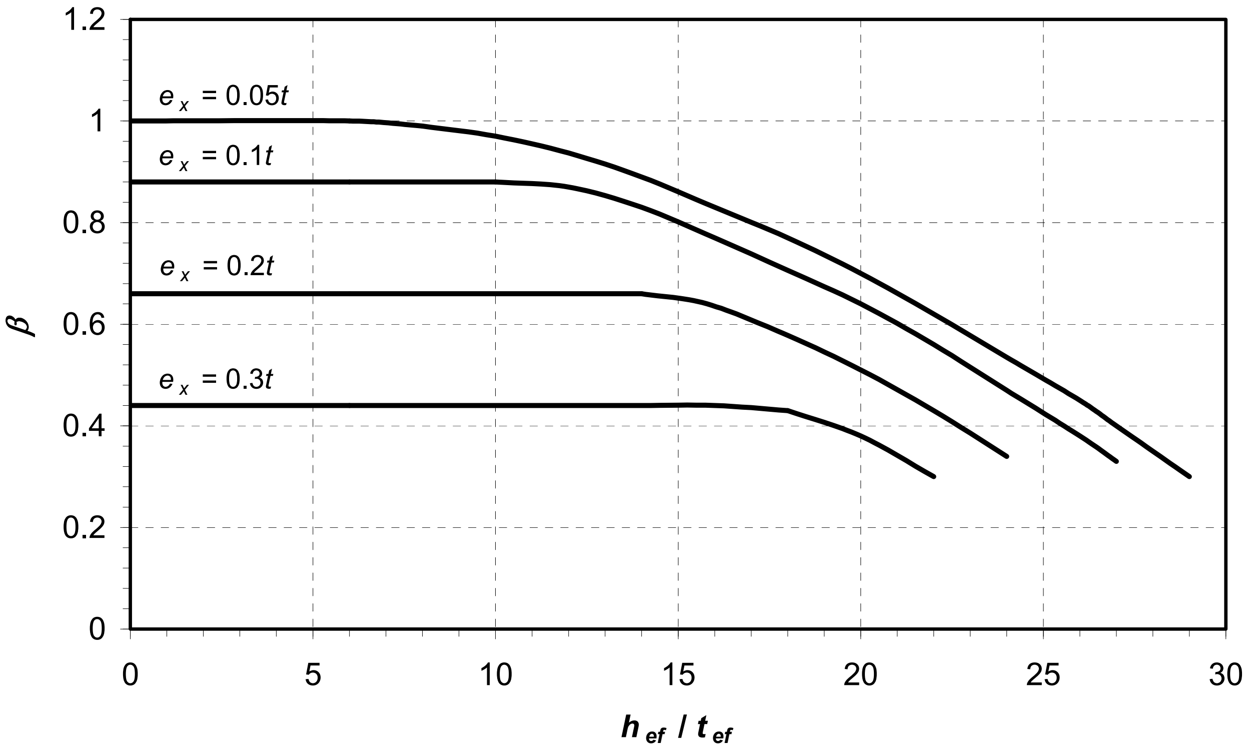
Bearing or crushing resistance per unit length

