EGT2
ENGINEERING TRIPOS PART IIA

Monday 9 May 2022 9.30 to 11.10

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet and at the top of each extra sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper
Graph Paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: Data Sheet for Question 2 (1 page)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) The thin-walled angle section shown in Fig. 1(a) has a uniform thickness t. Calculate the principal second moments of area of the cross-section. [50%]

(b) Figure 1(b) shows a cantilever of length l with a cross-section as in Fig. 1(a) and a Young's modulus E. A stiff plate is welded to the tip of the cantilever, to which a vertical force F is applied, acting downwards at an offset of $\frac{3}{2}b$ from the vertical leg of the cross-section.

(i) Determine the total deflection of point A at the tip of the cantilever and the intersection of both legs. [20%]

(ii) Determine the axial stress at point B at the root of the cantilever and the intersection of both legs. [30%]

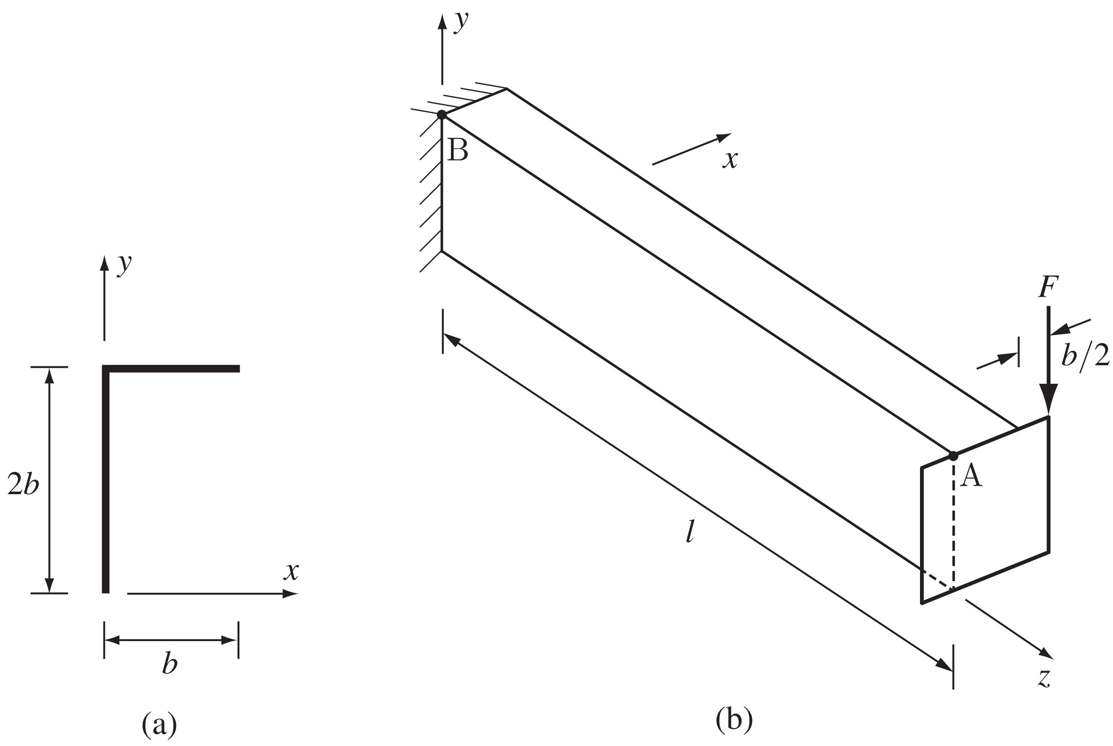

Fig. 1

2 (a) Figure 2(a) shows a beam over three spans. The beam is continuous over the supports, and the left and centre spans contain each a pin. The left span carries a concentrated force F and the centre span a distributed load w.

(i) Determine the influence line for the support reaction at C. [10%]

(ii) Use the influence line to determine the support reaction at C due to the given loading. [10%]

(b) Figure 2(b) shows a beam over three spans. The beam is simply supported at its left end, clamped at its right end and continuous over the two mid-span supports. The left span carries a distributed load w and the centre span two concentrated forces with $F = wl$. The beam has a uniform flexural rigidity EI.

