Version FC/3

EGT2
ENGINEERING TRIPOS PART IIA

Tuesday 6 May 2025 2 to 3.40

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: Data Sheet for Question 3 (2 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 Figure 1 shows an asymmetric steel cross-section obtained by cutting a universal column UC 305×305×158 cross-section. The dashed part of the top flange is missing. The asymmetric cross-section is mounted as a cantilever of length L, whereby the y-axis is pointing upwards and is loaded by its own weight.

(a) Provide an estimate for the St. Venant torsion constant of the cross-section. [10%]

(b) The restrained warping constant for torsion of an uncut I-beam is $\Gamma = \frac{d^2}{4} I_{yy}$. Estimate the warping constant of the considered cut I-beam by appropriately choosing d and I_{yy}. Justify your choices and explain your reasoning. [15%]

(c) Calculate the principal second moments of area and the principal axes of the cross-section. [55%]

(d) Determine at the root of the cantilever the axial stress at point A of the cross-section. [20%]

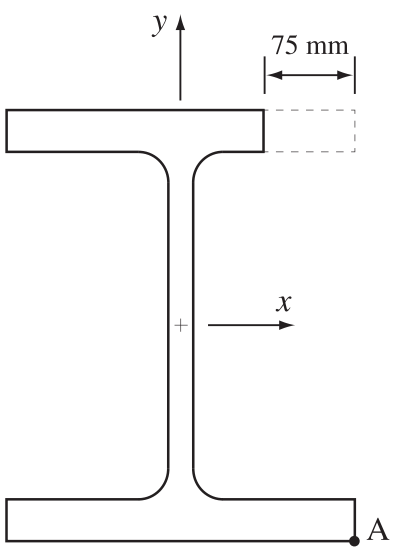


Fig. 1

2 (a) Figure 2(a) shows a version of the von Mises two-bar truss, and Fig. 2(b) indicates the initial geometry. Both bars have length, L, and are equally inclined to the horizontal by angle, α. Both bars are also rigid and connected together by a frictionless pin-joint. A pair of springs, each of linear stiffness, k [N/m], spans the supporting pin-joints. A vertical force, P, is applied to the central pin-joint, and increased from zero.

(i) Find *exact* expressions for the geometry of the deformed configuration in terms of the current inclined angle, $(1-\zeta)\alpha$, where ζ is a numerical factor. [15%]

(ii) Verify that P is equal to the following expression

$$P = 4kL\left[\sin\left((1-\zeta)\alpha\right) - \cos\alpha\tan\left((1-\zeta)\alpha\right)\right]$$

Plot the variation of $\bar{P} = P/(4kL)$ with ζ, indicating salient features. [40%]

(iii) When α is small, show that P is approximately equal to $2kL\alpha^3\zeta(1-\zeta)(2-\zeta)$. [10%]

(b) A rotational spring, of linear stiffness, c [Nm/rad], is added to the central pin-joint in Fig. 2(c), and becomes stressed relative to the initial configuration. Describe *qualitatively* how the variation of P with ζ from part a.ii changes according to different values of c. [35%]

