1.

(a)

Total infiltration during the first two-hour period is:

$$\int_{t_1}^{t_2} f \cdot dt = f_c(t_2 - t_1) - \frac{1}{K_f}(f_0 - f_c)\left(e^{-K_f t_2} - e^{-K_f t_1}\right)$$

$$= 2(2-0) - \frac{1}{1}(10-2)(e^{-2} - e^{0}) = 10.92 \text{ mm}$$

The total excess rain in the first two-hour period is: 10×2-10.92 = 9.08 mm

The total infiltration during the second two-hour period is:

$$\int_{t_1}^{t_2} f \cdot dt = f_c(t_2 - t_1) - \frac{1}{K_f}(f_0 - f_c)\left(e^{-K_f t_2} - e^{-K_f t_1}\right)$$

$$= 2(4-2) - \frac{1}{1}(10-2)\left(e^{-4} - e^{-2}\right) = 4.94 \text{ mm}$$

The total excess rain in the first two-hour period is: 10×2-4.94 = 15.06 mm

The total volume of the excess flow in the first two-hour period is:

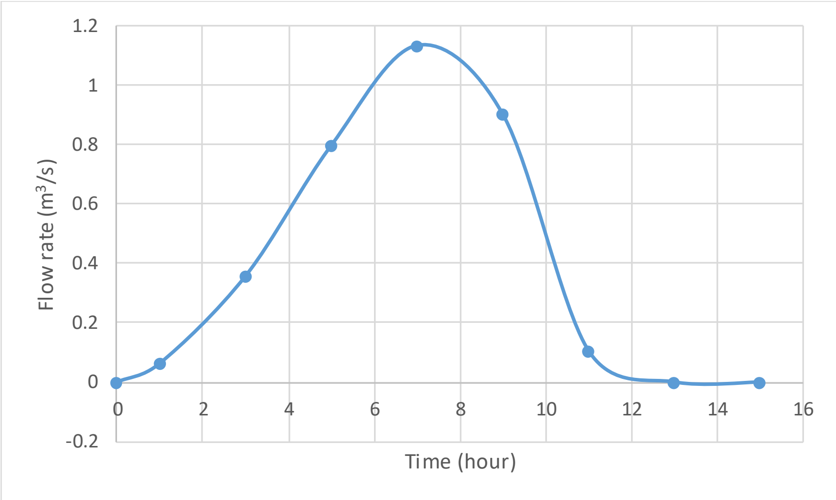
$$9.08 \times 10^{-3} \times 1 \times 10^{6} = 9080 \text{ m}^3$$

The total volume of the excess flow in the second two-hour period is:

$$15.06 \times 10^{-3} \times 1 \times 10^{6} = 15060 \text{ m}^3$$

The volume distribution of the excess flow due to the four-hour rain is:

Time (h)	0-2	2-4	4-6	6-8	8-10	10-12	12-14
First 2-hour rain (%)	5	20	30	40	5		0
First 2-hour volume (m^3)	454	1816	2724	3632	454		0
Second 2-hour rain (%)		5	20	30	40	5	5
Second 2-hour volume (m^3)		753	3012	4518	6024	753	0
Total volume (m^3)	454	2569	5736	8150	6478	753	
Total flow rate (m^3/s)	0.063	0.357	0.797	1.132	0.900	0.105	



(b)

The flow rate in the first one-hour period (2.5 $m^3 s^{-1}$) is entirely due to the first-hour 20 mm h^{-1} rain. Then, the second-hour 40 mm h^{-1} rain must produce a flow rate of 5.0 $m^3 s^{-1}$ in the second one-hour period.

The recorded flow rate in the second hour is 8.6 $m^3 s^{-1}$, so the contribution from the 1st-hour rain to the 2nd-hour flow rate must be (8.6-5.0) = 3.6 $m^3 s^{-1}$.

Similarly, the discharge in the third one-hour period generated by the 20 mm h^{-1} rain occurring in the first hour is (9.3-3.6×2) = 2.1 $m^3 s^{-1}$.

