Version FC/3

EGT2
ENGINEERING TRIPOS PART IIA

Monday 10 May 2021    9 to 10.40

**Module 3D4**

**STRUCTURAL ANALYSIS AND STABILITY**

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet and at the top of each answer sheet.*

**STATIONERY REQUIREMENTS**
Write on single-sided paper.

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**
CUED approved calculator allowed.
You are allowed access to the electronic version of the Engineering Data Books.

**10 minutes reading time is allowed for this paper at the start of the exam.**

**The time taken for scanning/uploading answers is 15 minutes.**

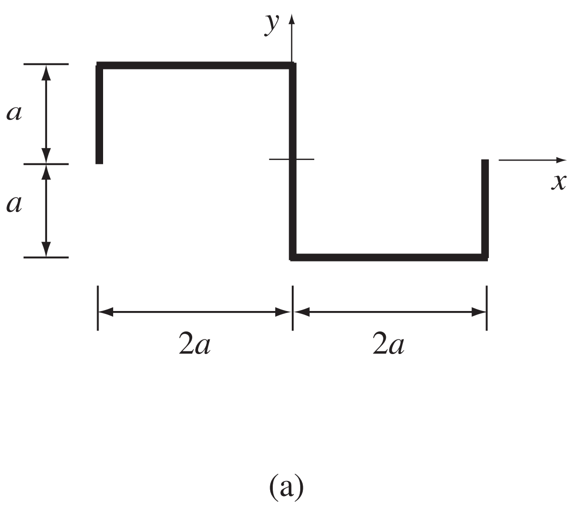
**Your script is to be uploaded as a single consolidated pdf containing all answers.**

1 (a) Figure 1(a) shows a thin-walled cross-section made from a sheet of uniform thickness $t$, density $\rho$ and elastic modulus $E$.

(i) Determine the principal second moments of area of the cross-section. [45%]

(ii) The section is mounted as a simply supported beam of length $L$, whereby the $y$-axis is pointing upwards and is loaded by its own weight. The shear centre of the cross-section is located at its centroid. Determine the maximum displacement of the simply supported beam. [20%]

(b) Figure 1(b) shows a thin-walled, channel-like cross-section with a uniform thickness $t$. Determine the shear centre of the cross-section. [35%]



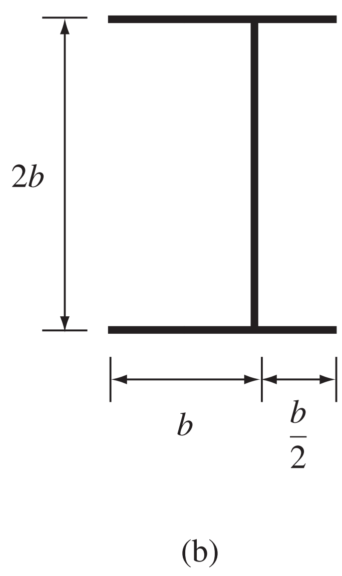




Fig. 1

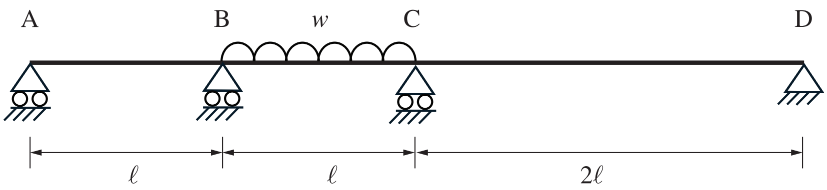
2 (a) Write down the statement of the reciprocal theorem in the form of an equation considering a structure of your choice for illustration. [15%]

(b) The three-span beam shown in Fig. 2 has a uniform stiffness $EI$ and is subjected to two different load cases. In the first load case shown in Fig. 2(a) the mid span carries a uniformly distributed load $w$. In the second load case shown in Fig. 2(b) the right span carries a point load $W$.