$$P_b = \frac{f_k t}{\gamma_m}$$

Buckling resistance per unit length

$$P_b = \frac{\beta f_k t}{\gamma_m}$$

Graph for capacity reduction factor β



Flexural resistance per unit length

$$M = \frac{f_{kx} Z}{\gamma_m}$$

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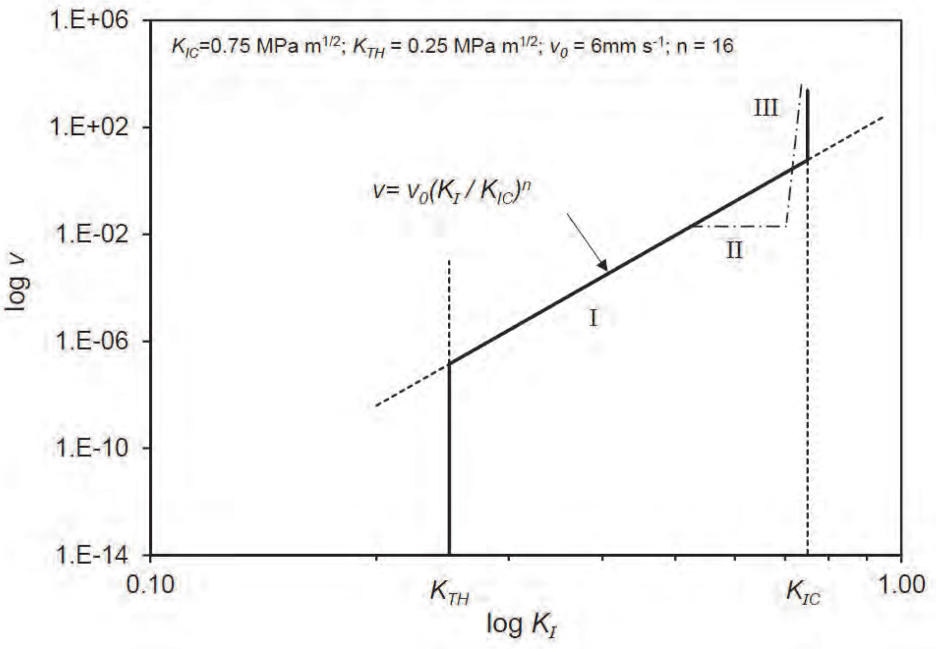
Explicit relationship between the flaw opening stress history and the initial flaw size:

$$\int_0^{t_f} \sigma^n(t)dt \approx \frac{2}{(n-2)\,v_0\,K_{IC}^{-n}\left(Y\sqrt{\pi}\right)^n a_i^{(n-2)/2}}$$

Idealised v–K relationship:



2-parameter Weibull distribution:

$$P_f = 1 - \exp\left[-kA\left(\sigma_f - f_{rk}\right)^m\right]$$

Stressed surface area factor (uniform stress):

$$\frac{\sigma_f}{\sigma_{A0}} = \left(\frac{A_0}{A_f}\right)^{1/m} = k_A$$

Load duration factor (constant stress history):

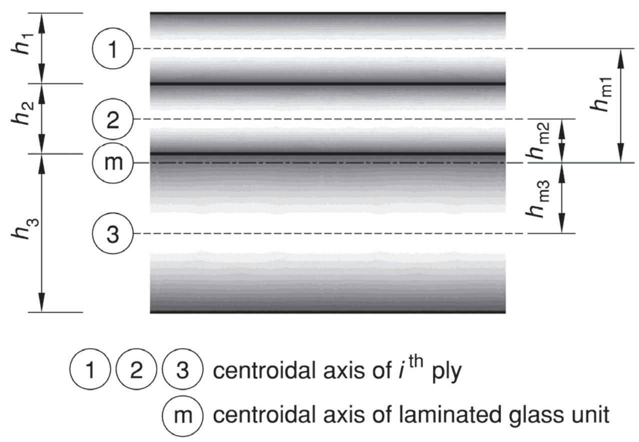
$$\frac{\sigma_f}{\sigma_{t0}} = \left(\frac{t_0}{t_f}\right)^{1/n} = k_{\mathrm{mod}}$$

Laminated glass equivalent thickness for bending deflection:

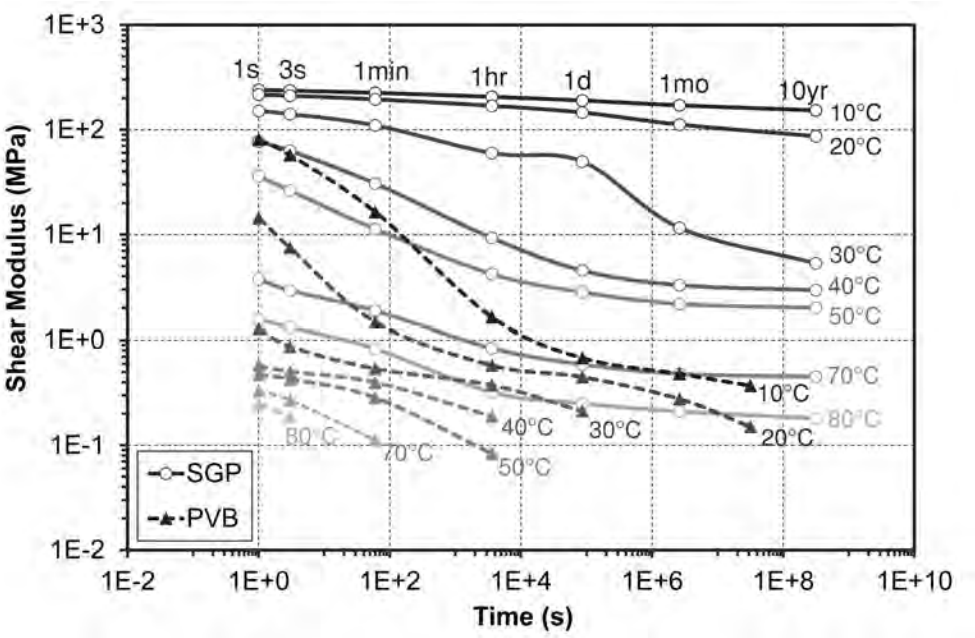
$$h_{eq,\delta} = \sqrt[3]{(1-\varpi)\sum_i h_i^{\,3} + \varpi\left(\sum_i h_i\right)^3}$$