Examiner's comments:

The question could be answered by directly superposing the given 2-hour hydrograph to construct the required 4-hour hydrograph. A few candidates first plotted the S-curve and then derived either the 1-hour or 4-hour hydrograph. It is OK to use the derived 1-hour or 4-hour hydrograph, but the solution in this way is quite time-consuming.

2.

(a)

$R_h = \frac{5\times2.5}{5+5} = 1.25$ m

For the river section:

$$C = 7.8\,ln\left(\frac{12.0\cdot R_h}{k_s}\right) = 7.8\,ln\left(\frac{12.0\cdot 1.25}{0.25}\right) = 31.9$$

$$Sb = \frac{\Delta}{1000}$$

For the cut-off channel:

$$C = 7.8\,ln\left(\frac{12.0\cdot R_h}{k_s}\right) = 7.8\,ln\left(\frac{12.0\cdot 1.25}{0.005}\right) = 62.4$$

$$Sb = \frac{\Delta}{400}$$

According to $U = C\sqrt{R_h S_b}$

The ratio is: $\frac{62.4\times\sqrt{1.25\times\frac{1}{400}}}{31.9\times\sqrt{1.25\times\frac{1}{1000}}} = 3.1$

(b)

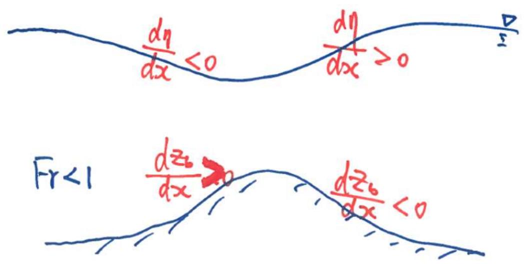
$$\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2}$$

$S_f = 0$, then $\frac{dh}{dx} = \frac{S_b}{1-Fr^2}$, N.B. $S_b = -\frac{dz_b}{dx}$, $\eta = z_b + h$

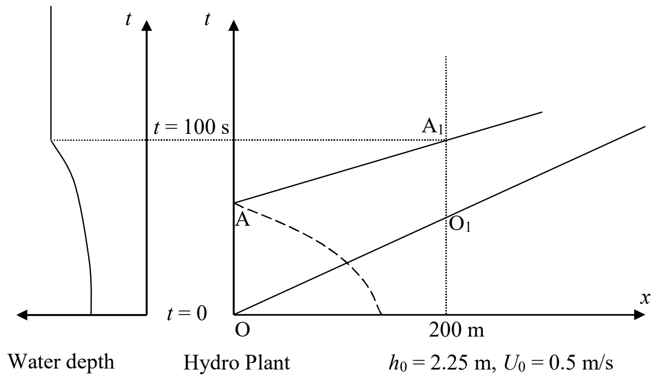
$$\frac{dh}{dx} - \frac{1}{1-Fr^2} S_b = 0 \Rightarrow \frac{dh}{dx} + \frac{1}{1-Fr^2}\frac{dz_b}{dx} = 0$$

$$\frac{dh}{dx} + \frac{1-Fr^2}{1-Fr^2}\frac{dz_b}{dx} + \frac{Fr^2}{1-Fr^2}\frac{dz_b}{dx} = 0$$

$$\Rightarrow \frac{d\eta}{dx} = -\frac{Fr^2}{1-Fr^2}\frac{dz_b}{dx}$$



(c)



We can easily prove that A_1 is in the disturbed zone. Hence, A must be on the vertical axis.

The positive line equation crossing A and A_1 is $\frac{dx}{dt} = U + \sqrt{gh}$

$$\frac{200}{100-t_A} = U_A + \sqrt{gh_A} \qquad \text{(i)}$$

Along the negative line between A and the intersection point on x axis, we have:

$$U_A - 2\sqrt{gh_A} = U_0 - 2\sqrt{gh_0} = -8.90 \qquad \text{(ii)}$$

Then, Equation (i) becomes:

$$\frac{200}{100-t_A} = -8.90 + 3\sqrt{gh_A} = -8.90 + 3\sqrt{9.81 \times (1.5 + 0.002t_A)^2}$$

$$\frac{200}{100-t_A} = -8.90 + 3\sqrt{9.81} \times (1.5 + 0.002t_A) = 0.0188t_A + 5.19$$

t_A = 69.2 s

So, $h_A = (1.5 + 0.002 \times 69.2)^2 = 2.68$ m,

From Equation (ii), $U_A = -8.90 + 2\sqrt{gh_A} = 1.35$ m/s

q = 3.62 m^2/s

Examiner's comments:

In (a), some candidates did not notice the different bed slopes of the meandering reach and the cut-off channel. In (b), several argued the water depth decrease over a hump, which alone could not explain the water surface drop.

