Version FC/4

EGT2
ENGINEERING TRIPOS PART IIA

Thursday 2 May 2024   9.30 to 11.10

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: Data Sheet for Question 2 (2 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

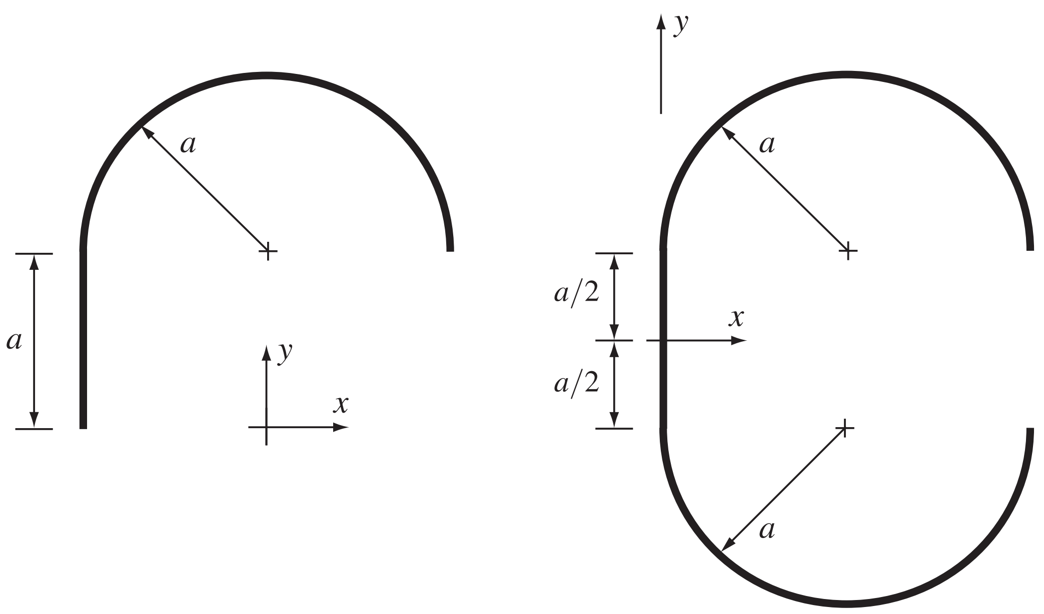
You may not remove any stationery from the Examination Room.

1 (a) Figure 1(a) shows a thin-walled cross-section consisting of one semicircular part of radius a and one straight part of length a. The cross-section has a uniform thickness t.

(i) Provide an estimate for the St. Venant torsion constant of the cross-section. [15%]

(ii) Calculate the principal second moments of area and the principal axes of the cross-section. [40%]

(b) Figure 1(b) shows a thin-walled cross-section consisting of two semicircular parts each of radius a and one straight part of length a. The cross-section has a uniform thickness t. Calculate the shear centre of the cross-section. [45%]



(a) (b)

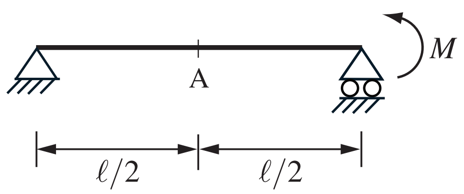
Fig. 1

2 (a) Figure 2(a) shows a simply supported beam with a moment M applied at its right end. Determine the deflection at A using the reciprocal theorem together with deflection coefficients taken from the Structures Data Book. [15%]