Laminated glass equivalent thickness for bending stress:

$$h_{eq,\sigma} = \sqrt{\frac{\left(h_{eq,\delta}\right)^3}{\left(h_i + 2\varpi h_{m,i}\right)}}$$



G(t) of PVB and SGP interlayers:



Glass design strength:

$$f_{gd} = \frac{k_{\mathrm{mod}}\, k_A\, f_{gk}}{\gamma_{mA}} + \frac{f_{rk}}{\gamma_{mV}}$$

Stress-history (load duration) interaction equation:

$$\frac{\sigma_{1,\,S}}{f_{gd\,,\,S}} + \frac{\sigma_{1,\,M}}{f_{gd\,,\,M}} + \frac{\sigma_{1,\,L}}{f_{gd\,,\,L}} \;\le\; 1$$

Empirical stress concentration for bolted connections:

$$K_t = 1.5 + 1.25\left(\frac{H}{d} - 1\right) - 0.0675\left(\frac{H}{d} - 1\right)^2$$

where

$$K_t = \frac{\sigma_{\max}\left(H - d\right)t}{P}$$

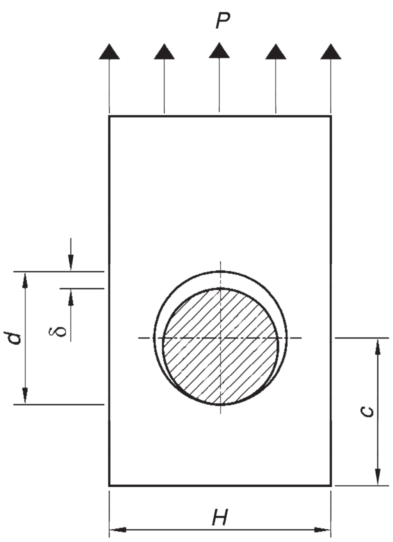


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Table 1.1 Span versus depth ratio

Structural system	Span/effective depth ratio	
	EC2*	
	high	light
1. Simply supported beam, one-way or two-way spanning simply supported slab	14	20
2. End span of continuous beam or one-way continuous slab or two-way spanning slab continuous over one long side	18	26
3. Interior span of beam or one-way or two-way spanning slab	20	30
4. Slab supported on columns without beams (flat slab), based on longer span	17	24
5. Cantilever	6	8

highly stressed $\rho = 1.5\%$ and lightly stressed $\rho = 0.5\%$ (slabs are normally assumed to be lightly stressed) *Table 7.4N, NA.5 [1.2]

Table 1.2 Minimum member sizes and cover (to main reinforcement) for initial design of continuous members

Member	Fire resistance	Minimum dimension, mm		
		4 hours	2 hours	1 hour
Columns fully exposed to fire	width	450	300	200
Beams	width cover	240 70	200 50	200 45
Slabs with plain soffit	thickness cover	170 45	125 35	100 35

Extracts from Table 4.1 [1.1]

