

1.

(a) The Rational Method expresses a relationship between rainfall intensity and catchment area, as independent variables, and the peak flood discharge resulting from the rainfall, as the dependent variable. The Rational method predicts the peak runoff according to the formula: $Q=CiA$, where C is a runoff coefficient, i is the rainfall intensity, and A is the sub catchment area.

The Rational Method is one of the most commonly used procedures for calculating peak flows from small drainages less than 200 acres, so that the rainfall can be assumed to be uniform.

(b) i. The open channel flow equation is not linear, giving nonlinear relationship between the flow rate and water depth. If the flow is confined in the river channel, then a larger rainfall leads to a higher flow velocity according to Manning formula, hence a shorter time of concentration.

ii. If, subject to heavy rainfall, the river bank bursts and water overflows to the floodplain, rather than restricted to the main channel, then the large flow resistance on the floodplain leads to slower flow.

(c.i) The infiltration rate at the end of the first hour is:

$$f = f_c + (f_0 - f_c)e^{-K_f t} = 2 + (10 - 2)e^{-1 \times 1} = 4.94 \text{ mm/hr}$$

Although this may not be needed, it can be used to double-check whether the following integration is correct. It can also be used for calculating the infiltration in the third hour by regarding the new f_0 to be 4.94 mm/hr.

The infiltration in the first hour is:

$$\begin{aligned} \int_0^T f \cdot dt &= f_c(T - 0) - \frac{1}{K_f}(f_0 - f_c)(e^{-K_f T} - 1) \\ &= 2 - \frac{1}{1}(10 - 2)(e^{-1} - 1) = 2 + 5.06 = 7.06 \text{ mm} \end{aligned}$$

The infiltration in the second hour is zero, because of zero rainfall.

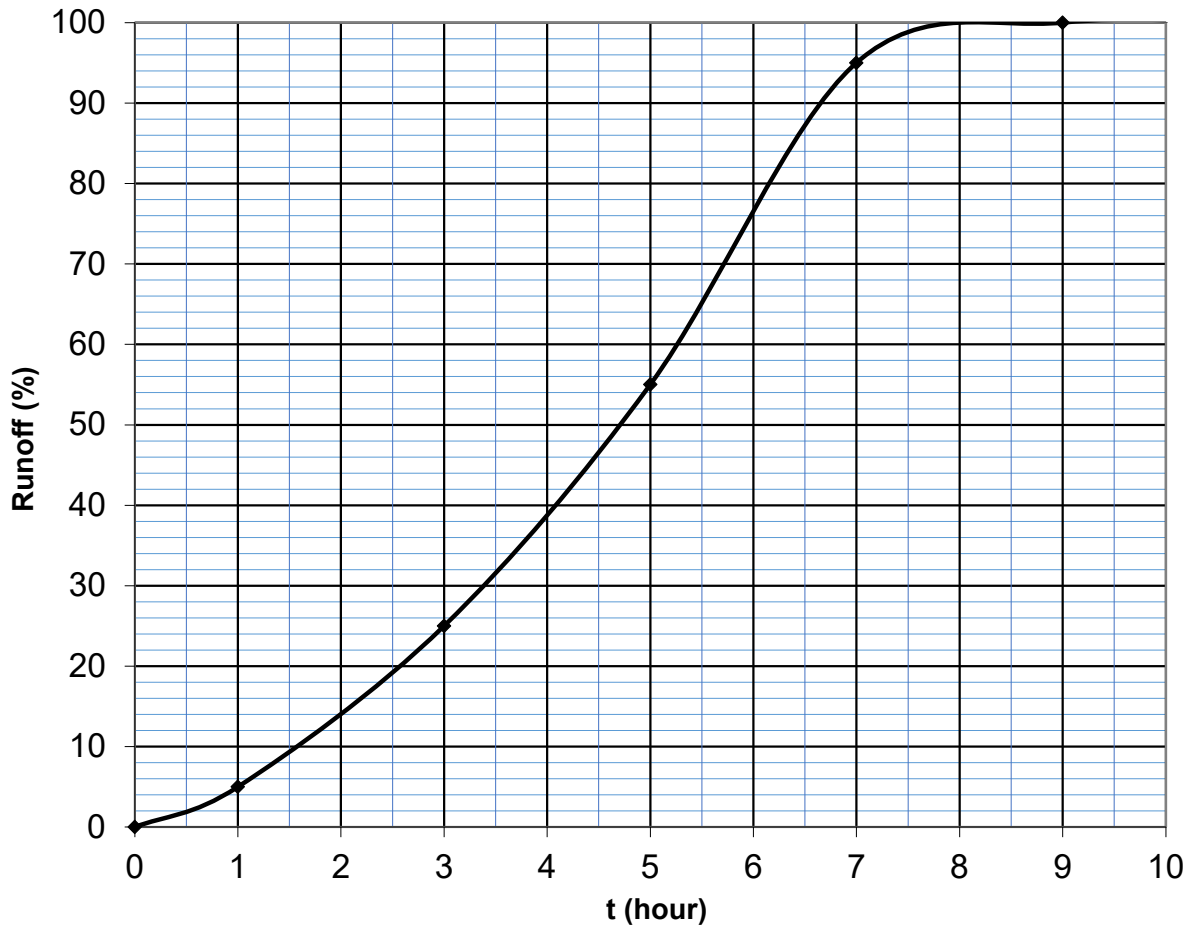
The infiltration in the third hour is:

