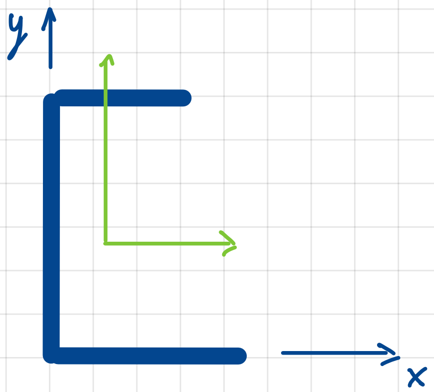
Q1 a)

$$J = \frac{1}{3}\left(2\ell t^3 + 4\ell t^3 + 3\ell t^3\right) = 3\ell t^3$$

Warping is important because two flanges will bend when torsion applied leading to shear forces. These shear forces in the two flanges will carry some of the torque.

b)



$$x_s = \frac{1}{9\ell t}\left(\ell \cdot 2\ell t + \frac{3}{2}\ell \cdot 3\ell t\right) = \frac{13}{18}\ell \approx 0.72\ell$$

$$y_s = \frac{1}{9\ell t}\left(4\ell \cdot 2\ell t + 2\ell \cdot 4\ell t\right) = \frac{16}{9}\ell \approx 1.78\ell$$

$$I_{xx} = \frac{(4\ell)^3 t}{12} + 4\ell t(2\ell)^2 + 2\ell t(4\ell)^2 = \frac{160}{3}\ell^3 t \approx 53.33\ell^3 t$$

$$I_{yy} = \frac{(2\ell)^3 t}{12} + \ell^2 \cdot 2\ell t + \frac{(3\ell)^3 t}{12} + \left(\frac{3}{2}\ell\right)^2 3\ell t$$

$$= \frac{35\ell^3 t}{3} \approx 11.67\ell^3 t$$

$$I_{xy} = 4\ell \cdot \ell \cdot 2\ell t = 8\ell^3 t$$

$$I_{xy} = I_{st} + 9\ell t\left(\frac{13}{18}\ell\right)\left(\frac{16}{9}\ell\right) = -\frac{32}{9}\ell^3 t$$

$$I_{xx} = I_{ss} + 9lt \cdot \left(\frac{16}{9} l\right)^2 \Rightarrow I_{ss} = \frac{224}{9} l^3 t$$

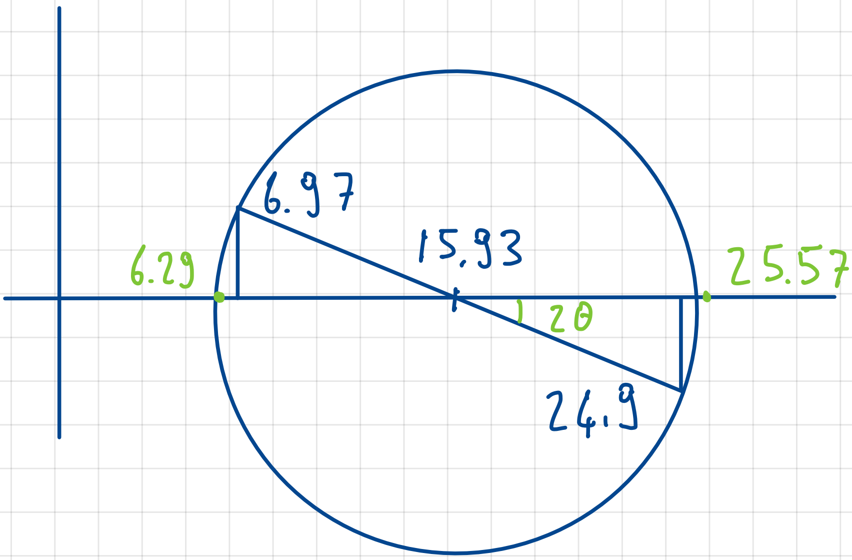
$$I_{yy} = I_{tt} + 9lt \cdot \left(\frac{13}{18} l\right)^2 \Rightarrow I_{tt} = \frac{251}{36} l^3 t$$

$$I_{xy} = I_{st} + 9lt\left(\frac{13}{18} l\right)\left(\frac{16}{9} l\right) = -\frac{32}{9} l^3 t$$

