2018 IIA 3D5 - Water Engin1eering Dr D. Liang

1.

(a) Thiessen's polygons attach a representative area to each rain gauge. The size of the representative area (a polygon) is based on how close each gauge is to the others surrounding it. It is a method of determining the uneven distribution of rainfall over a catchment according to measurements at rain gauges.

(b) Field capacity corresponds to the superior limit of available water and represents the moisture of the soil after drainage of the water contained in the macropores by gravity action.

(c.i)

The total infiltration during the first two-hour period is:

$$\int_{t_1}^{t_2} f \cdot dt = f_c(t_2 - t_1) - \frac{1}{K_f}(f_0 - f_c)\left(e^{-K_f t_2} - e^{-K_f t_1}\right)$$

$$= 2(2-0) - \frac{1}{1}(20-2)\left(e^{-2} - e^{0}\right) = 19.56 \text{ mm}$$

The excess rain in the first two-hour period is: 30×2-19.56 = 40.44 mm

This corresponds to 20.22 mm/h rainfall excess.

The total infiltration during the second two-hour period is:

$$\int_{t_1}^{t_2} f \cdot dt = f_c(t_2 - t_1) - \frac{1}{K_f}(f_0 - f_c)\left(e^{-K_f t_2} - e^{-K_f t_1}\right)$$

$$= 2(4-2) - \frac{1}{1}(20-2)\left(e^{-4} - e^{-2}\right) = 6.11 \text{ mm}$$

The excess rain in the first two-hour period is: 30×2-6.11 = 53.89 mm

This corresponds to 26.95 mm/h rainfall excess.

(c.ii)

The total volume of the excess flow in the first two-hour period is:

$$40.44 \times 10^{-3} \times 6 \times 10^{6} = 2.426 \times 10^{5} \text{ m}^3$$

The total volume of the excess flow in the second two-hour period is:

$$53.89 \times 10^{-3} \times 6 \times 10^{6} = 3.233 \times 10^{5} \text{ m}^3$$

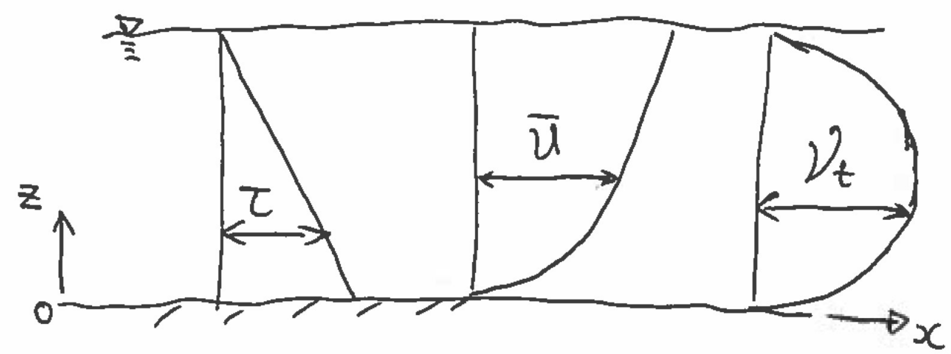
The volume distribution of the excess flow due to the four-hour rain is:

Time (h)	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16
First 2-hour rain (%)	3	18	35	27	12	5	0	0
First 2-hour volume (×10^4 m^3)	0.73	4.37	8.49	6.55	2.91	1.21	0	0
Second 2-hour rain (%)	0	3	18	35	27	12	5	0
Second 2-hour volume (×10^4 m^3)	0	0.97	5.82	11.32	8.73	3.88	1.62	0
Total volume (m^3)	0.73	5.34	14.31	17.87	11.64			

The peak discharge happens during 6-8 hours:

$$\frac{17.87 \times 10^{4}}{2 \times 3600} = 24.8 \text{ m}^3/\text{s}$$

(d)



Suspended sediment concentration follows the Rouse profile.

(e)

The bed layer is a thin layer on top of the immobile bed. Within the bed layer, sediment transport occurs in the form of bed load. Above this layer, the sediment transport occurs in the form of suspended load.

