3D8 Cribs (April 2014)

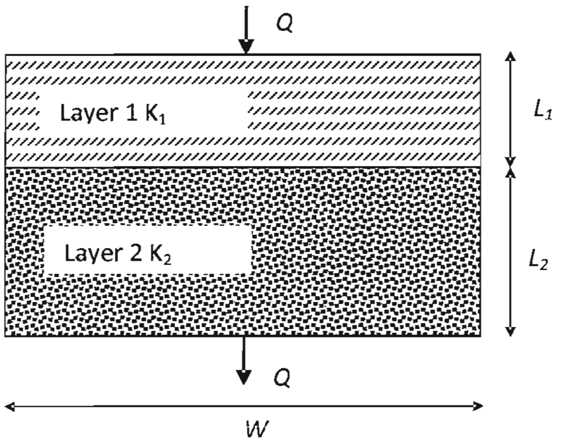
Q1. a) Pressure exerted by the fluid that is present within the void space of a porous media is measured in the units of kPa. Due to the interconnected nature of the void space, the fluid pressure increases hydrostatically with depth. It affects the inter-granular stresses between the particles of the porous media.

Pressure head is a convenient way by which civil engineers express the fluid pressure. It can be defined as the level to which the fluid will rise if a standpipe were to be inserted at a given location within the porous media. It is measured in length units such as 'm'.

Potential head is the pressure head measured above a datum. The height of a given point above the datum is called elevation head. Potential head is defined as the sum of pressure head and the elevation head. Elevation head is taken as positive if the point of interest is above the datum, otherwise it is taken as negative.

[10%]

b) Consider vertical flow through a soil deposit of two horizontal layers with differing hydraulic conductivity.



Assume unit length into the page. Continuity of mass flow rate requires that the volumetric flow rate of water through layer 1 be the same as that through layer 2.

$$Q = Q_1 = Q_2$$

$$A = A_1 = A_2 = v \text{ constant}$$

$$\Delta\bar{h}_1 \neq \Delta\bar{h}_2 \qquad \text{but} \qquad \Delta\bar{h}_1 + \Delta\bar{h}_2 = \Delta\bar{h}$$

Darcy's Law for layer 1 gives the specific discharge:

$$v = \frac{Q}{W} = K_1 \frac{\Delta\bar{h}_1}{L_1}; \qquad \Delta\bar{h}_1 = v\frac{L_1}{K_1}$$

where $\Delta\bar{h}_1$ is the difference in potential head between the top and bottom of layer 1. Similarly for layer 2:

$$\Delta\bar{h}_2 = v\frac{L_2}{K_2}$$

$$\Delta\bar{h}_1 + \Delta\bar{h}_2 = v\left[\frac{L_1}{K_1} + \frac{L_2}{K_2}\right]$$

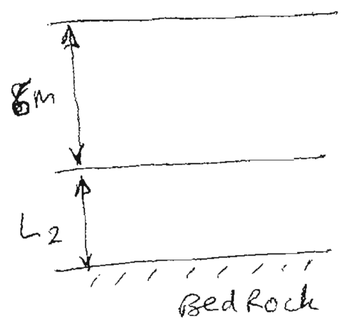
$$v = \frac{L_1 + L_2}{\left(\frac{L_1}{K_1} + \frac{L_2}{K_2}\right)} \left(\frac{\Delta \bar{h}_1 + \Delta \bar{h}_2}{L_1 + L_2}\right)$$

but $\left(\frac{\Delta \bar{h}_1 + \Delta \bar{h}_2}{L_1 + L_2}\right)$ is the average or effective vertical hydraulic gradient across the whole deposit, and so the effective vertical hydraulic conductivity of the deposit is:

$$K_{vertical} = \frac{L_1 + L_2}{\left(\frac{L_1}{K_1} + \frac{L_2}{K_2}\right)}$$

[20%]

1 c)



$$K_v = \frac{6 + L_2}{\frac{6}{2.05\times10^{-4}} + \frac{L_2}{3\times10^{-3}}} = \frac{6+L_2}{8.5\times3600}$$

$$\therefore \frac{L_2}{3\times10^{-3}} = 8.5\times3600 - \frac{6}{2.05\times10^{-4}}$$