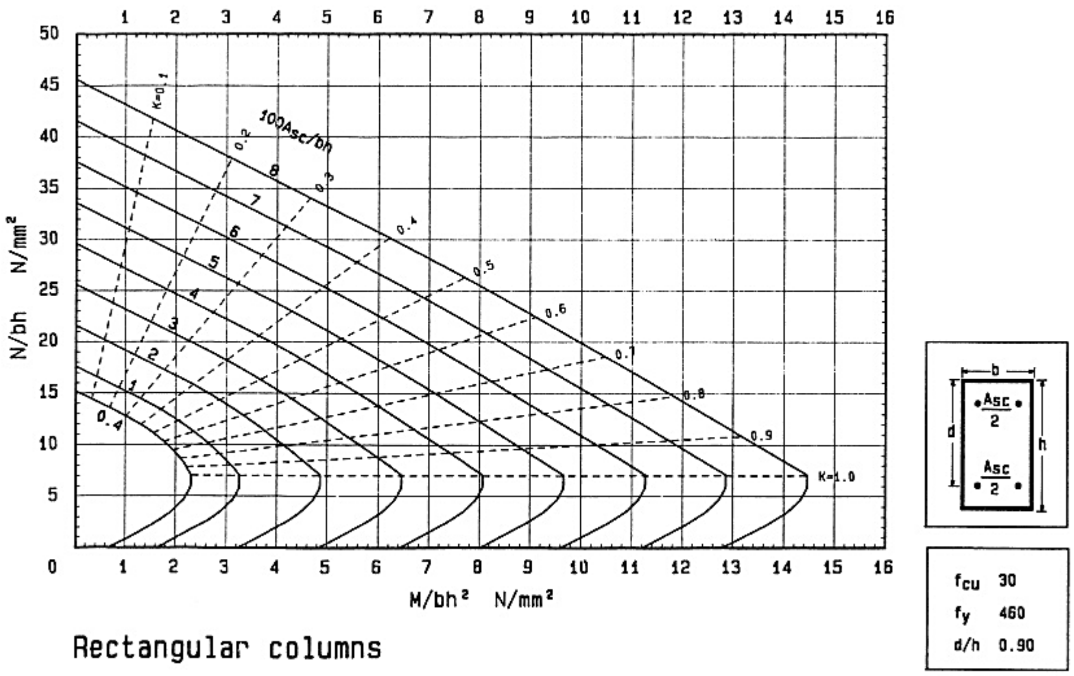

Fig 1.1 Interaction diagram from [1.3]

[1.1] Manual for the design of reinforced concrete building structures to EC2, IStructE, ICE, March 2000 - FM 507
[1.2] Eurocode 2: Design of concrete structures, EN 1992-1-1:2004, UK National Annex –NA to BS EN 1992-1-1:2004
[1.3] Structural design. Extracts from British Standards for Students of Structural design. PP7312:2002, BSi

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Flexure

Under-reinforced – singly reinforced

$$M_u = \frac{A_s f_y d(1 - 0.5x/d)}{\gamma_s}$$

$$\frac{x}{d} = \frac{\gamma_c A_s f_y}{\gamma_s 0.6 f_{cu} bd}$$

if x/d =0.5

$$M_u = 0.225 f_{cu} bd^2 / \gamma_c$$

Balanced section

$$\rho_b = \frac{A_s}{bd} = \frac{\gamma_s 0.6 f_{cu}}{\gamma_c f_y} \cdot \frac{\varepsilon_{cu}}{\varepsilon_y + \varepsilon_{cu}}$$

Shear

Without internal stirrups

$$V_{Rd,c} = [\frac{0.18}{\gamma_c} k(100\rho_1 f_{ck})^{1/3}] b_w d \geq (0.035 k^{3/2} f_{ck}^{1/2}) b_w d$$

where: f_{ck} is the characteristic concrete compressive cylinder strength (MPa).

$k = 1 + \sqrt{200/d} \leq 2.0$ (d in mm)

$\rho_l = A_s/b_w d \leq 0.02$

With internal stirrups

- Concrete resistance

$$V_{Rd,\max} = f_{c,\max}(b_w 0.9d)/(\cot\theta + \tan\theta)$$

where: $f_{c,\max} = 0.6(1 - f_{ck}/250) f_{cd}$

- Shear stirrup resistance

$$V_{Rd,s} = A_{sw} f_y (0.9d)(\cot\theta)/(s\gamma_s)$$

Columns – axial loading only

$$\sigma_u = 0.6 \frac{f_{cu}}{\gamma_c} + \rho_c \frac{f_y}{\gamma_s}$$

Standard steel diameters (in mm)

6, 8, 10, 12, 16, 20, 25, 32 and 40

3D3 2024
Numerical answers

1(b)(i) $N = 774$kN
1(b)(ii) $P_d = 170$kN
2(b) $t \approx 333$mm
4(b)(i) 3kN; 4.3kN