2

(a) Assume the Manning roughness coefficient is the same for different flow rates.
When depth is 0.8 m:

$$U = \frac{3.6}{3 \times 0.8} = 1.5 \text{ m/s}$$

$$R_h = \frac{3 \times 0.8}{3 + 2 \times 0.8} = 0.522 \text{ m}$$

$$U = C\sqrt{R_{h1} S_b} \qquad \text{so } C = \frac{1.5}{\sqrt{0.522 \times 0.001}} = 65.65$$

$$C = \frac{1}{n} \cdot R_h^{1/6} \qquad \text{so } n = \frac{1}{C} \cdot R_h^{1/6} = \frac{0.522^{1/6}}{65.65} = 0.0137$$

When depth is 1.5 m:

$$R_h = \frac{3 \times 1.5}{3 + 2 \times 1.5} = 0.75 \text{ m}$$

$$C = \frac{1}{n} \cdot R_h^{1/6} = \frac{0.75^{1/6}}{0.0137} = 69.58$$

$$U = C\sqrt{R_{h1} S_b} = 69.58\sqrt{0.75 \times 0.001} = 1.91 \text{ m/s}$$

$$Q = 1.91 \times 4.5 = 8.60 \text{ m}^3\text{/s}$$

Assume the roughness height is the same for different flows.
When depth is 0.8 m:

According to $C = 7.8 \ln\left(\frac{12.0 \cdot R_h}{k_s}\right)$

$$65.65 = 7.8 \ln\left(\frac{12.0 \cdot 0.522}{k_s}\right)$$

$$k_s = 0.00139 \text{ mm}$$

When depth is 1.5 m:

$$C = 7.8 \ln\left(\frac{12.0 \cdot R_h}{k_s}\right) = 7.8 \ln\left(\frac{12.0 \cdot 0.75}{0.00139}\right) = 68.45$$

$$U = C\sqrt{R_{h1} S_b} = 68.45\sqrt{0.75 \times 0.001} = 1.875 \text{ m/s}$$

$$Q = 1.875 \times 4.5 = 8.44 \text{ m}^3\text{/s}$$

(b)

Continuity eq. $\quad 2 \times 50 U_1 = 1.5 \times (50 - 5 \times 2.8) U_2$

$$U_1 = 0.54 U_2$$

Energy eq. $\quad 2 + \frac{U_1^2}{2g} = 1.5 + \frac{U_2^2}{2g}$

$$U_2 = 3.72 \text{ m/s}$$

$$Q = 1.5 \times (50 - 5 \times 2.8) \times 3.72 = 200.95 \text{ m}^3\text{/s}$$

(c)

Starting from $\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2}$, with $S_f = 0$

$$\frac{dh}{dx} = \frac{S_b}{1-Fr^2} = \frac{-1}{1-Fr^2}\frac{dz_b}{dx} = \left(\frac{-Fr^2}{1-Fr^2} - 1\right)\frac{dz_b}{dx}$$

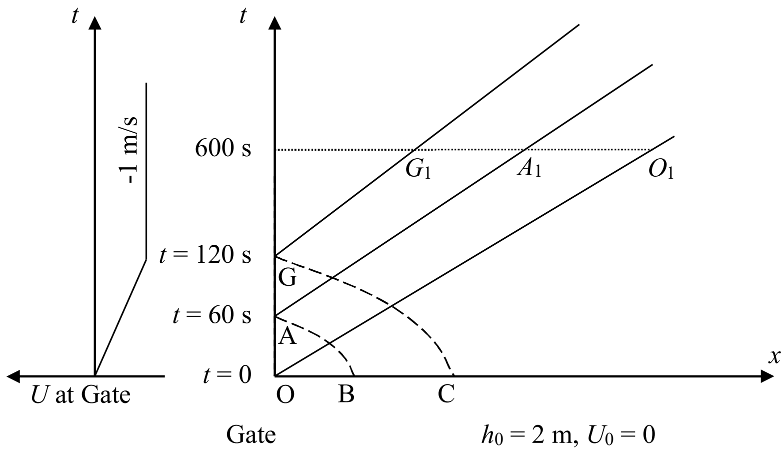
$$\frac{dh}{dx} + \frac{dz_b}{dx} = \left(\frac{-Fr^2}{1-Fr^2}\right)\frac{dz_b}{dx}$$

$$\frac{d\eta}{dx} = \left(\frac{-Fr^2}{1-Fr^2}\right)\frac{dz_b}{dx}$$

On the upstream side of the hump, $\frac{dz_b}{dx} > 0$, so the water level decreases along the flow.

On the downstream side of the hump, $\frac{dz_b}{dx} < 0$, so the water level increases along the flow.