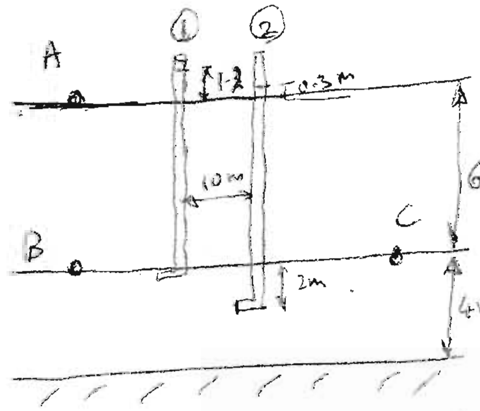
$$= 1331.707$$

$$L_2 = 3\times10^{-3} \times 1331.707 = 3.9951$$

$$L_2 \simeq 4.0 \text{ m}$$

[30%]

1 d)



$\bar{h}_1 = 4 + 6 + 1.2 = 11.2$ m

$\bar{h}_2 = 2 + 2 + 6 + 0.3 = 10.3$ m

$\Delta\bar{h} = 0.9$ m

$i = \frac{\Delta h}{\Delta s} = \frac{0.9}{10} = 0.09$

Fastest way for contaminant to reach from A to C is Diffusion from A to B ; Advection from B to C.

Diffusion from A to B: in silt.

Use $\frac{C}{C_0} = erfc\left(\frac{z}{\sqrt{4 D_d^* t}}\right)$

For traces to appear at silt/sand interface.

$\frac{C}{C_0} \approx 0 = erfc\left(\frac{z}{\sqrt{4 D_d^* t}}\right)$

$\Rightarrow \quad \frac{z}{\sqrt{4 D_d^* t}} = 3 \quad ; \quad z = 6\,m$

$\therefore \quad \sqrt{4 D_d^* t} = \frac{6}{3} = 2.$

$4 D_d^* t = 4$

$t = \frac{1}{D_d^*} = \frac{1}{5.8 \times 10^{-6}} = 1.996$ days

$\simeq \underline{\underline{2\text{ days}}}$

Advection from B to C in sand:-

Distance BC = 150 m. ; $e = 0.6$

$n = \frac{e}{1+e} = 0.375$

$v = K_{sand} \times i = 3 \times 10^{-3} \times 0.09 = 2.7 \times 10^{-4}$ m/s.

Intrinsic velocity $v_a = \frac{v}{n} = \frac{2.7 \times 10^{-4}}{0.375} = 7.2 \times 10^{-4}$ m/s.

$\therefore$ Time taken $= \frac{150}{7.2 \times 10^{-4}} = \underline{\underline{2.41}}$ days

$\therefore$ Total time from A to C = 4.41 days.

Time of diffusion from A to B is comparable to time for diffusion from B to C. The hydraulic conductivity of silt is smaller, therefore horizontal advection in silt will not be dominant, relatively speaking.

[40%]

Q 2 a) Specific heat capacity cp or more commonly Specific heat is defined as the amount of heat energy that is required to change the temperature by a given amount. It has the units of kJ/kg/°K.

Volumetric Heat Capacity (VHC) is defined as the ability of a given volume of soil to store heat energy for given change in temperature. This can be calculated as;

$$VHC = \rho\, c_p$$

where ρ is the density of soil. If the soil is dry we use the dry density, if it is fully saturated we use the saturated density of soil. VHC has the units of J/m^3/K.

The thermal diffusivity of a material α can be viewed as the ease with which heat can flow through the soil. It is defined as the ratio of the thermal conductivity to the volumetric heat capacity.