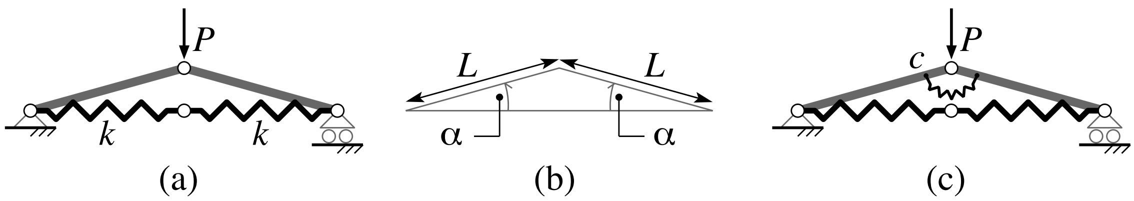


Fig. 2

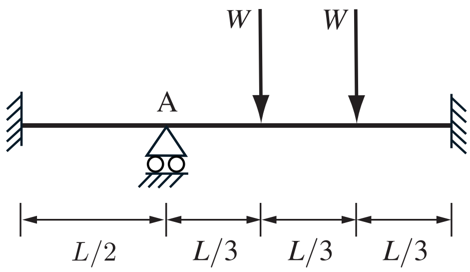
3 (a) Figure 3(a) shows a beam with two spans. The beam is clamped at its left and right ends and continuous over the support A. The right span carries two forces W. The beam has a uniform flexural stiffness EI. Use the displacement method to determine the rotation over support A. [25%]

(b) Figures 3(b), 3(c) and 3(d) show three cranked frames, each of which consists of two beams. In all three frames, the support A is clamped. The beams are on one plane, and the force W is orthogonal to that plane. The flexural stiffness of the beams is EI, and their torsional stiffness is $GJ = 2EI$.

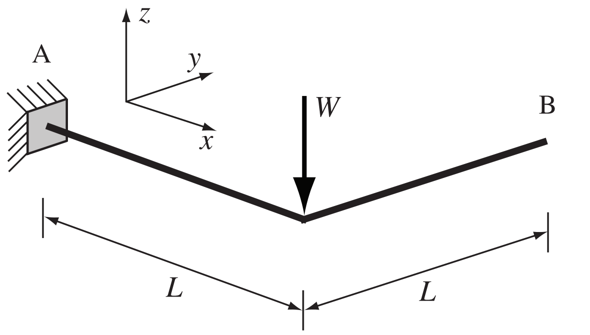
(i) Determine the tip deflection of the structure shown in Fig. 3(b). [10%]

(ii) Determine the reaction force at the support B of the structure shown in Fig. 3(c). [30%]

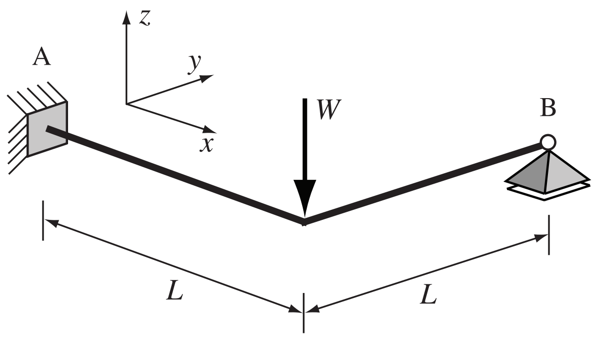
(iii) Determine the reaction forces at the supports B and C of the structure shown in Fig. 3(d). [35%]



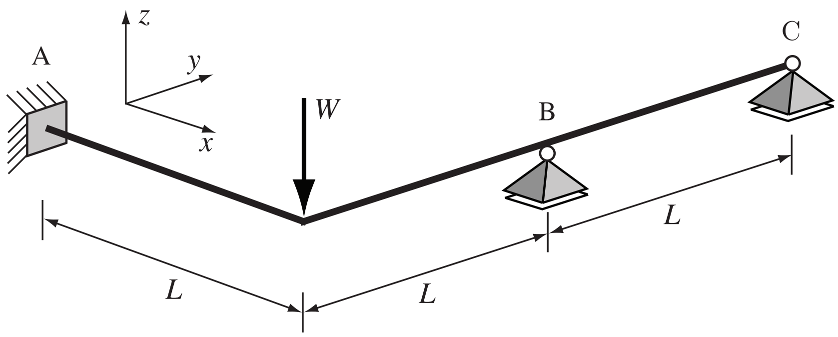
(a)



(b)



(c)



(d)