(d)



The positive characteristic starting from O divides the affected/unaffected regions.

Positive line OO_1 is straight: $\frac{dx}{dt} = U_O + \sqrt{gh_O}$

$$\frac{x_{O1} - 0}{600} = 0 + \sqrt{9.81 \times 2.0} \quad \Rightarrow \quad x_{o1} = 2657.7 \text{ m}$$

A point on the graph is ($x_{o1} = 2657.7$ m, $h_{o1} = 2$ m).

U_A = -0.5 m/s. Draw negative line through line A, according the –ve relationship:

$$-0.5 - 2\sqrt{9.81 \cdot h_A} = 0 - 2\sqrt{9.81 \times 2} \Rightarrow h_A = 1.78 \text{ m}$$

Positive line AA_1 is straight: $\frac{dx}{dt} = U_A + \sqrt{gh_A}$

$$\frac{x_{A1} - 0}{600 - 60} = -0.5 + \sqrt{9.81 \times 1.78} \quad \Rightarrow \quad x_{A1} = 1986.5 \text{ m}$$

A point on the graph is ($x_{A1} = 1986.5$ m, $h_{A1} = 1.78$ m).

U_G = -1 m/s. Draw negative line through line G, according the –ve relationship:

$$-1 - 2\sqrt{9.81 \cdot h_G} = 0 - 2\sqrt{9.81 \times 2} \Rightarrow h_G = 1.57 \text{ m}$$

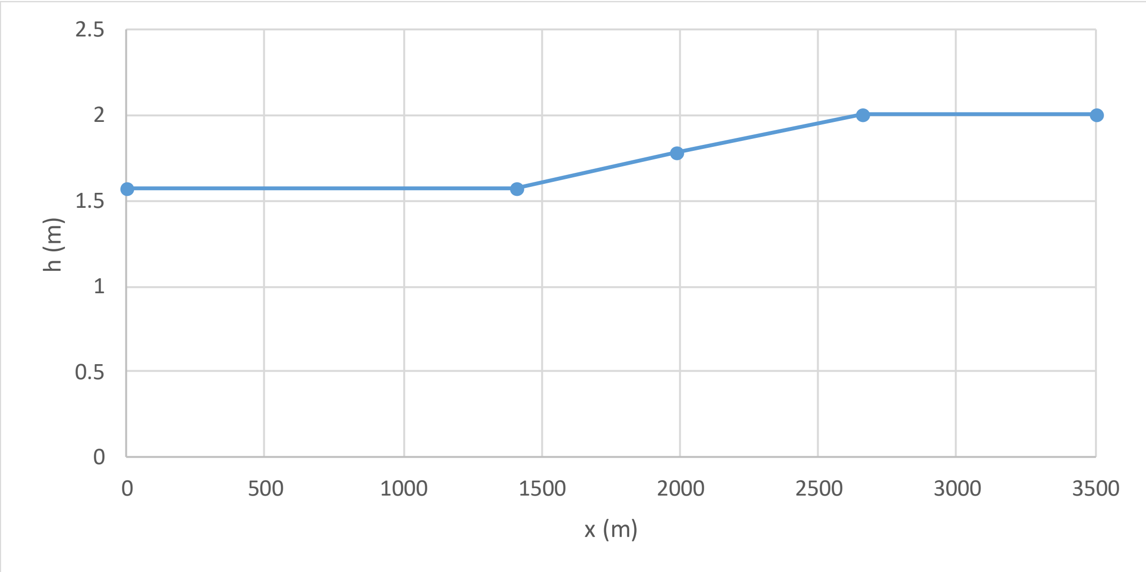
Positive line GG_1 is straight: $\frac{dx}{dt} = U_G + \sqrt{gh_G}$

$$\frac{x_{G1} - 0}{600 - 120} = -1 + \sqrt{9.81 \times 1.57.} \quad \Rightarrow \quad x_{G1} = 1403.8 \text{ m}$$

A point on the graph is ($x_{G1} = 1403.8$ m, h_{G1} = 1.57 m).

10 min after the opening of the gate, the water depth in front of the gate is 1.57m.
The length over which the water depth varies of 1.57 m to 2 m is:

$$x_{o1} - x_{G1} = 2657.7 - 1403.8 = 1254 \text{ m}$$



3

(a.i) The Froude number in an irregular channel is $Fr = U \Big/ \sqrt{g\frac{A}{B}}$.