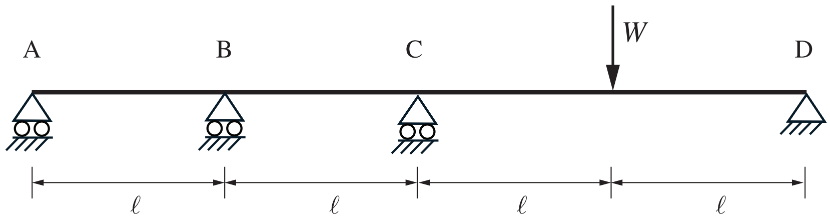
(i) Sketch the bending moment diagrams corresponding to each of the load cases, while marking their salient features. No numerical computations are necessary. [20%]

(ii) Sketch the influence line for the support reaction force at A. Based on this influence line choose a combination of the two load cases to obtain maximum and minimum support reactions. No numerical computations are necessary. [10%]

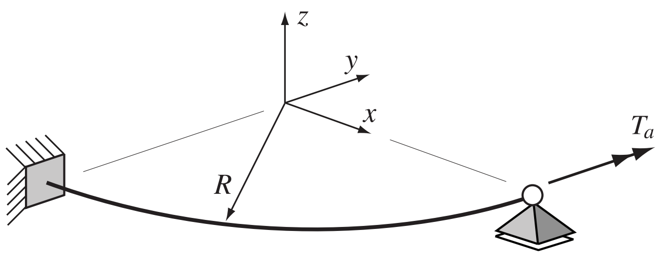
(c) The circular propped cantilever shown in Fig. 2(c) lies in the $x$-$y$ plane and has radius $R$. The flexural stiffness of the cantilever is $EI$ and its torsional stiffness is $GJ = 2EI$. A torque $T_a$ is applied at its propped end. Determine the reaction force at the propped support and the reactions at the built-in support. [55%]




(a)




(b)




(c)

Fig. 2

3 (a) Figure 3 shows a pin-jointed assembly of four rigid rods connected by a central linear elastic spring with spring constant $k$. The boundary conditions are as shown. Lateral loads $C$ and $T$ are applied at the roller-supported ends as shown.

(i) Assuming displacements are small, determine an expression for the total potential energy in the form

$$\Pi(\mathbf{w}) = \frac{1}{2}\mathbf{w}^T\mathbf{K}_{tot}\mathbf{w}$$

where $\mathbf{w} = \begin{pmatrix} w_1 & w_2 \end{pmatrix}^T$ are the displacements at the ends of the spring as shown. [30%]

(ii) When $T = kL/2$, determine the critical value of $C$ that will cause lateral instability. [20%]

(b) The formula for the critical value of equal-and-opposite end-moments which will cause lateral-torsional buckling of an I-beam of length $L$ contains a factor to account for the effect of warping restraint. That factor is:

$$\left(1 + \frac{\pi^2}{L^2}\frac{E\Gamma}{GJ}\right)^{1/2}$$

where $J$ is the St. Venant torsion constant and $\Gamma = ID^2/4$ is a warping constant with $I$ being the second moment of area of the beam cross-section about its minor axis and $D$ being the distance between flange centroids.

(i) Explain why warping restraint is still relevant, even if the ends of the beam are free to warp. [20%]

(ii) Determine the critical value of equal and opposite major-axis end-moments that will cause lateral-torsional buckling of an 8 m long beam with $457 \times 191 \times 98$ UB section. [30%]

