Version FC/3

EGT2
ENGINEERING TRIPOS PART IIA

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Tuesday 9 May 2023 9.30 to 11.10

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**Module 3D4**

**STRUCTURAL ANALYSIS AND STABILITY**

*Answer not more than* ***three*** *questions.*

*All questions carry the same number of marks.*

*The* ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number* ***not*** *your name on the cover sheet.*

**STATIONERY REQUIREMENTS**
Single-sided script paper
Graph Paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**
CUED approved calculator allowed
Attachment: Data Sheet for Question 2 (2 pages)
Engineering Data Book

**10 minutes reading time is allowed for this paper at the start of the exam.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

**You may not remove any stationery from the Examination Room.**

1 The thin-walled channel-like cross-section shown in Fig. 1 has a uniform thickness $t$ and elastic modulus $E$.

(a) Determine the St. Venant torsion constant of the cross-section and explain whether restrained warping torsion is important or not. [15%]

(b) Calculate the principal second moments of area of the cross-section. [40%]

(c) Estimate the approximate position of the shear-centre of the cross-section and explain your reasoning. Provide a sketch indicating the relative position of the shear-centre in relation to the cross-section. [15%]

(d) A simply supported beam with length $l$ and the given cross-section carries a concentrated force of magnitude $F$. The force acts in the negative $y$-direction at the shear-centre at midspan of the beam. Calculate the maximum deflection of the shear-centre of the beam. [30%]

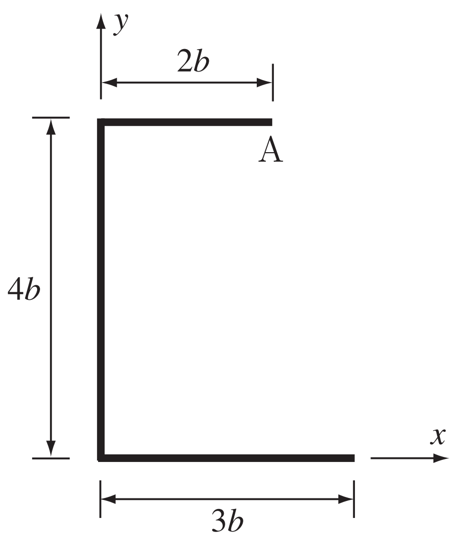


Fig. 1

2 (a) The three-span beam shown in Fig. 2(a) has a uniform stiffness $EI$. The left support is clamped and the centre span carries a uniformly distributed load $w$.

(i) Sketch the bending moment diagram while marking salient features. [10%]

(ii) Estimate the largest sagging moment at the centre span by providing a maximum and a minimum value. Explain your reasoning. [15%]

(b) Figure 2(b) shows a frame consisting of two beams that are rigidly connected at B. The beam $AB$ carries a concentrated vertical load $F$ at its mid-span and the flexural stiffness of the two beams is $EI$.

(i) Assume that the axial displacements in both beams can be neglected. Calculate the moment at the joint B. (Datasheet attached.) [35%]

(ii) Assume that the axial stiffness of the beam BC can be neglected and the axial stiffness of the beam AB is $EA$. Calculate the stiffness matrix of the structure. [40%]

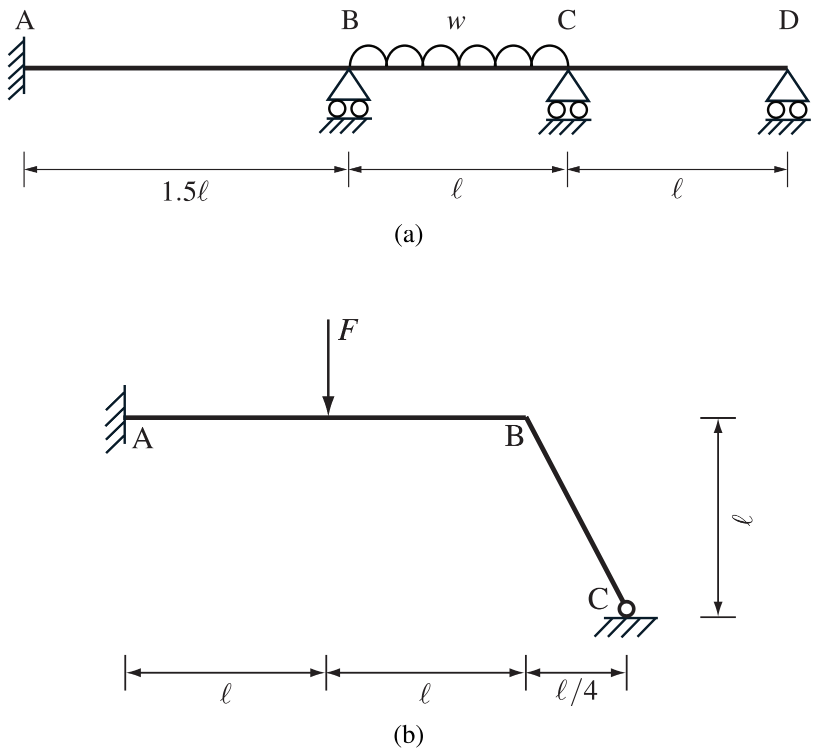


Fig. 2

3 (a) Assuming a lateral deflected shape $w(x) = x^2(L - x)$, use Rayleigh's quotient to estimate the axial load that will cause flexural buckling of a cantilever of length $L$, clamped at $x = 0$, propped at $x = L$ and flexural rigidity $EI$. [30%]

(b) Do you expect your answer in part a) to be an over-estimate or an under-estimate of the true buckling load? Explain your reasoning. [10%]

(c) According to your estimate in part a), what is the effective length of a propped cantilever? [10%]

(d) Explain briefly how the Perry-Robertson formula deals with inelastic material behaviour. [10%]

(e) The Perry-Robertson formula for column buckling is

$$(\sigma_Y - \sigma_{cr})(\sigma_E - \sigma_{cr}) = \eta\sigma_{cr}\sigma_E$$

where $\sigma_Y$ is the material yield stress, $\sigma_E$ is the Euler stress, $\sigma_{cr}$ is the stress at the section centroid when the column carries the critical buckling load $P_{cr}$ and $\eta$ is a dimensionless imperfection parameter.