The width of the water surface is: $B = 3 + 2\times 2\frac{1}{\tan 30^\circ} = 3 + 2\times 3.464 = 9.93$ m

The area of the cross section is: $A = \frac{(3+9.93)\times 2}{2} = 12.93$ m^2

Flow velocity: $U = \frac{48}{12.93} = 3.71$ m/s

Celerity of shallow waves: $C = \sqrt{g\frac{A}{B}} = \sqrt{9.81\times\frac{12.93}{9.93}} = 3.57$ m/s

Hence, the Fr is bigger than 1 and the flow is supercritical.

(a.ii) The wetted perimeter is: $P_h = 3 + 2\times\frac{2}{\sin 30^\circ} = 11$ m

The hydraulic radius is: $R_h = \frac{A}{P_h} = \frac{12.93}{11} = 1.175$ m

According to the Chezy formula $U = C\sqrt{R_h S_b}$, the Chezy coefficient is:

$$C = \frac{U}{\sqrt{R_h S_b}} = \frac{3.71}{\sqrt{1.175\times 0.004}} = 54.12$$

According to the Manning formula $C = \frac{1}{n}R_h^{1/6}$, the Manning coefficient is:

$$n = \frac{1}{C}R_h^{1/6} = \frac{1.175^{1/6}}{54.12} = 0.019$$

(b.i and b.ii)

Given $d = 0.2$ mm, then $d_* = d\cdot\left(\frac{g(s-1)}{\nu^2}\right)^{1/3} = 0.0002\times\left(\frac{9.81(2.65-1)}{10^{-12}}\right)^{1/3} = 5.06$

$$w_s = \frac{\nu}{d}\left[\sqrt{10.36^2 + 1.049\cdot d_*^3} - 10.36\right] = \frac{10^{-6}}{0.0002}\left[\sqrt{10.36^2 + 1.049\cdot 5.06^3} - 10.36\right]$$

$$w_s = 0.026 \text{ m/s}$$

$$\theta_c = \frac{0.30}{1+1.2d_*} + 0.055\left[1 - \exp(-0.02d_*)\right] = 0.04242 + 0.00529 = 0.0477$$

In the summer,

The hydraulic radius is: $R_h = \frac{A}{P_h} = \frac{5\times 1}{5 + 2\times 1} = 0.714$ m

The total bed shear stress is: $\tau_b = \rho g R_h S_b = 9810\times 0.714\times 0.0005 = 3.50$ Pa

The total shear velocity is: $u_* = \sqrt{\frac{\tau_b}{\rho}} = \sqrt{\frac{3.50}{1000}} = 0.059$ m/s

The total Chezy coefficient is: $C = 7.8\ln\left(\frac{12.0\cdot R_h}{k_s}\right) = 7.8\ln\left(\frac{12.0\cdot 0.714}{0.02}\right) = 47.27$

The flow velocity is: $U = C\sqrt{R_h S_b} = 47.27\times\sqrt{0.714\times 0.0005} = 0.89$ m/s

The grain-related Chezy coefficient is: $C' = 7.8\ln\left(\frac{12.0 \times 0.714}{3 \times 0.0002}\right) = 74.62$

The grain-related shear stress is: $\tau_b' = \rho g \frac{U^2}{C'^2} = 9810 \frac{0.89^2}{74.62^2} = 1.396 \text{ Pa}$

$$\theta' = \frac{\tau_b'}{g(\rho_s - \rho)d} = \frac{1.396}{9.81(2650 - 1000)0.2 \times 10^{-3}} = 0.43$$

$$T = \frac{\theta' - \theta_c}{\theta_c} = \frac{0.43 - 0.0477}{0.0477} = 8.01$$

$$\bar{c}(a) = 0.015\frac{d \cdot T^{1.5}}{a \cdot d_*^{0.3}} = 0.015\frac{0.2 \times 10^{-3} \cdot 8.01^{1.5}}{2 \times 10^{-2} \cdot 5.06^{0.3}} = 0.00209 = 5.54 \text{ kg/m}^3$$

The grain-related shear velocity is: $u_*' = \sqrt{\frac{\tau_b'}{\rho}} = \sqrt{\frac{1.396}{1000}} = 0.037 \text{ m/s}$

$$a/h = \frac{2 \times 10^{-2}}{1} = 0.02 \text{ , } w_s/(\kappa u_*) = \frac{0.026}{0.4 \times 0.059} = 1.10$$