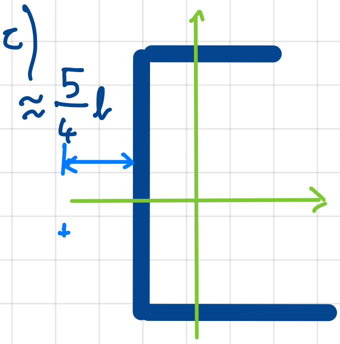
$I_{ss} \approx 24.9\, l^3 t$ $\qquad I_{tt} = 6.97 l^3 t$ $\qquad I_{st} = -3.56$

Eigenvalues $\quad I_{s's'} = 25.57\, l^3 t \qquad I_{t't'} = 6.29\, l^3 t$

$$\begin{pmatrix} -5.23 \\ 1 \end{pmatrix}, \begin{pmatrix} 0.19 \\ 1 \end{pmatrix}$$

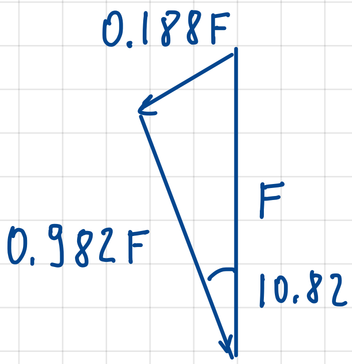


c)



Shear centre should be slightly below the horizontal axis at a distance about $\frac{1}{4}(2l + 3l) = \frac{5}{4} l$

d) According to the Data book $\dfrac{F l^3}{48 E I}$



Displacements

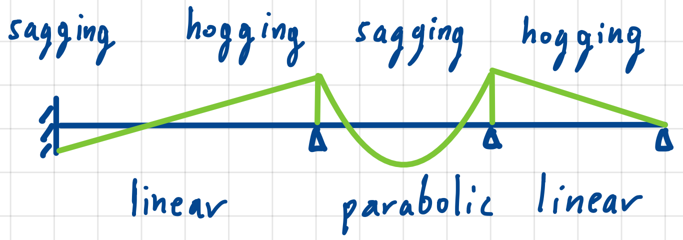
$\swarrow$ $$\frac{0.188\, F l^3}{48 E\, 6.29\, l^3 t} = 6.23 \cdot 10^{-4} \frac{F l^3}{E l^3 t}$$

$\searrow$ $$\frac{0.982\, F l^3}{48 E\, 25.57\, l^3 t} = 8.0 \cdot 10^{-4} \frac{F l^3}{E l^3 t}$$

Total displacement: $1.014 \cdot 10^{-3} \dfrac{F l^3}{E l^3 t}$

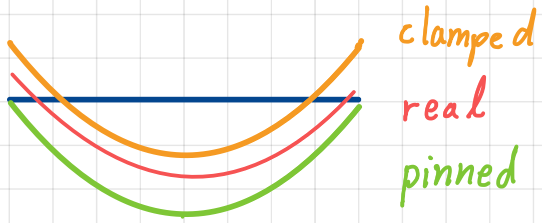
Q2 a)

i)



ii) Assuming both sides pinned: $\dfrac{wl^2}{8}$

Assuming bot sides clamped: $\dfrac{wl^2}{8} - \dfrac{wl^2}{12} = \dfrac{wl^2}{24}$



b) System has kinematic degree of indeterminacy one

Length of incl. beam $1.03l$

Stiffness of joint B:

$$K = \frac{4EI}{2l} + \frac{3EI}{1.03l} = 4.91\frac{EI}{l}$$

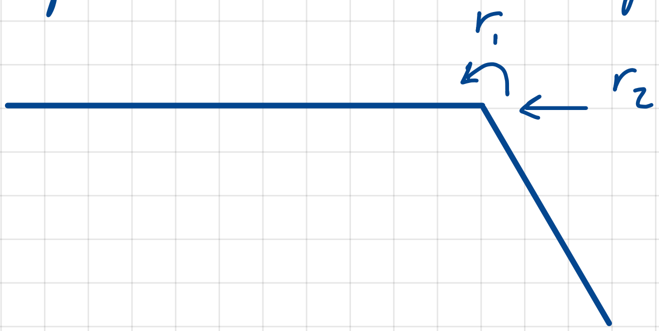
Moment due to F: $\dfrac{2Fl}{8} = \dfrac{Fl}{4}$