Fig. 3

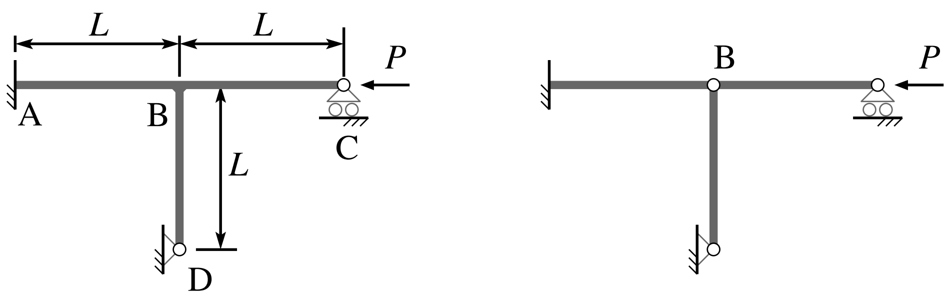
(TURN OVER

4 Figure 4(a) indicates a subframe structure, where the horizontal beam, AC, of length $2L$, is rigidly connected in the middle at B to the vertical beam, BD, of length L. Both beams have a linear bending stiffness, EI, and are free of stresses initially. The support at A is built-in, at C is a horizontal roller, and at D is pinned. A horizontal force, P, is applied at C, where the subframe deforms only within the plane of the page. Both beams may be considered to be axially rigid, and Table 1 indicates s and c stability-related functions.

(a) Calculate the value of P at which the subframe buckles within its plane. [50%]

(b) Determine the corresponding rotations of joints A, B, C and D and sketch the buckling mode-shape. [20%]

(c) A pin-joint is introduced at B in Fig. 4(b) before loading again. Determine the new buckling value of P. [30%]



(a) (b)

Fig. 4

P/P_E	s	c
0.0	4.0000	0.5000
0.2	3.7297	0.5550
0.4	3.4439	0.6242
0.6	3.1403	0.7136
0.8	2.8159	0.8330
1.0	2.4674	1.0000
1.2	2.0901	1.2487
1.4	1.6782	1.6557
1.6	1.2240	2.4348
1.8	0.7170	4.4969
2.0	0.1428	24.6841
2.2	-0.5194	-7.5107
2.4	-1.3006	-3.3703
2.6	-2.2490	-2.2312
2.8	-3.4449	-1.7081
3.0	-5.0320	-1.4157

Table 1

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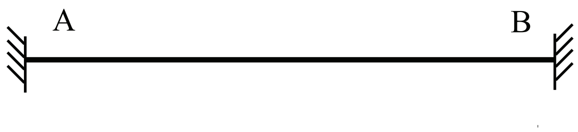
3D4: STRUCTURAL ANALYSIS AND STABILITY
Data Sheet for Question 3: Stiffness Matrices.

Notation and sign convention

Beam type I



$$
\begin{bmatrix} P_A \\ S_A \\ M_A \\ P_B \\ S_B \\ M_B \end{bmatrix}
=
\begin{bmatrix}
\frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\
0 & \frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} & 0 & -\frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} \\
0 & -\frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{6\,EI}{L^2} & \frac{2\,EI}{L} \\
-\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\
0 & -\frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} & 0 & \frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} \\
0 & -\frac{6\,EI}{L^2} & \frac{2\,EI}{L} & 0 & \frac{6\,EI}{L^2} & \frac{4\,EI}{L}
\end{bmatrix}
\begin{bmatrix} u_A \\ w_A \\ \phi_A \\ u_B \\ w_B \\ \phi_B \end{bmatrix}
$$

Beam type II

$$\begin{bmatrix} P_A \\ S_A \\ M_A \\ P_B \\ S_B \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{3EI}{L^3} & -\frac{3EI}{L^2} & 0 & -\frac{3EI}{L^3} & 0 \\ 0 & -\frac{3EI}{L^2} & \frac{3EI}{L} & 0 & \frac{3EI}{L^2} & 0 \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{3EI}{L^3} & \frac{3EI}{L^2} & 0 & \frac{3EI}{L^3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_A \\ w_A \\ \phi_A \\ u_B \\ w_B \\ \phi_B \end{bmatrix}$$