From the Table in the data sheet,

$$I_1 = 0.646 + (0.310 - 0.646)\frac{1.10 - 1.0}{1.5 - 1.0} = 0.579$$

$$-I_2 = 1.448 + (0.873 - 1.448)\frac{1.1 - 1.0}{1.5 - 1.0} = 1.333$$

$$q_s = \int_a^h \bar{c}(z)\bar{u}(z)dz = 11.6 \cdot u_* \cdot \bar{c}(a) \cdot a \cdot \left[I_1 \ln\left(\frac{30h}{k_s}\right) + I_2\right]$$

$$q_s = 11.6 \cdot 0.059 \cdot 5.54 \cdot 0.02 \cdot \left[0.579\ln\left(\frac{30 \times 1}{0.02}\right) \text{-} 1.333\right] = 0.22 \text{ kg/(m s)}$$

Over the whole width: $q_s B = 0.22 \times 5 = 1.10$ kg/s

In the winter,

The hydraulic radius is: $R_h = \frac{A}{P_h} = \frac{5 \times 2}{5 + 2 \times 2} = 1.11$ m

The total bed shear stress is: $\tau_b = \rho g R_h S_b = 9810 \times 1.11 \times 0.0005 = 5.44 \text{ Pa}$

The total shear velocity is: $u_* = \sqrt{\frac{\tau_b}{\rho}} = \sqrt{\frac{5.44}{1000}} = 0.074 \text{ m/s}$

The total Chezy coefficient is: $C = 7.8\ln\left(\frac{12.0 \cdot R_h}{k_s}\right) = 7.8\ln\left(\frac{12.0 \cdot 1.11}{0.02}\right) = 50.71$

The flow velocity is: $U = C\sqrt{R_h S_b} = 50.71 \times \sqrt{1.11 \times 0.0005} = 1.195 \text{ m/s}$

The grain-related Chezy coefficient is: $C' = 7.8\ln\left(\frac{12.0 \times 1.11}{3 \times 0.0002}\right) = 78.06$

The grain-related shear stress is: $\tau_b' = \rho g \frac{U^2}{C'^2} = 9810 \frac{1.195^2}{78.06^2} = 2.30 \text{ Pa}$

$$\theta' = \frac{\tau_b'}{g(\rho_s - \rho)d} = \frac{2.30}{9.81(2650 - 1000)0.2 \times 10^{-3}} = 0.71$$

$$T = \frac{\theta' - \theta_c}{\theta_c} = \frac{0.71 - 0.0477}{0.0477} = 13.88$$

$$\bar{c}(a) = 0.015 \frac{d \cdot T^{1.5}}{a \cdot d_*^{0.3}} = 0.015 \frac{0.2 \times 10^{-3} \cdot 13.88^{1.5}}{2 \times 10^{-2} \cdot 5.06^{0.3}} = 0.00477 = 12.64 \ \text{kg/m}^3$$

The grain-related shear velocity is: $u_*' = \sqrt{\frac{\tau_b'}{\rho}} = \sqrt{\frac{2.30}{1000}} = 0.048 \ \text{m/s}$

$$a / h = \frac{2 \times 10^{-2}}{2} = 0.01 \ \text{m}, \quad w_s / (\kappa u_*) = \frac{0.026}{0.4 \times 0.074} = 0.88$$

From the Table in the data sheet,

$$I_1 = 2.174 + (0.788 - 2.174) \frac{0.88 - 0.6}{1.0 - 0.6} = 1.20$$

$$-I_2 = 4.254 + (2.107 - 4.254) \frac{0.88 - 0.6}{1.0 - 0.6} = 2.75$$

$$q_s = \int_a^h \bar{c}(z) \bar{u}(z) dz = 11.6 \cdot u_* \cdot \bar{c}(a) \cdot a \cdot \left[I_1 \ln\left(\frac{30h}{k_s}\right) + I_2 \right]$$

$$q_s = 11.6 \cdot 0.074 \cdot 12.64 \cdot 0.02 \cdot \left[1.20 \ln\left(\frac{30 \times 2}{0.02}\right) - 2.75 \right] = 1.49 \ \text{kg/(m s)}$$

Over the whole width: $q_s B = 1.49 \times 5 = 7.45$ kg/s

(b.iii)

In the summer, h = 1 m, U = 0.89 m/s, u_* = 0.059 m/s, A = 5 m^2.