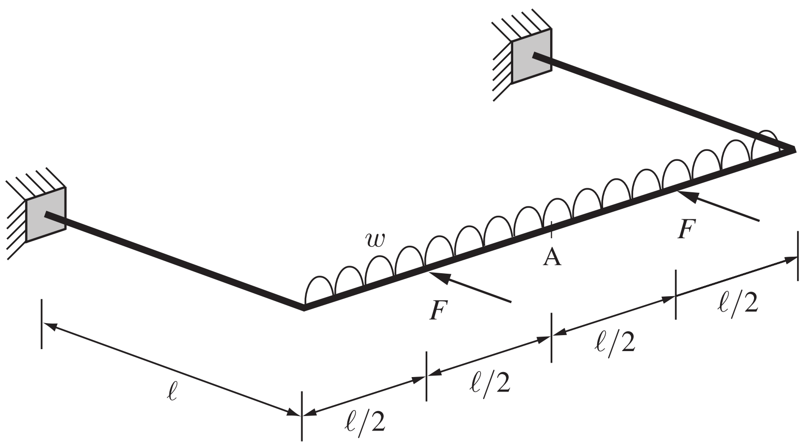
(b) Figure 2(b) shows a grillage consisting of three beams that is built in to a rigid wall. The grillage is loaded with a distributed load w and two concentrated loads F. The distributed load acts orthogonal to the plane of the grillage, whereas the concentrated forces act in the plane of the grillage. The flexural stiffness of all members is EI, their torsional stiffness is $GJ = 2EI$ and their axial stiffness EA can be assumed as infinite.

(i) Compute the bending moment at A due to the distributed loading w. [40%]

(ii) Calculate the bending moment at A due to the in-plane forces F. [45%]



(a)



(b)

Fig. 2

3 (a) Figure 3(a.i) shows an unloaded horizontal rigid rod of length, L, connected to ground at pin-joints by a vertical spring of linear stiffness, k [Nm^{-1}], and by a torsional spring of linear stiffness, c, [Nm rad^{-1}]. A horizontal force, P, is applied to the right end, and increased from zero until the rod buckles in-plane, as shown in Fig. 3(a.ii), where the angle of rotation of the rod is θ.

(i) Verify that the critical buckling load for P is given by $c/L + kL$. [10%]

(ii) Show that, when $kL = (1/3)(c/L)$, the initial post-buckling variation of P with θ is constant. [15%]

(iii) Sketch the variation of P with θ when $kL = 0.4c/L$, noting salient features. [15%]

(b) A light, horizontal rigid rod, AB, of length, a, is connected to another light, horizontal rigid rod, BC, of length, b, by a pin-joint, as shown in Fig. 4(a.i). The pin-jointed end, A, is connected to ground by a torsional spring of linear stiffness, c, [Nm rad^{-1}], and the pin-jointed end, C, is connected to ground by horizontal roller supports. A horizontal force, P, is applied at C, and increased from zero until the arrangement buckles in-plane, as shown in Fig. 4(a.ii). Show that the critical buckling load for P is given by [20%]

$$\frac{cb}{a(a+b)}$$

(c) The rod, AB, is now replaced by a light, uniform beam, AB, which has a bending stiffness, EI, [Nm^2], and length, a. The beam is axially rigid and is built-in at A, as shown in Fig. 4(b.i). A horizontal force, P, is applied at C and increased from zero until the arrangement buckles in-plane, as shown in Fig. 4(b.ii). Show that the critical buckling load for P is given by the solution of the equation

$$\tan(\alpha a) = \alpha(a+b), \quad \text{where} \quad \alpha^2 = \frac{P}{EI}$$

Do not solve this equation. [40%]