$$\Rightarrow \quad r = \frac{Fl}{4}\,\frac{l}{4.9EI} = 0.051\frac{Fl^2}{EI}$$

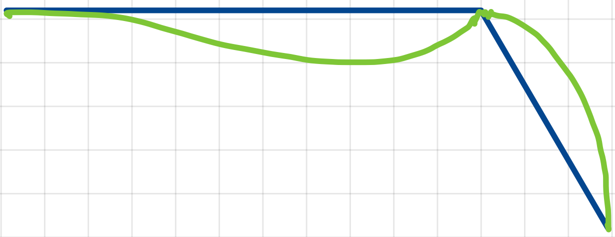
Moment at B

$$\frac{Fl}{4} - 0.051\frac{Fl^2}{EI}\cdot\frac{4EI}{2l} = 0.148\,Fl$$

c) System has kinematic degree of indeterminacy two

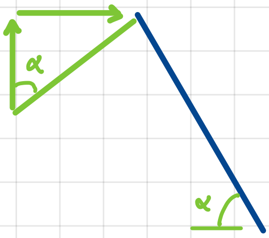


$r_1 = 1$ and $r_2 = 0$



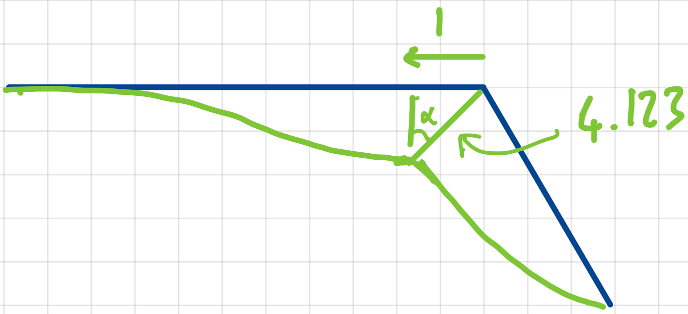
$$K_{11} = \frac{4EI}{2\ell} + \frac{3EI}{1.03\ell} = 4.91\,\frac{EI}{\ell} \quad \text{(like in (i))}$$

$$K_{12} = \frac{3EI}{(1.03\ell)^2}\sin(14.04°) = 0.686\,\frac{EI}{\ell^2}$$



$\alpha = 14.04°$

$r_1 = 1$ and $r_2 = 0$



$$K_{22} = \frac{EA}{2\ell} + \frac{3EI}{(1.03\ell)^3}\sin(14.04) = 0.5\,\frac{EA}{\ell} + 0.67\,\frac{EI}{\ell^3}$$

3D4 2023,

Q3. (a) $U = \frac{1}{2}\int_0^L EI\left(\frac{d^2\phi}{dx^2}\right)^2 dx$ lat. Strain Energy

$\phi = x^2(L-x) = x^2L - x^3$

$\phi' = 2xL - 3x^2$

$\phi'' = 2L - 6x = 2(L-3x)$

$(\phi'')^2 = 4(L-3x)^2 = 4(L^2 - 6xL + 9x^2)$

$U = \frac{1}{2} EI\, 4 \int_0^L (L^2 - 6xL + 9x^2)\, dx$

$= 2EI\left[L^2x - 3x^2L + \frac{9x^3}{3}\right]_0^L$

$= 2EI\,L^3\,[1 - 3 + 3] = \underline{\underline{2EIL^3}}$

$W = \frac{1}{2}\int_0^L (\phi')^2 dx = \frac{1}{2}\int_0^L (2xL - 3x^2)^2 dx$ End shortening