$$\begin{aligned} \int_0^T f \cdot dt &= f_c(T - 0) - \frac{1}{K_f}(f_0 - f_c)(e^{-K_f T} - 1) \\ &= 2 - \frac{1}{1}(4.94 - 2)(e^{-1} - 1) = 2 + 1.86 = 3.86 \text{ mm} \end{aligned}$$

Hence, the rainfall excess is 2.94 mm in the 1st hour, 0 in the 2nd hour, 6.14 in the 3rd hour.

(c.ii) Construct the S curve

Time (hour)	0	1	3	5	7	9	11	13
Runoff (%)	0	5	25	55	95	100	100	100



Need to find the runoff proportions due to a one-hour uniform rainfall excess.

Time (h)	0.0	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5
S-curve (%)	0.0	2.0	9.5	19.2	31.5	44.2	65.0	86.4	98.8	100.0	100.0	100.0
1-hour shifted S-curve (%)	0.0	0.0	2.0	9.5	19.2	31.5	44.2	65.0	86.4	98.8	100.0	100.0
Subtract the 2 S-curves (%)	0.0	2.0	7.5	9.7	12.3	12.7	20.8	21.4	12.4	1.2	0.0	0.0

The first and last rows in the above table gives the outflow hydrograph for 1-hour rain.

Time (h)	0.0	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5
S-curve (%)	0.0	2.0	9.5	19.2	31.5	44.2	65.0	86.4	98.8	100.0	100.0	100.0
1-hour shifted S-curve (%)	0.0	0.0	2.0	9.5	19.2	31.5	44.2	65.0	86.4	98.8	100.0	100.0
Subtract the 2 S-curves (%)	0.0	2.0	7.5	9.7	12.3	12.7	20.8	21.4	12.4	1.2	0.0	0.0
Contribution from 1st-hour 2.94mm (mm)	0.00	0.06	0.22	0.29	0.36	0.37	0.61	0.63	0.36	0.04	0.00	0.00
Contribution from 2nd-hour 6.14mm (mm)	0.00	0.00	0.00	0.12	0.46	0.60	0.76	0.78	1.28	1.31	0.76	0.07
Total (mm)	0.00	0.06	0.22	0.41	0.82	0.97	1.37	1.41	1.64	1.35	0.76	0.07
Total (m3/s)	0.000	0.002	0.006	0.011	0.023	0.027	0.038	0.039	0.046	0.037	0.021	0.002

The peak flow rate occurs between 7 hr and 8 hr.

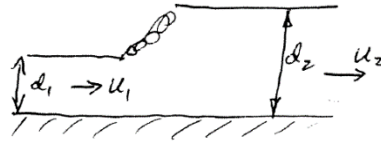
The peak flow rate 7.5 hr after the rain starts is:

$$\frac{(1.28+0.36) \times 10^{-3} \times 10^5}{3600} = 0.0456 \text{ m}^3/\text{s} = 45.6 \text{ litre/s}$$

2.

(a) Consider how small disturbances propagate in the flow. As illustrated in the graph in the lecture notes, the small disturbances generated downstream of the jump cannot propagate upstream into the supercritical flow region. Therefore, they will accumulate and form a large change in the water depth, which is called a hydraulic jump.

(b)



Continuity $u_1 d_1 = u_2 d_2 \quad \text{--- (1)}$

Momentum $\frac{\rho g d_1^2}{2} - \frac{\rho g d_2^2}{2} = \rho u_2^2 d_2 - \rho u_1^2 d_1$

substitute from (1) for u_2

$$\frac{\rho g}{2} (d_1^2 - d_2^2) = \rho \frac{u_1^2 d_1}{d_2} (d_1 - d_2)$$

$$\therefore \frac{g}{2} (d_1 + d_2) = \frac{u_1^2 d_1}{d_2} = u_1 u_2$$

(c)