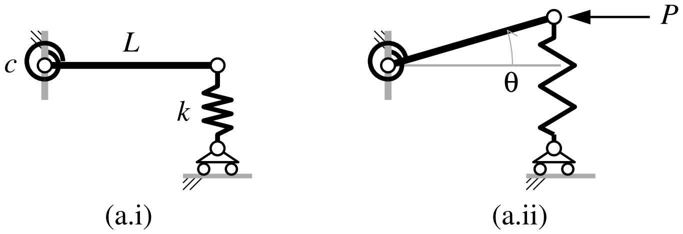


Fig. 3

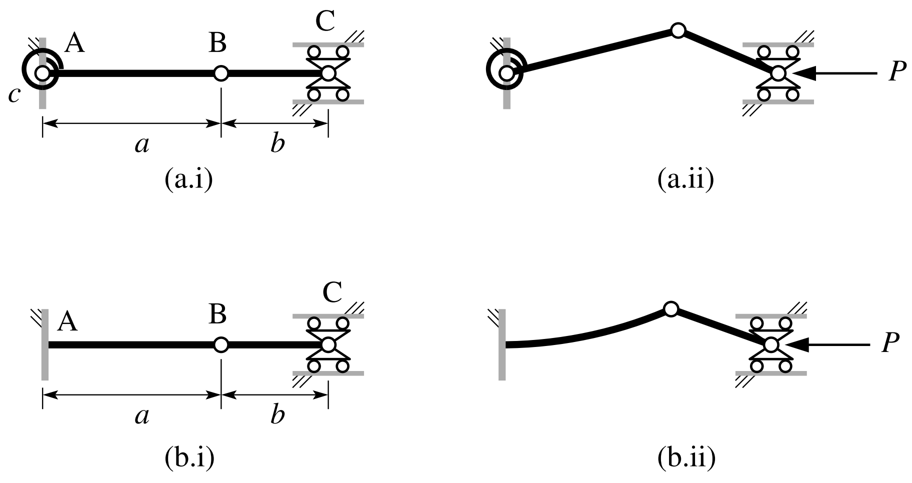


Fig. 4

4 Figure 5 shows a uniform beam, AB, of length, L, and bending stiffness, EI. The beam is rigidly built-in at end A, where $x = 0$, and is simply-supported at end B, where $x = L$. The beam is also supported on an elastic foundation of constant stiffness per unit length, k, [(Nm^{-1}) m^{-1}], which resists transverse deflections of the beam, w, vertically upwards, when a horizontal compressive force, P is applied to the end, B.

(a) Show that the strain energy stored in bending, the work performed by P, and the strain energy stored in the foundation are respectively given by [30%]

$$\frac{1}{2}\int_0^L EI\left(\frac{\mathrm{d}^2w}{\mathrm{d}x^2}\right)^2 \mathrm{d}x, \quad \frac{1}{2}P\int_0^L \left(\frac{\mathrm{d}w}{\mathrm{d}x}\right)^2 \mathrm{d}x, \quad \frac{1}{2}k\int_0^L w^2\mathrm{d}x$$

(b) The in-plane deflected shape of a deformed beam is represented by the polynomial series

$$w = \sum_{n=0}^{i} a_i \left(\frac{x}{L}\right)^n$$

(i) Show that the minimum number of terms, i, required to satisfy the kinematical and statical boundary conditions is $i = 4$ when allowing for a single arbitrary constant. [25%]

(ii) From the result in part b(i), calculate an approximate value for the buckling load of P by the Rayleigh-Ritz method. [40%]

(iii) Comment on how to improve the accuracy of your answer in part b(ii). [5%]

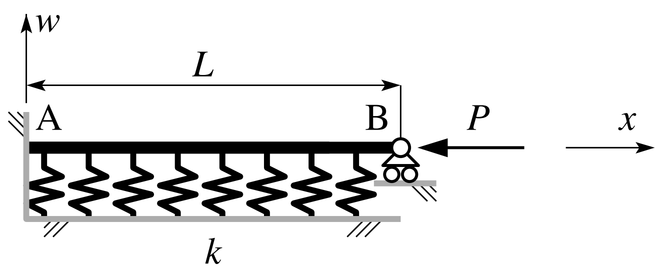


Fig. 5

END OF PAPER

Data Sheet for Question 2: Stiffness Matrices.

Notation and sign convention

Beam type I

$$\begin{bmatrix} P_A \\ S_A \\ M_A \\ P_B \\ S_B \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} & 0 & -\frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{6\,EI}{L^2} & \frac{2\,EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} & 0 & \frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} \\ 0 & -\frac{6\,EI}{L^2} & \frac{2\,EI}{L} & 0 & \frac{6\,EI}{L^2} & \frac{4\,EI}{L} \end{bmatrix} \begin{bmatrix} u_A \\ w_A \\ \phi_A \\ u_B \\ w_B \\ \phi_B \end{bmatrix}$$

Beam type II

$$\begin{bmatrix} P_A \\ S_A \\ M_A \\ P_B \\ S_B \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{3EI}{L^3} & -\frac{3EI}{L^2} & 0 & -\frac{3EI}{L^3} & 0 \\ 0 & -\frac{3EI}{L^2} & \frac{3EI}{L} & 0 & \frac{3EI}{L^2} & 0 \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{3EI}{L^3} & \frac{3EI}{L^2} & 0 & \frac{3EI}{L^3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_A \\ w_A \\ \phi_A \\ u_B \\ w_B \\ \phi_B \end{bmatrix}$$