(3)

(a.i) Given d = 0.2 mm, then $d_* = d \cdot \left(\frac{g(s-1)}{\nu^2}\right)^{1/3} = 0.0002 \times \left(\frac{9.81(2.65-1)}{10^{-12}}\right)^{1/3} = 5.06$

$$w_s = \frac{\nu}{d}\left[\sqrt{10.36^2 + 1.049 \cdot {d_*}^3} - 10.36\right] = \frac{10^{-6}}{0.0002}\left[\sqrt{10.36^2 + 1.049 \cdot 5.06^3} - 10.36\right]$$

$w_s = 0.026$ m/s

$$\frac{\bar{c}(z)}{\bar{c}(a)} = \left(\frac{h-z}{z} \cdot \frac{a}{h-a}\right)^{\frac{w_s}{\kappa u_*}}$$

$$\frac{0.0001}{0.1} = \left(\frac{3-2}{2} \cdot \frac{0.01}{3-0.01}\right)^{\frac{0.026}{0.4u_*}}$$

u_* = 0.06 m/s = $\sqrt{ghS_b}$

S_b = 1.23 × 10^{-4}

(a.ii) $\frac{u_*}{w_s} = 2.3$, $\frac{u_* d}{\upsilon} = 12$

Using Liu's Diagram, Transition Zone (between Dunes and Antidunes).

(a.iii)

$$\bar{c}(x, y) = \frac{\dot{M}/h}{U\sqrt{4\pi\frac{x}{U}D_y}} exp\left(-\frac{y^2}{4D_y x/U}\right)$$

$$D_y = D_{ty} = 0.15hu_* = 0.15 \times 3 \times 0.06 = 0.027$$

$$0.5 = 2 \times \frac{5/3}{U\sqrt{4\pi\frac{150}{U}0.027}} exp(-0)$$

U = 0.87 m/s

From $U = C\sqrt{R_h S_b}$

$$0.87 = C\sqrt{3 \times 1.23 \times 10^{-4}}$$

$$C = 45.3 = 7.8\, ln\left(\frac{12.0 \times 3}{k_s}\right)$$

K_s = 0.11 m

(b)

Hydraulic Design Charts

Darcy-Weisbach equation: $H_f = \lambda \cdot \frac{L}{D} \cdot \frac{U^2}{2g}$

Colebrook-White formula: $\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{k_s}{3.7D} + \frac{2.51}{Re\sqrt{\lambda}}\right)$

The friction factor can be eliminated by combining these two equations.

Substituting $\sqrt{\lambda} = \frac{\sqrt{2gS_f D}}{U}$ **and** $Re = \frac{UD}{\nu}$ **into Colebrook-White formula:**

$$U = -2\sqrt{2gS_f D} \cdot \log_{10}\left(\frac{k_s}{3.7D} + \frac{2.51\nu}{D\sqrt{2gS_f D}}\right)$$

ν for water depends on temperature; g = 9.81 m²/s; k_s depends on the pipe material. If ν, g and k_s are fixed, then the above equation gives the relationship between three variables: U, S_f and D. The equation $Q = U\pi D^2/4$ **introduces one extra variable: Q. So, these 2 equations contain 4 variables in total. Given any two of the variables, the other two can be determined.**

Examiner's comments:

Some forgot to consider the influence of the image source in (a.iii). The pipe design charts in (b) were generally well explained, but a few did not make it clear that the roughness height, fluid viscosity coefficient and gravitational acceleration were deemed constants in each chart.

4.