$S_f = 0$, then $\frac{dh}{dx} = \frac{S_b}{1 - Fr^2}$, N.B. $S_b = -\frac{dz_b}{dx}$
 $\eta = z_b + h$

$$\therefore \frac{dh}{dx} + \frac{1}{1 - Fr^2} \frac{dz_b}{dx} = 0 \Rightarrow \left(\frac{dh}{dx} + \frac{dz_b}{dx} \right) + \frac{Fr^2}{1 - Fr^2} \frac{dz_b}{dx} = 0$$

$$\Rightarrow \frac{d\eta}{dx} = -\frac{Fr^2}{1 - Fr^2} \frac{dz_b}{dx}$$



$Fr < 1$



(d)

The flow velocity is $U = \frac{Q}{bh} = \frac{10}{2(2+0.08x)} = \frac{5}{2+0.08x}$

The small disturbance travel upstream at a velocity $\sqrt{gh} - U = \sqrt{9.81 \times 2} - \frac{5}{2+0.08x}$

$$\frac{dx}{dt} = 4.43 - \frac{5}{2+0.08x}$$

$$dt = \frac{dx}{4.43 - \frac{5}{2+0.08x}}$$

$$dt = \frac{0.08x+2}{0.3544x+3.86} dx = \frac{0.08x+0.8713+1.1287}{0.3544x+3.86} dx = 0.226dx + \frac{1.1287}{0.3544x+3.86} dx$$

$$t = \int_0^{100} 0.226dx + \frac{1.1287}{0.3544x+3.86} dx = 0.226 \times 100 + \int_0^{100} \frac{3.185}{x+10.89} dx$$

$$t = 22.6 + 3.185 \ln(x + 10.89) \Big|_0^{100} = 22.6 + 3.185 \times (\ln 110.89 - \ln 10.89)$$

$$t = 22.6 + 3.185 \times (4.71 - 2.39) = 22.6 + 7.39 = 30.0 \text{ s}$$

3

$$(a) \quad C = 7.8 \ln \left(\frac{12.0 \cdot R_h}{k_s} \right) = 7.8 \ln \left(\frac{12.0 \times 1}{0.006} \right) = 59.29$$

$$U = C \sqrt{R_h S_b} = 59.29 \sqrt{1 \times 0.0005} = 1.33 \text{ m/s}$$

$$q = Uh = 1.33 \text{ m}^2/\text{s}$$

$$(b) \quad \text{Given } d = 3 \text{ mm, then } d_* = d \cdot \left(\frac{g(s-1)}{\nu^2} \right)^{\frac{1}{3}} = 0.003 \times \left(\frac{9.81(2.65-1)}{10^{-12}} \right)^{\frac{1}{3}} = 75.9$$

Find the critical Shields parameter.

$$\theta_c = \frac{0.30}{1+1.2d_*} + 0.055[1 - \exp(-0.02d_*)] = \frac{0.30}{92.08} + 0.055[1 - 0.219] = 0.0462$$

$$\text{The actual shear stress is: } \tau_b = \rho g R_h S_f = 9810 \times 1 \times 0.0005 = 4.91 \text{ Pa}$$

$$\theta = \frac{\tau_b}{g(\rho_s - \rho)d} = \frac{1 \times 0.0005}{(2.65-1) \times 0.003} = 0.101$$

$$\text{Meyer-Peter and Müller} \quad \frac{q_b}{\sqrt{g(s-1) \cdot d^3}} = 8 \left[\left(\frac{C}{C'} \right)^{1.5} \theta - 0.047 \right]^{1.5}$$

$$q_b = 8 \times [0.101 - 0.047]^{1.5} \sqrt{9.81(2.65-1) \cdot 0.003^3} = 6.64 \times 10^{-5}$$

$$q_b = 6.64 \times 10^{-5} \times 2650 = 0.176 \text{ kg}/(\text{m s})$$

(c)

$$\theta = \frac{\tau_b}{g(\rho_s - \rho)d} = \frac{1 \times 0.0005}{(2.65-1) \times 0.003} = 0.101$$

$$\text{Zyserman and Fredsøe } \bar{c}(2d) = \frac{0.331 \cdot (\theta' - 0.045)^{1.75}}{1 + 0.72 \cdot (\theta' - 0.045)^{1.75}} = 2.12 \times 10^{-3}$$

$$2.12 \times 10^{-3} \times 2650 = 5.63 \text{ kg}/\text{m}^3$$

$$\frac{\bar{c}(z)}{\bar{c}(a)} = \left(\frac{h-z}{z} \cdot \frac{a}{h-a} \right)^{\frac{w_s}{\kappa u_*}}$$

$$u_* = \sqrt{ghS_b} = \sqrt{9.81 \times 1 \times 0.0005} = 0.07 \text{ m/s}$$

$$\text{Given } d = 3 \text{ mm, } d_* = d \cdot \left(\frac{g(s-1)}{\nu^2} \right)^{\frac{1}{3}} = 0.001 \times \left(\frac{9.81(2.65-1)}{10^{-12}} \right)^{\frac{1}{3}} = 75.9$$