$D_x = D_L + D_{tx} = (5.86 + 0.15) h u_* = 6.01 \times 1.0 \times 0.059 = 0.35 \ \text{m}^2\text{/s}$

This is one-dimensional instant release problem. Assume the substance is conservative.

$$\bar{c}(x,t) = \frac{M / A}{(4\pi t)^{1/2} \sqrt{D_x}} \exp\left(- \frac{(x - Ut)^2}{4 D_x t} \right)$$

When t = 30 min = 3600 s, the peak concentration is:

$$\frac{M / 5}{(4\pi \times 3600)^{1/2} \sqrt{0.35}} = 0.1$$

$$M = 62.9 \ \text{kg}$$

4.

(a) It is defined to be the ratio of the rate of advection of a physical quantity by the flow to the rate of diffusion of the same quantity driven by an appropriate gradient. In the context of species or mass transfer, the Péclet number is the product of the Reynolds number and the Schmidt number

(b.i)

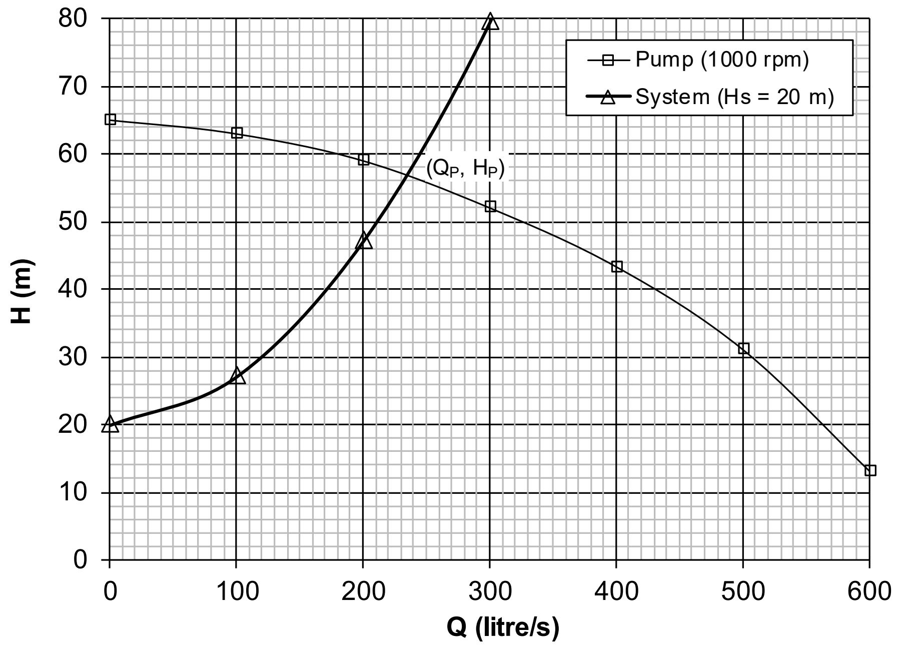
Static lift = 140 – 120 = 20 m.

Relative roughness height: $\frac{k_s}{D} = 0.0001$

The head of the system is: $H = 20 + \left(20 + \lambda\frac{L}{D}\right)\frac{U^2}{2g} = 20 + \left(20 + \lambda\frac{1000}{0.3}\right)\frac{U^2}{2\times 9.81}$

Q (litre/s)	0	100	200	300	400	500	600
Pump (1000 rpm)	65	63	59	52	43.3	31	13
U (m/s)	0	1.42	2.83	4.25	5.66	7.08	8.49
Re	0	4.2E+5	8.5E+5	1.3E+6	1.7E+6	2.1E+6	2.5E+6
λ	0	0.015	0.0139	0.0134	0.0131	0.0130	0.0129
System H (m)	20	27.1	47.1	79.4	124.0	181.7	251.6

Plot the pump curve and system curve, which cross at the duty point:
$Q = 235$ l/s



(b.ii)

The static lift is 30 m in the winter. When delivering 235 l/s flow rate, the total head is:

$$H_2 = 30 + \left(20 + \lambda\frac{L}{D}\right)\frac{U^2}{2g} = 30 + \left(20 + \lambda\frac{1000}{0.3}\right)\frac{U^2}{2\times 9.81}$$