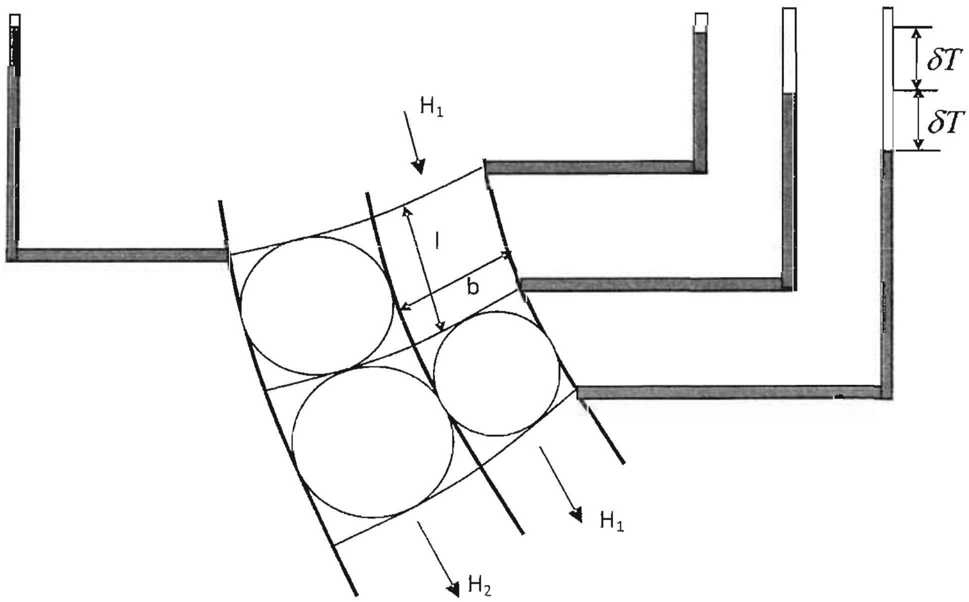
$$\alpha = \frac{\lambda}{VHC} = \frac{\lambda}{\rho\, c_p}$$

Thermal diffusivity α has the units of m^2/s. [15%]

2b) Consider plane flow in a unit section one metre into the page for an isotropic homogenous porous media. The heat flownet will be composed of:

- **Heat flow lines,** representing the average trajectory of heat flow; c.f. streamlines in fluids
- **Isotherms,** of δT
- **Boundary conditions**

There can be no heat flow along an isotherm and therefore heat flow lines and isotherms meet at right angles (i.e. orthogonal to each other). Isotherms cannot cross each other. Heat flow lines cannot cross each other and we can think of heat flow lines as boundaries of a flow tube. From continuity, the heat flow rate within each flow tube must be constant.



Applying Fourier's law to the element of breadth *b* and length *l* :

$$H_1 = \lambda A \frac{\delta T}{l}$$

so:

$$H_1 = b \times 1 \times \lambda \times \frac{\delta T}{l}$$

$$= \lambda \times \delta T \quad \text{, if } b = l$$

For convenience a flow net of curvilinear squares is usually drawn with *b=l*, by adopting simple trail and error solutions. If the isotherms are drawn with equal drops in temperature then $\delta T = {\Delta T}/{N_T}$, where ΔT is the total change in temperature along the heat flow tube from a source of heat to a sink and N_T is the number of drops in temperature. The heat flux rate per flow tube is:

$$H = \lambda \times \frac{\Delta T}{N_T} \quad \text{per heat flow tube.}$$

So a heat flow net of curvilinear squares will have constant temperature differences and the same heat flow in each tube. Curvilinear squares have edges crossing at right angles, and an inscribing circle would just touch all four sides.

$$H = H_1 = H_2$$

If there are N_f flow tubes then the total heat flow quantity will be:

$$H = \lambda \Delta T \frac{N_f}{N_T} \qquad \text{W/m length}$$

[25%]

2 c) i) See heat flow net on next page.

ii) Temperatures interpolated from flow net

$T_A = 54°C$

$T_B = 40°C$ [20%].

$T_C = 52°C$ (using symmetry) $= T_C'$

iii) $H = \lambda \Delta T \frac{N_f}{N_T}$ $\lambda = \rho C_p \alpha = 19.3 \times \frac{10^3}{9.81} \times 1400 \times 1.162 \times 10^{-6}$

$= 3.2 \text{ W/m/°C}$

$\therefore H = 3.2 \times (70-10) \times \frac{2 \times 14}{5} = 1075.2 \text{ W/m}$

$= 1.075 \text{ kW/m}$. [20%]

