Version FC/3

EGT2
ENGINEERING TRIPOS PART IIA

Tuesday 29 April 2014 2 to 3.30

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Figure 1(a) shows an unsymmetrical solid cross-section. Compute the principal second moments of area of the cross-section. [40%]

(b) Figure 1(b) shows a thin-walled channel-like open cross-section with a constant thickness t.

(i) Comment whether restrained warping torsion is important for this cross-section. For a beam with such a cross-section, how could its torsional stiffness be increased? [20%]

(ii) Compute the shear centre of the cross-section. [40%]

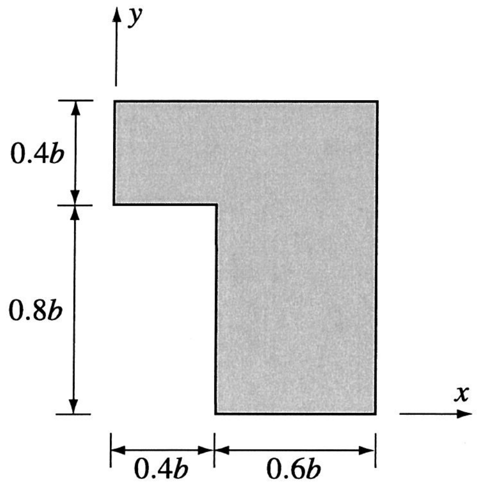

Fig. 1(a)

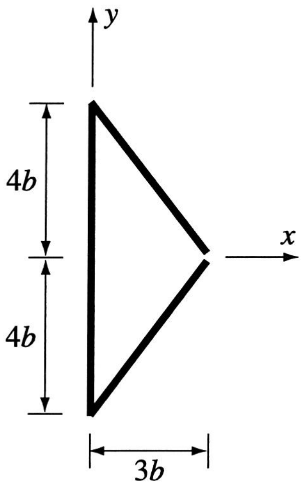

Fig. 1(b)

2 Figure 2 shows a cantilever which lies in the $x-y$ plane and has a constant radius R. The flexural stiffness of the cantilever is EI and its torsional stiffness is $GJ = 3EI$. At its free end the cantilever is supported by a cable pointing in the z-direction. The axial stiffness of the cable is $EA = 10EI/R^2$. The cantilever is loaded at its free end by a force F acting in the negative z-direction. The structure has one redundancy which is designated as a unit tension in the cable.

(a) Compute the axial force in the cable. [50%]

(b) For the reaction force at the support to which the cable is attached, first sketch and then determine the equation of the influence line corresponding to a transverse rolling point load on the cantilever. [30%]

(c) Due to fabrication errors the cable has been produced 0.1% too short. Therefore, the connection between the cantilever and cable was achieved by forcing the two members into place. Compute the axial force in the cable due to both the load F and the lack of fit between the cantilever and the cable. [20%]

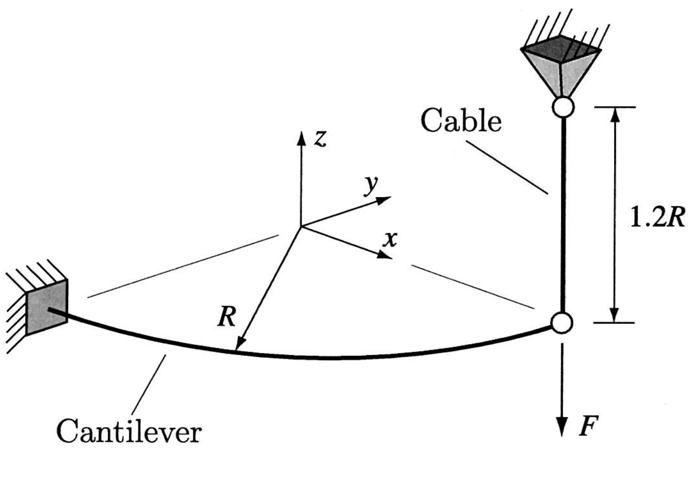

Fig. 2

3 (a) Write an expression for the total potential energy Π (as a function of the deflection x) of a spring of stiffness k loaded by a constant force W. Illustrate your answer with a graph of $\Pi(x)$. [10%]

(b) A linear spring has constant stiffness k and is initially stress-free. It is then slowly compressed by a distance x by a hydraulic jack, at which point the jack is exerting a load W on the spring.

(i) What was the work done by the jack, expressed as a function of W and x? [10%]

(ii) Does your answer to part (i) above still hold if the spring has a nonlinear stiffness? [10%]

(c) The mechanism shown in Fig. 3(a) consists of a light rigid triangular panel ABC on a hinge support at C. The triangle has a right angle at C and sides AC and BC are of equal length R. The panel is connected from B to a point D on a vertical wall by a linear spring of constant stiffness k. The spring connection at the wall is such that the spring always remains horizontal, even as the point D moves vertically along the wall. The spring is unstressed when BC is vertical.

A vertical load F is applied at A such that the panel rotates by an angle α about C, as shown in Fig. 3(b).