Here, U = 3.33 m/s, Re = 1E6, λ = 0.0137, so

$H_2 = 67.0$ m

The new duty point is $Q_2 = 235$ l/s, $H_2 = 67.0$ m. The parabolic line through the origin and this new duty point is

$$H = \frac{H_2}{{Q_2}^2} Q^2 = \frac{67}{235^2} Q^2 = 0.00121 Q^2$$

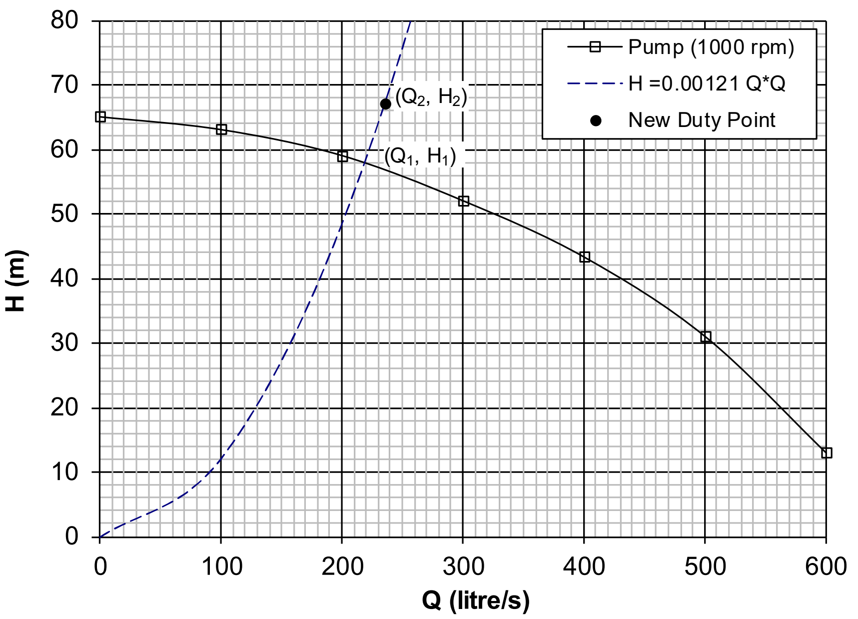
It crosses the pump curve (1000 rpm) at:

$$Q_1 = 220 \text{ l/s}, H_1 = 59.0 \text{ m}$$

According to either $\frac{Q_2}{N_2} = \frac{Q_1}{N_1}$ or $\frac{H_2}{{N_2}^2} = \frac{H_1}{{N_1}^2}$, where $N_1 = 1000$ rpm, we get N_2.

$$\frac{235}{N_2} = \frac{220}{1000} \quad \Rightarrow \quad N_2 = 1068 \text{ rpm}$$

$$\frac{67.0}{{N_p}^2} = \frac{59.0}{1000^2} \quad \Rightarrow \quad N_2 = 1065 \text{ rpm}$$



Comments on Questions

Q1 Hydrology and uniform flows
Some candidates derived the 1-hour unit hydrograph in part (c.ii), which is a bit time-consuming. A quicker way is to directly use the given 2-hour hydrograph. Lots of candidates were confused with the meaning of the 'bed layer'. Some wrongly interpreted it as the stationary sediment layer on the bed or the layer that defines the bed roughness height.

Q2 Open channel flows
There are different ways to answer part (a). A few candidates assumed a constant Chezy coefficient. Although this is not wrong, it does not offer a good estimate of the discharge. Hardly anyone was able to answer part (c) correctly. Quite a few did not even notice the difference between the water depth and the water level. To be able to answer part (c), the bed slope and the water depth variation should be combined to derive the change in the water level.

Q3 Sediment transport
In answering parts (a.i) and (b.i), a few candidates mistakenly used the formulae for infinitely-wide rectangular channels. Generally, the bed friction and Chezy coefficient should be calculated using the hydraulic radius rather than the water depth, and the shallow wave celerity is sqrt(gA/B) rather than sqrt(gh), where B is the channel width at the free surface. Some candidates were not careful with the units of the sediment transport formulae. The formulae in the datasheet give the volumetric near-bed concentration of the suspended sediment.

Q4 Pipeline-pump systems
The common mistake lies in finding the dynamically similar points on the characteristics of a variable-speed pump in part (b)(ii). The solution does not require the complete system characteristic in the winter, as the flow rate is already known. It requires a parabola from the origin to the new duty point.

DL