$= \frac{1}{2}\int_0^L (4x^2L^2 - 12x^3L + 9x^4)\, dx$

$= \frac{1}{2}\left[\frac{4x^3L^2}{3} - \frac{12x^4L}{4} + \frac{9x^5}{5}\right]_0^L$

$= \frac{1}{2}L^5\left[\frac{4}{3} - 3 + \frac{9}{5}\right]$

$= \frac{1}{2(15)}L^5\,[20 - 45 + 27] = \underline{\underline{\frac{L^5}{15}}}$

[30%]

Rayleigh Quotient $P_{cr} \simeq \frac{U}{W} = \frac{2EIL^3}{L^5/15} = \frac{30\,EI}{L^2}$ (True value $\approx \frac{20\,EI}{L^2}$)

b) Overestimate. Deflected shape assumed is not an eigenvector, and energy goes flat in eigendirection first.

[10%]

c) $P_{Euler} = \frac{\pi^2 EI}{L^2}$, $P_{cr} = \frac{\pi^2 EI}{L_{eff}^2} = \frac{30\,EI}{L^2}$

$\therefore \frac{L_{eff}^2}{L^2} = \frac{\pi^2}{30}$ $L_e = \frac{\pi}{\sqrt{30}}L \simeq \underline{\underline{0.574\,L}}$ (True value $\simeq 0.7L$)

[10%]

3D4 2023 Q3 cont'd

d) Perry-Robertson deals with inelastic behaviour by halting the moment σ_y is experienced.
i.e. stop at first yield

e)

Euler column. $L = 4m$.

CHS $D = 168.3\,mm$ $D_{mid\,wall} = 155.8\,mm$.

$t = 12.5\,mm$

$I = 1868\,cm^4 = 1868 \times 10^{-8}\,m^4$ Structures Data Book.

$A = 61.2\,cm^2 = 61.2 \times 10^{-4}\,m^2$

$$P_E = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (210 \times 10^9\,N/m^2)(1868 \times 10^{-8}\,m^4)}{(4m)^2}$$

$$= \underline{\underline{2419.8\ kN}}$$

$$\sigma_E = \frac{2.420 \times 10^6}{61.2 \times 10^{-4}} = \underline{\underline{395.4\,MPa}}$$

$\sigma_y = 275\,MPa$.

$$\eta = 0.003\frac{L}{r} \qquad r = \sqrt{\frac{I}{A}} = \sqrt{\frac{1868 \times 10^{-8}}{61.2 \times 10^{-4}}}$$

$$\eta = 0.003\left(\frac{4}{0.0552}\right) = \underline{0.2174} \qquad = 0.0552\,m \quad (= 55.2\,mm)$$

$$(\sigma_y - \sigma_{cr})(\sigma_E - \sigma_{cr}) = \eta\, \sigma_{cr}\sigma_E$$

$$\left(1 - \frac{\sigma_{cr}}{\sigma_y}\right)\left(\frac{\sigma_E}{\sigma_y} - \frac{\sigma_{cr}}{\sigma_y}\right) = \eta \frac{\sigma_{cr}}{\sigma_y}\frac{\sigma_E}{\sigma_y}$$

$$(1-c)(e-c) = \eta c e \qquad c = \frac{\sigma_{cr}}{\sigma_y} \quad e = \frac{\sigma_E}{\sigma_y}$$

$$e - c - ec + c^2 = \eta c e$$

$$c^2 - (1 + e + \eta e)c + e = 0$$

$$c^2 - \Phi c + e = 0, \qquad c = \frac{\Phi \pm \sqrt{\Phi^2 - 4e}}{2}$$

$$e = \frac{395.4}{275} = 1.438 \qquad \Phi = 1 + e(1+\eta) = 1 + 1.438(1 + 0.2174)$$

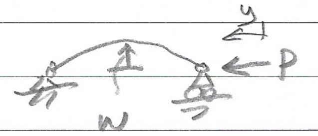
$$= 2.7506$$

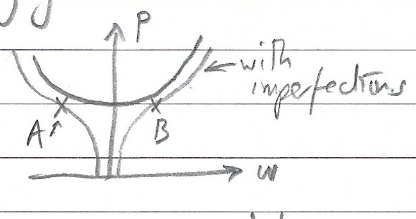
$$c = \left[2.7506 \pm \sqrt{(2.7506)^2 - 4(1.438)}\right]/2 = 0.702$$