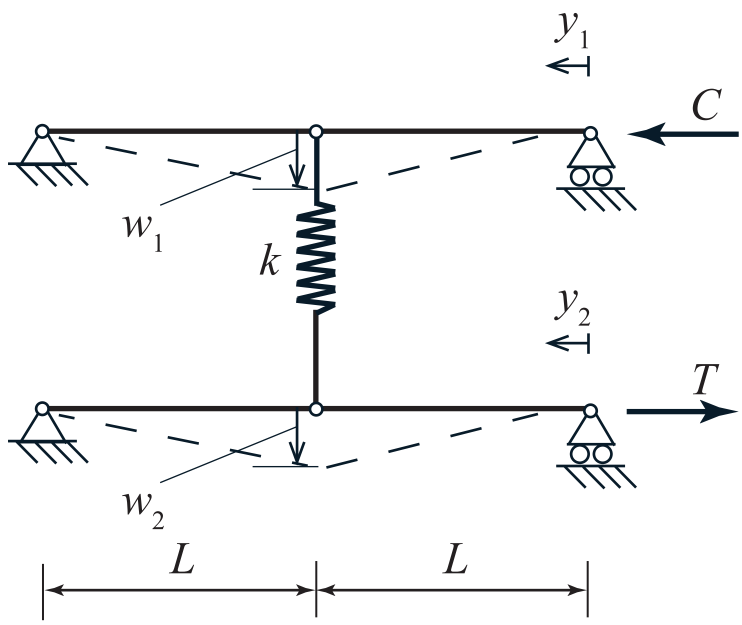



Fig. 3

4 (a) Define what is meant by *effective length* for column buckling. [10%]

(b) For a column in a building, explain why it may often be appropriate to use an effective buckling length equal to the column's actual length, rather than some smaller value that was deduced by elastic subframe analysis. [15%]

(c) Figure 4 shows a rigidly-jointed frame consisting of a continuous beam ABCD supported by columns BE and CF. The support conditions are as shown in the diagram. All members have length $L$ and flexural rigidity $EI$ with respect to bending within the plane of the diagram. Bending out of the plane of the paper is prevented. Equal vertical loads $P$ are applied at B and C as shown.

(i) Determine a $2 \times 2$ tangent stiffness matrix relating the moments and rotations at B and C using $s$ and $c$ stability functions. [25%]

(ii) Using $s$ and $c$ functions in Table 1, estimate the value of $P$ that will cause in-plane flexural buckling of the frame. [25%]

(iii) Provide a clear sketch of the buckling mode, and indicate the magnitudes of the rotations of all joints expressed as a proportion of the rotation at B. [25%]

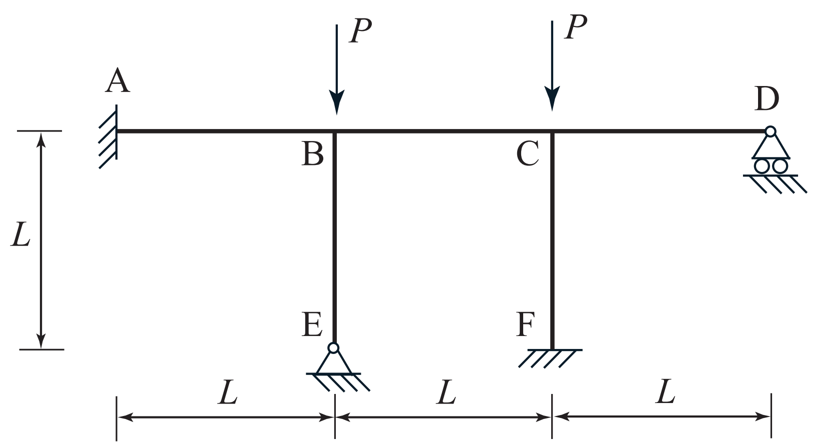



Fig. 4

Table 1. $s$ and $c$ stability functions for an Euler column. $P$ is the axial load and $P_E$ is the Euler load.

| $P/P_E$ | $s$ | $c$ |
|---|---|---|
| 0.0 | 4.0000 | 0.5000 |
| 0.2 | 3.7297 | 0.5550 |
| 0.4 | 3.4439 | 0.6242 |
| 0.6 | 3.1403 | 0.7136 |
| 0.8 | 2.8159 | 0.8330 |
| 1.0 | 2.4674 | 1.0000 |
| 1.2 | 2.0901 | 1.2487 |
| 1.4 | 1.6782 | 1.6557 |
| 1.6 | 1.2240 | 2.4348 |
| 1.8 | 0.7170 | 4.4969 |
| 2.0 | 0.1428 | 24.6841 |
| 2.2 | -0.5194 | -7.5107 |
| 2.4 | -1.3006 | -3.3703 |
| 2.6 | -2.2490 | -2.2312 |
| 2.8 | -3.4449 | -1.7081 |
| 3.0 | -5.0320 | -1.4157 |

**END OF PAPER**