(i) Use the displacement method to determine the rotations of the joints over supports B and C. [50%]

(ii) Determine the moments over supports B and C. [20%]

(iii) Sketch the bending moment diagram. [10%]

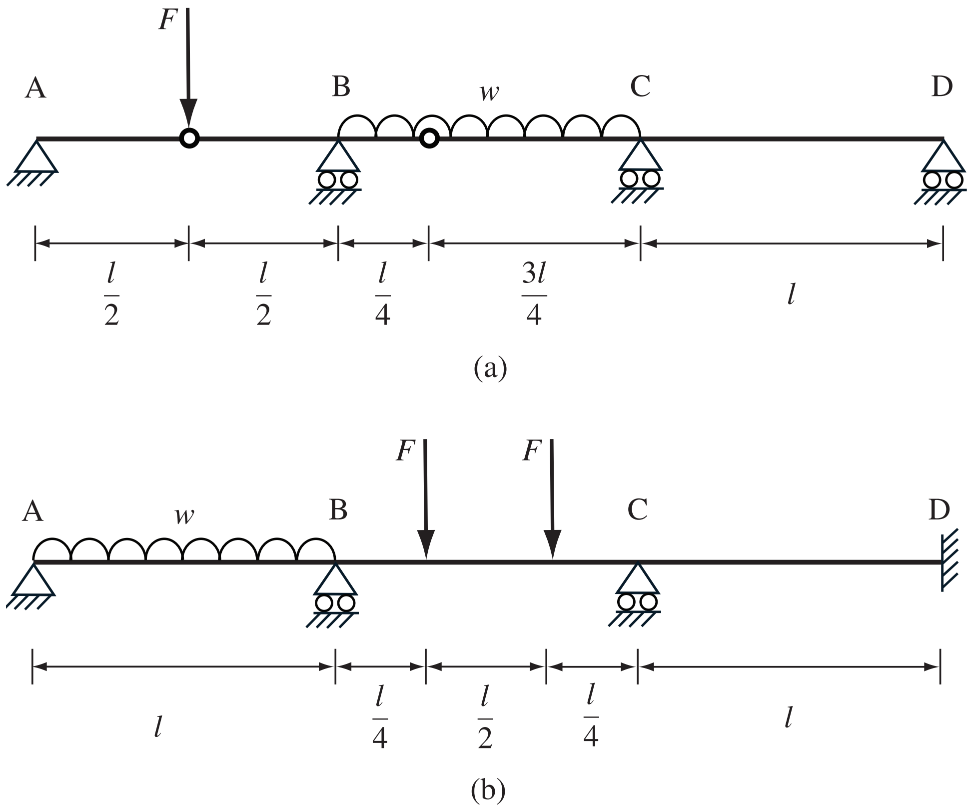

Fig. 2

3 (a) The formula for the critical value of equal-and-opposite end-moments which will cause lateral-torsional buckling of an I-beam of length L contains a factor to account for the effect of warping restraint. That factor is

$$\left(1 + \frac{\pi^2}{L^2}\frac{E\Gamma}{GJ}\right)^{1/2}$$

where J is the St. Venant torsion constant and $\Gamma = ID^2/4$ is the warping constant with I being the second moment of area of the beam cross-section about its minor axis and D being the distance between flange centroids.

Determine the critical value of equal-and-opposite major-axis end-moments that will cause lateral-torsional buckling of a 6 m steel beam with $533 \times 210 \times 122$ UB section. [40%]

(b) Figure 3 shows a column ABC carrying a vertical load P at its knee B. The support conditions are as shown in the diagram. All members have length L and flexural rigidity EI with respect to bending within the plane of the diagram. Bending out of the plane of the diagram is prevented.

(i) Determine the 2×2 tangent stiffness matrix relating the moments and rotations at B and C using s and c stability functions. [20%]

(ii) Using the table of s and c stability functions provided, estimate the smallest value of P that will cause in-plane flexural buckling of the frame. [20%]

(iii) Provide a sketch of the buckling mode, and indicate the magnitudes of the rotations of all joints expressed as a proportion of the rotation at B. [20%]

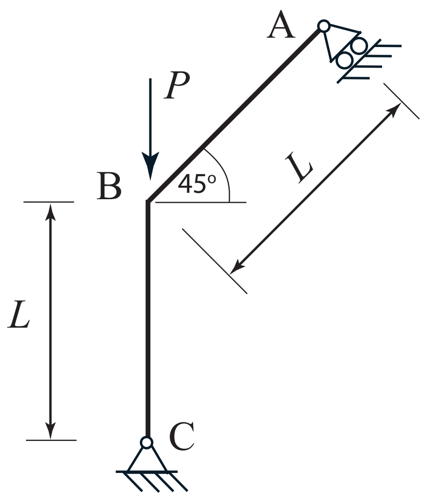

Fig. 3

Table 1. s and c stability functions for an Euler column. P is the axial load and P_E is the Euler load.