(a)

Relative roughness height: $\frac{k_s}{D} = 0.0002$

Assuming hydraulically rough, λ = 0.01375

$$30 = \left(\lambda \frac{L}{D}\right)\frac{U^2}{2g} = \left(0.01375 \frac{1000}{0.225}\right)\frac{U^2}{2\times 9.81}$$

U = 3.1 m/s

$$\text{Re} = \frac{3.1\times 0.225}{10^{-6}} = 7\times 10^5,$$

Refined friction factor is: λ = 0.0153

$$30 = \left(\lambda \frac{L}{D}\right)\frac{U^2}{2g} = \left(0.0153 \frac{1000}{0.225}\right)\frac{U^2}{2\times 9.81}$$

U = 2.94 m/s

$$\text{Re} = \frac{2.94\times 0.225}{10^{-6}} = 6.6\times 10^5,$$

Accept.

Q = 116.8 litre/s

(b) When the flow rate is 200 litre/s,

U = 5.03 m/s, Re = 1.13×10^6

Friction factor is: λ = 0.0148

Required pump head: $-30 + \left(\lambda \frac{L}{D}\right)\frac{U^2}{2g} = 54.8$ m

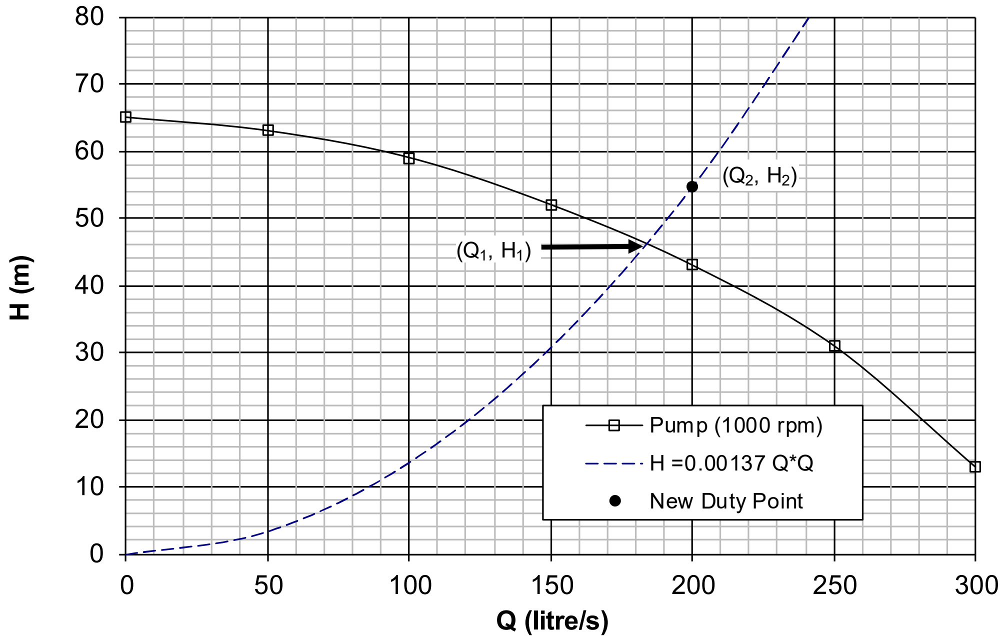
So, Q_2 = 200 litre/s, H_2 = 54.8 m

From the figure below, Q_1 = 184 litre/s, H_1 = 46 m

According to either $\frac{Q_2}{N_2} = \frac{Q_1}{N_1}$ or $\frac{H_2}{{N_2}^2} = \frac{H_1}{{N_1}^2}$, where N_1 = 1000 rpm, we get N_2.

$$\frac{200}{N_2} = \frac{184}{1000} \Rightarrow N_2 = 1087 \text{ rpm}$$

$$\frac{54.8}{{N_2}^2} = \frac{46.0}{1000^2} \Rightarrow N_2 = 1091 \text{ rpm}$$



Examiner's comments:

A few simply did many trial-and-error calculations in (a), rather than designing an iterative procedure. Most of the mistakes in (b) arose from the wrong homologous operation points of the pump. Quite a few plotted the system curve, which was not necessary.