$\therefore \sigma_{cr} = 0.702(275) = 193\,MPa$. $\therefore P_{cr} = (193 \times 10^6)(61.2 \times 10^{-4}) = \underline{\underline{1181\,kN}}$

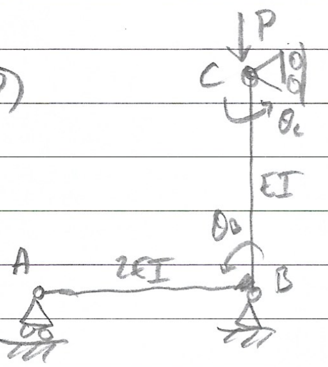
3D4 2023

Q4. a). For slope of load-deflection curve to be stiffness, need load and deflection to be conjugate (Same place, same direction (for point loads)) (same shape for distributed loads).
Often, in studies of buckling, we plot axial load versus lateral deflection, so not conjugate.

eg 



At point A, slope is negative!!
B slope is positive.
But these are same case, just reflected vertically.
Stiffness at condition A is $\underline{\underline{\text{NOT}}}$ negative.
Problem is P and w $\underline{\underline{\text{NOT}}}$ conjugate.
(P is conjugate to y).

b) 

We can write down stiffness matrix wrt θ_B, θ_C by inspection.

$$\begin{bmatrix} M_C \\ M_B \end{bmatrix} = \frac{EI}{L} \underbrace{\begin{bmatrix} s & sc \\ sc & s \end{bmatrix}}_{\text{from BC}} \begin{bmatrix} \theta_C \\ \theta_B \end{bmatrix}$$

+ 3 (2) from AB

far end pinned

2EI, not EI.

$$\therefore \begin{bmatrix} M_C \\ M_B \end{bmatrix} = \frac{EI}{L} \begin{bmatrix} s & sc \\ sc & s+6 \end{bmatrix} \begin{bmatrix} \theta_C \\ \theta_B \end{bmatrix}$$

Now $M_C = 0$ no applied moment,

so $M_C = 0 = \frac{EI}{L}\left[s\theta_C + sc\theta_B \right] \Rightarrow \theta_C = -c\theta_B$

$$M_B = \frac{EI}{L}\left[sc\theta_C + (s+6)\theta_B \right] = \frac{EI}{L}\left[-sc^2 + s + 6 \right]\theta_B$$

so rotational stiffness at B is $\frac{EI}{L}\left[s(1-c^2) + 6 \right]$

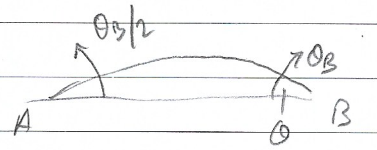
3D4 2023

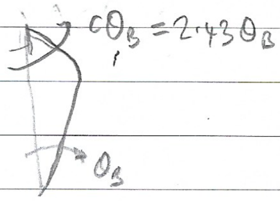
Q4 contd, b) ii) For instability, need $s(1-c^2) = -6$

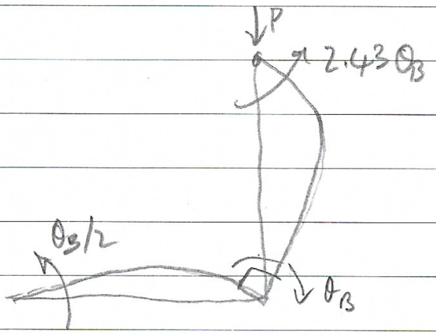
P/P_E	s	c	$s(1-c^2)$
1	2.4674	1	0
1.2	2.0901	1.2487	−1.17
1.4	1.6782	1.6557	
1.6	1.2240	2.4348	−6.03 ← accept this $P/P_E = 1.6$.

$\therefore \quad P_{cr} \simeq 1.6\left(\frac{\pi^2 EI}{L^2}\right)$

b) iii)







or