Assume that $\eta = 0.003L/r$, where $L$ is the column length and $r$ is the radius of gyration of the column section with respect to bending about its minor axis. Determine the axial design load of an Euler column of length $L = 4$ m which has a circular hollow section of 168.3 mm external diameter and 12.5 mm wall thickness made of steel with yield stress of 275 MPa. [40%]

4 (a) The slope of a graph of load against deflection often corresponds to a stiffness. Explain why this might not always be the case and give an example. [20%]

(b) Figure 3 shows a frame ABC carrying a vertical load $P$ applied at C. The support conditions are as shown in the diagram. Members AB and BC each have length $L$. Each may be considered to be axially rigid. The flexural rigidities of members AB and BC with respect to bending within the plane of the diagram are $2EI$ and $EI$ respectively. Bending out of the plane of the diagram is prevented.

(i) Derive an expression for the rotational stiffness at B in terms of the $s$ and $c$ stability functions. [30%]

(ii) Using the table of $s$ and $c$ functions provided, estimate the smallest value of $P$ that will cause in-plane flexural buckling of the frame. [30%]

(iii) Provide a clear sketch of the buckling mode, and indicate the magnitudes of the rotations of all joints expressed as a proportion of the rotation at B. [20%]

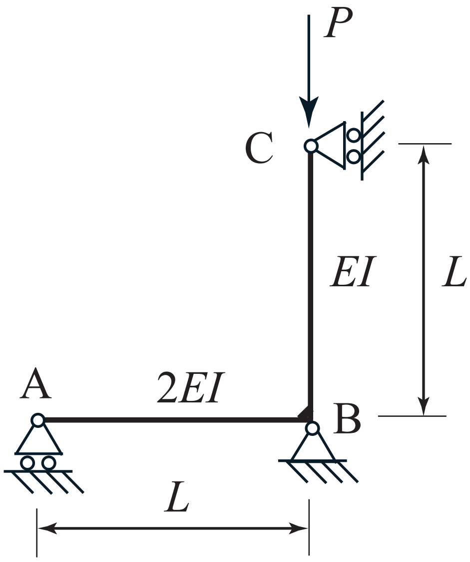


Fig. 3

Table 1. $s$ and $c$ stability functions for an Euler column. $P$ is the axial load and $P_E$ is the Euler load.

| $P/P_E$ | $s$ | $c$ |
|---|---|---|
| 0.0 | 4.0000 | 0.5000 |
| 0.2 | 3.7297 | 0.5550 |
| 0.4 | 3.4439 | 0.6242 |
| 0.6 | 3.1403 | 0.7136 |
| 0.8 | 2.8159 | 0.8330 |
| 1.0 | 2.4674 | 1.0000 |
| 1.2 | 2.0901 | 1.2487 |
| 1.4 | 1.6782 | 1.6557 |
| 1.6 | 1.2240 | 2.4348 |
| 1.8 | 0.7170 | 4.4969 |
| 2.0 | 0.1428 | 24.6841 |
| 2.2 | -0.5194 | -7.5107 |
| 2.4 | -1.3006 | -3.3703 |
| 2.6 | -2.2490 | -2.2312 |
| 2.8 | -3.4449 | -1.7081 |
| 3.0 | -5.0320 | -1.4157 |

**END OF PAPER**

**Data Sheet for Question 2: Stiffness Matrices.**

**Notation and sign convention**


**Beam type I**


$$\begin{bmatrix} P_A \\ S_A \\ M_A \\ P_B \\ S_B \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} & 0 & -\frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{6\,EI}{L^2} & \frac{2\,EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} & 0 & \frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} \\ 0 & -\frac{6\,EI}{L^2} & \frac{2\,EI}{L} & 0 & \frac{6\,EI}{L^2} & \frac{4\,EI}{L} \end{bmatrix} \begin{bmatrix} u_A \\ w_A \\ \phi_A \\ u_B \\ w_B \\ \phi_B \end{bmatrix}$$

**Beam type II**


$$
\begin{bmatrix} P_A \\ S_A \\ M_A \\ P_B \\ S_B \\ M_B \end{bmatrix}
=
\begin{bmatrix}
\frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\
0 & \frac{3EI}{L^3} & -\frac{3EI}{L^2} & 0 & -\frac{3EI}{L^3} & 0 \\
0 & -\frac{3EI}{L^2} & \frac{3EI}{L} & 0 & \frac{3EI}{L^2} & 0 \\
-\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\
0 & -\frac{3EI}{L^3} & \frac{3EI}{L^2} & 0 & \frac{3EI}{L^3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} u_A \\ w_A \\ \phi_A \\ u_B \\ w_B \\ \phi_B \end{bmatrix}
$$