$$w_s = \frac{\nu}{d} \left[\sqrt{10.36^2 + 1.049 \cdot d_*^3} - 10.36 \right] = \frac{10^{-6}}{0.003} \left[\sqrt{10.36^2 + 1.049 \times 75.9^3} - 10.36 \right]$$

$$w_s = 0.22 \text{ m/s}$$

$$\text{Rouse exponent is } \frac{0.22}{0.4 \times 0.07} = 7.86$$

$$\frac{\bar{c}(0.0012)}{\bar{c}(0.006)} = \left(\frac{1-0.012}{0.012} \cdot \frac{0.006}{1-0.006} \right)^{\frac{0.22}{0.4 \times 0.07}}$$

$$\bar{c}(0.0012) = 0.024 \text{ kg/m}^3$$

(d) 2D instantaneous release problem.

$$u_* = \sqrt{ghS_b} = \sqrt{9.81 \times 1 \times 0.0005} = 0.07 \text{ m/s}$$

$$D_x = D_{tx} + D_L = (0.15 + 5.86)hu_* = 0.414 \text{ m}^2/\text{s}$$

$$D_y = D_{ty} = 0.15hu_* = 0.0105 \text{ m}^2/\text{s}$$

$$\text{Consider the image source } \bar{c}(x, y, t) = 2 \frac{M/h}{4\pi t \sqrt{D_x D_y}} \exp\left(-\frac{(x-Ut)^2}{4D_x t} - \frac{y^2}{4D_y t}\right)$$

The maximum concentration occurs at $t = 300/U = 225.56 \text{ s}$

$$\bar{c}(300, 0, 225.56) = 2 \times \frac{\frac{2}{1}}{4\pi \times 225.56 \times \sqrt{0.414 \times 0.0105}} \exp(-0 - 0) = 0.0214 \text{ kg/m}^3$$

4.

(a) When the fluid just enters the channel, the thickness of the boundary layer gradually increases from zero moving in the direction of fluid flow and eventually reaches the water surface and fills the entire channel. This region from the entrance of the channel to the point where the boundary layer covers the entire channel is termed as the hydrodynamic entrance region. In this region, the velocity profile develops and thus the flow is called the hydrodynamically developing flow. After this region, the velocity profile is fully developed and continues unchanged. This region is called the hydrodynamically fully developed region, which is the focus of study in open channel flows.

For the flow over an airplane's wing, the boundary layer can freely develop. There is no pressure gradient in the streamwise direction.

(b)

- i. Neglect bed slope and bed friction.
- ii. Channel is infinitely long.
- iii. Flow is subcritical.
- iv. Flow is initially uniform.

(c.i)

$$\text{Energy equation: } 37 = \left(\lambda \frac{L}{D} + \sum \zeta \right) \frac{U^2}{2g} = \left(\lambda \frac{1500}{0.45} + 0.5 + 6 \times 0.25 + 1 \right) \frac{U^2}{2 \times 9.81}$$

$$\text{Relative roughness height: } \frac{k_s}{D} = \frac{0.225}{450} = 0.0005$$

Assume that the flow is hydraulically rough, $\lambda=0.0167$

$$\text{Energy equation: } 37 = \left(0.0167 \times \frac{1500}{0.45} + 3.0 \right) \frac{U^2}{2 \times 9.81} \Rightarrow U = 3.52 \text{ m/s}$$

$$\text{Then, } Re = 3.52 \times 0.45 \times 10^6 = 1.6 \times 10^6 \Rightarrow \lambda = 0.0169$$

$$\text{Energy equation: } 37 = \left(0.0169 \times \frac{1500}{0.45} + 3.0 \right) \frac{U^2}{2 \times 9.81} \Rightarrow U = 3.50 \text{ m/s}$$

This is close to the previous estimate, so it is accepted.

$$\text{The flow rate is } \frac{\pi D^2}{4} U = 0.556 \text{ m}^3/\text{s}$$

(c.ii)

$$\text{System curve: } H = 37 + \left(\lambda \frac{L}{D} + \sum \zeta \right) \frac{U^2}{2g} = 37 + \left(\lambda \frac{1500}{0.45} + 3 \right) \frac{U^2}{2 \times 9.81}$$

Construct the system curve using Moody diagram as:

Q (m ³ /s)	0	0.2	0.4	0.6	0.8
U (m/s)	0.00	1.26	2.52	3.77	5.03
Re	0.0E+00	5.7E+05	1.1E+06	1.7E+06	2.3E+06
Lamda	0.0000	0.0175	0.0171	0.0170	0.0169
System (m)	37.0	41.9	56.4	80.3	113.6
Pump (m)	100	91	75	53	24

Find the intersection between the system curve and pump curve.

We have the flow rate to be 0.49 m³/s.