$\frac{P}{P_E}$	s	c
0.0	4.0000	0.5000
0.2	3.7297	0.5550
0.4	3.4439	0.6242
0.6	3.1403	0.7136
0.8	2.8159	0.8330
1.0	2.4674	1.0000
1.2	2.0901	1.2487
1.4	1.6782	1.6557
1.6	1.2240	2.4348
1.8	0.7170	4.4969
2.0	0.1428	24.6841
2.2	-0.5194	-7.5107
2.4	-1.3006	-3.3703
2.6	-2.2490	-2.2312
2.8	-3.4449	-1.7081
3.0	-5.0320	-1.4157

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4 (a) The rigid rod of length L shown in Fig. 4 is pinned at its base. At the top it is connected to a linear elastic spring of spring constant k. This spring stays always horizontal. When the spring is unstressed the rod makes an angle θ_0 with the vertical. The rod carries a vertical load P.

(i) Write an expression for the total potential energy of the system as a function of the rod angle θ (and θ is not necessarily small). [10%]

(ii) Derive an equation for the equilibrium paths $P(\theta)$. [10%]

(iii) Sketch the equilibrium paths on a graph of P against θ, for the case $\theta_0 = 0$ and a few values of θ_0 nearby. [10%]

(iv) Given θ_0, derive an expression for the value $\theta = \theta_{max}$ at which the peak load P_{max} occurs. Hence find an expression for P_{max} as a function of θ_{max}. [20%]

(v) Briefly explain what implications this analysis has regarding the assessment of structural stability by small deflection eigenvalue analysis. [10%]

(b) A tapering column of length L is simply supported at each end. Its flexural rigidity EI varies linearly with distance along the column, such that $EI = a + bx$. Estimate the buckling load of the column, assuming the buckling mode shape $\phi(x)$ is quadratic. Comment on how your estimate relates to the Euler load. [40%]

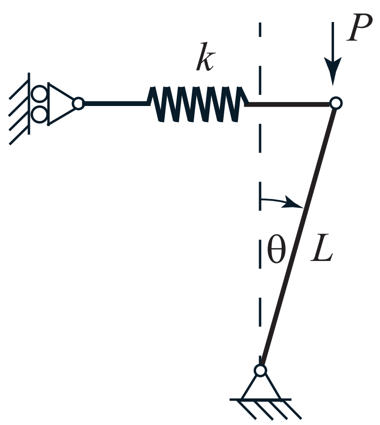

Fig. 4

END OF PAPER

Data Sheet for Question 2: Stiffness Matrices.

Notation and sign convention

Beam type I

$$\begin{bmatrix} S_A \\ M_A \\ S_B \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} & -\frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{4EI}{L} & \frac{6\,EI}{L^2} & \frac{2\,EI}{L} \\ -\frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} & \frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} \\ -\frac{6\,EI}{L^2} & \frac{2\,EI}{L} & \frac{6\,EI}{L^2} & \frac{4\,EI}{L} \end{bmatrix} \begin{bmatrix} w_A \\ \phi_A \\ w_B \\ \phi_B \end{bmatrix}$$

Beam type II

$$\begin{bmatrix} S_A \\ M_A \\ S_B \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{3\,EI}{L^3} & -\frac{3\,EI}{L^2} & -\frac{3\,EI}{L^3} & 0 \\ -\frac{3EI}{L^2} & \frac{3EI}{L} & \frac{3\,EI}{L^2} & 0 \\ -\frac{3\,EI}{L^3} & \frac{3\,EI}{L^2} & \frac{3\,EI}{L^3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_A \\ \phi_A \\ w_B \\ \phi_B \end{bmatrix}$$