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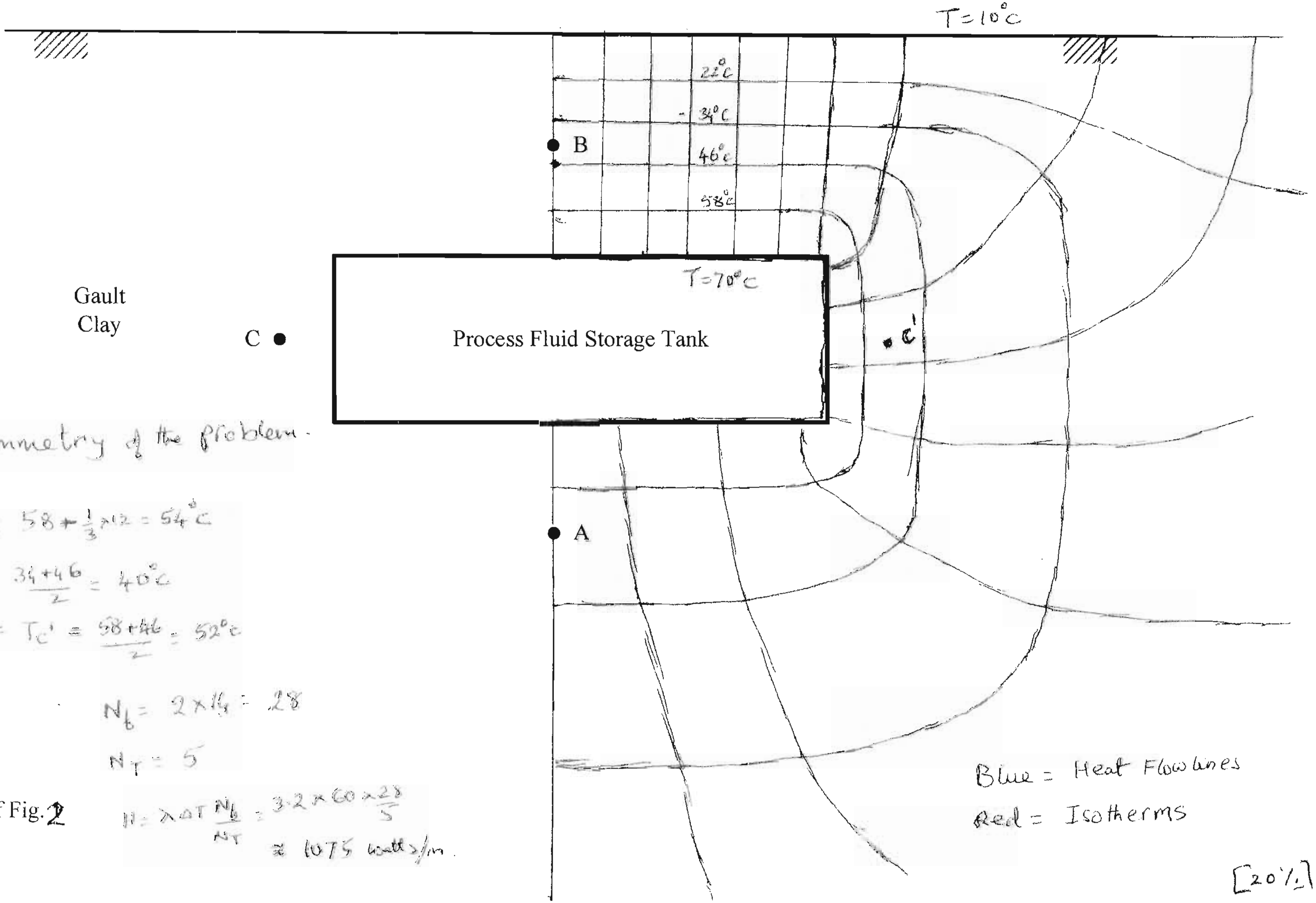


Use Symmetry of the problem.

$T_A = 58 + \frac{1}{3} \times 12 = 54°C$

$T_B = \frac{34+46}{2} = 40°C$

$T_C = T_{C'} = \frac{58+46}{2} = 52°C$

$N_f = 2 \times 14 = 28$

$N_T = 5$

$H = \lambda \Delta T \frac{N_f}{N_T} = \frac{3.2 \times 60 \times 28}{5}$

≈ 1075 watts/m

Extra copy of Fig. 2

[20%]

3 (a) Stack effect Stack effect is caused by temperature differences inside and outside. A temperature difference implies a density difference, and if the internal air is warmer than that outside, it will be less dense. Assuming there are no adverse wind conditions, the internal air will thus tend to rise, flowing out of upper openings, and drawing external air through lower openings.