(i) Plot the equilibrium paths on a graph of F versus α and determine the stability of each path. [40%]

(ii) If instead the spring was stress-free when BC makes a small angle α_0 with the vertical, describe qualitatively the behaviour of the system when the load F is increased from zero, considering both cases of α_0 positive and negative. [30%]

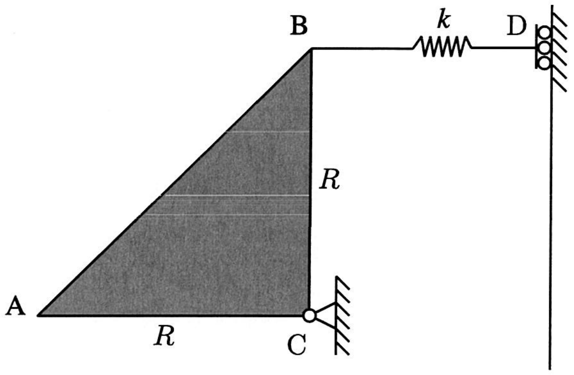

Fig. 3(a)

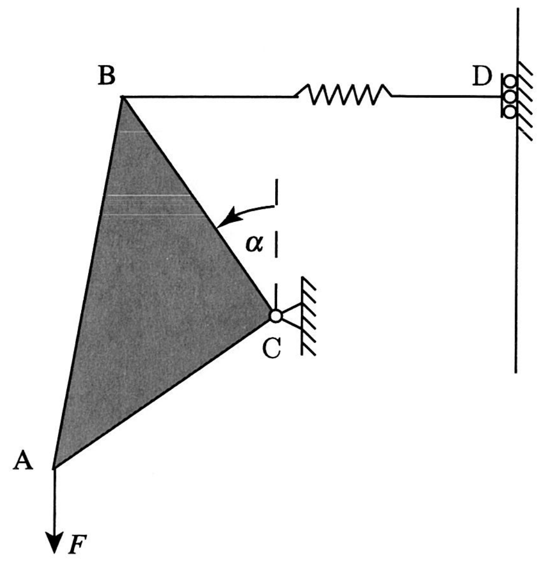

Fig. 3(b)

(TURN OVER

4 (a) For the frames shown in Fig. 4(a) and Fig. 4(b), write down the 2×2 stiffness matrices that relate the rotations at the points A and B to any applied moments at these points. The matrices should be populated by terms involving some combination of k, s and c, where $k = EI/L$ and s and c are the stiffness and carry-over stability functions. All members have the same flexural rigidity EI and length L. All columns carry an axial load P, and all beams carry no axial load. All members are fully braced against out of plane behaviour. [30%]

(b) All members of the frame shown in Fig. 4(a) are 4 m long steel beams with 610×229 UB 125 section with their webs in the plane of the paper. A table of stiffness and carry-over stability functions is provided in Table 1, where P_E is the Euler load.

(i) Determine the effective length of the column AB with respect to in-plane buckling and the corresponding critical load P. [30%]

(ii) Sketch the buckling mode shape, clearly indicating all joint rotations as a proportion of θ_A. [20%]

(c) Starting from the 2×2 stiffness matrix relating the end rotations and end moments of a simple Euler strut for the no-sway case, show how a 3×3 matrix may be constructed to include the possibility of sway. [20%]

P/P_E	s	c
0.0	4.00	0.50
1.0	2.47	1.00
1.1	2.28	1.11
1.2	2.09	1.25
1.3	1.89	1.42
1.4	1.68	1.66
1.5	1.46	1.97
1.6	1.22	2.43
1.7	0.98	3.17
1.8	0.72	4.50
1.9	0.44	7.66

P/P_E	s	c
2.0	0.14	24.68
2.1	-0.18	-21.07
2.2	-0.52	-7.51
2.3	-0.89	-4.62
2.4	-1.30	-3.37
2.5	-1.75	-2.67
2.6	-2.25	-2.23
2.7	-2.81	-1.93
2.8	-3.44	-1.71
2.9	-4.18	-1.54
3.0	-5.03	-1.42

Table 1

 (cont.

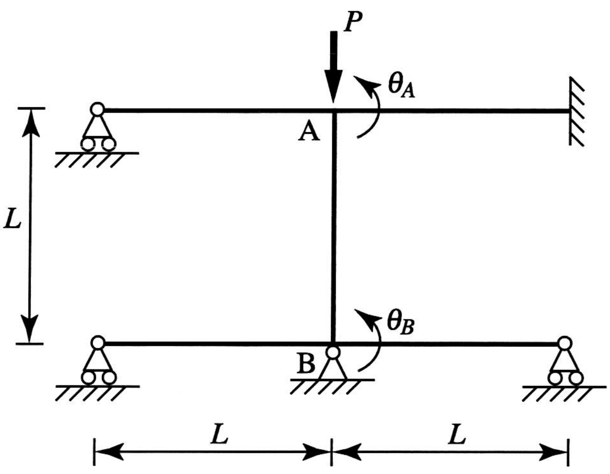

Fig. 4(a)

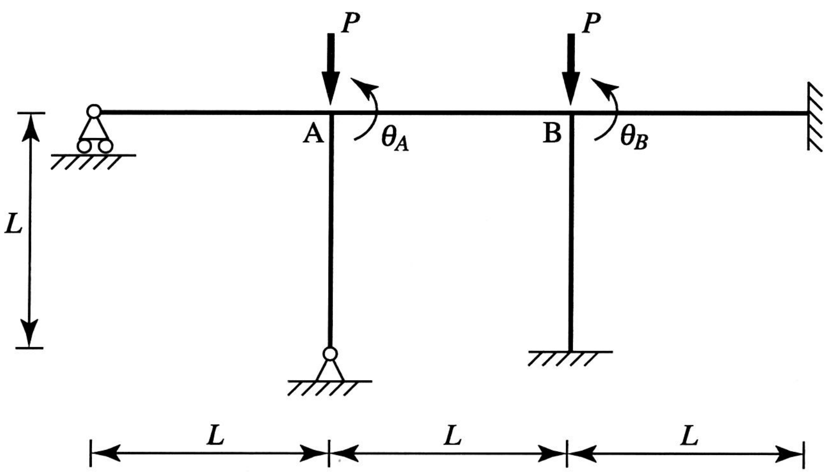

Fig. 4(b)

END OF PAPER

THIS PAGE IS BLANK