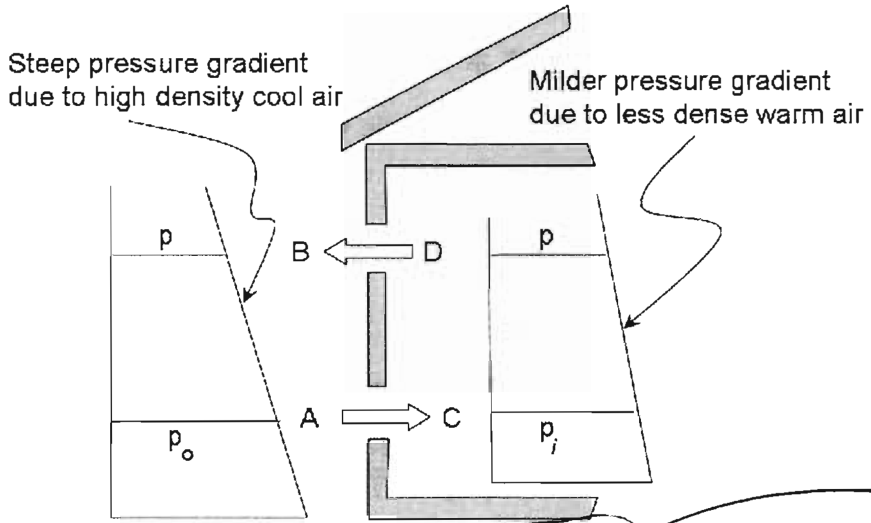


Figure 1: The buoyancy-driven stack effect (after CIBSE Guide A Figure 4.9)

The equations for stack effect can be derived by consideration of Figure 1, which shows a tall room with two openings, one at the top and one at the bottom.

The external temperature is T_o (in Kelvin) and the internal temperature is T_i everywhere through the room.

Assume that that the air is moving so slowly that kinetic energy terms (i.e. dynamic wind pressures) can be neglected everywhere except across the orifices. The pressure $p_o(H)$ outside the upper opening is thus:

$$p_o(H) = p_o - \rho_o g H \tag{1}$$

where ρ_o is the density of the external air at T_o, g is the gravitational constant and H is the distance separating the 2 openings. Similarly, inside the room, the pressure inside the upper window is:

$$p_i(H) = p_i - \rho_i g H \tag{2}$$

where ρ_i is the density of the internal air at T_i. The pressure differences across the bottom and top opening respectively are:

$$\Delta p_b = p_o - p_i$$
$$\Delta p_t = p_i(H) - p_o(H) = (p_i - \rho_i g H) - (p_o - \rho_o g H) = p_i - p_o + (\rho_o - \rho_i) g H$$

Thus, the total pressure difference between the two openings is:

$$\Delta p_s = \Delta p_b + \Delta p_t = (\rho_o - \rho_i) g H \tag{3}$$

Assuming that the air expands linearly with temperature, the density decreases as the reciprocal of temperature, thus

$$\rho \propto \frac{1}{T} \tag{4}$$

and taking conditions at say 273K as a benchmark, then

$$\frac{\rho}{\rho_{273}} = \frac{273}{T} \tag{5}$$

whence

$$\rho = \rho_{273} \frac{273}{T} \tag{6}$$

The buoyancy-driven pressure difference Δp_s driving the flow may thus be written

$$\Delta p_s = \rho_{273} g H \left(\frac{273}{T_o} - \frac{273}{T_i} \right) \tag{7}$$

Equating this to the pressure losses through the two orifices, we obtain

$$\Delta p = \frac{1}{2} \rho \frac{Q^2}{C_{orif1}^2 A_1^2} + \frac{1}{2} \rho \frac{Q^2}{C_{orif2}^2 A_2^2} \tag{8}$$

which rearranges to give a flow

$$Q = \frac{A_1 A_2}{\sqrt{A_1^2 + A_2^2}} C_{\text{orif}} \sqrt{\frac{2 \Delta p_s}{\rho}} \tag{9}$$

with Δp_s given by Equation 7.

This equation predicts the flow rate one might obtain from a stack-driven flow through two orifices separated by a height H.

3 (b)

Area $= 100\ m^2$ ht $= 10\ m$ $A_1 = 0.8\ m^2$

$A_2 = 0.4\ m^2$

$\Rightarrow$ Volume $= 1000\ m^3$

$H = 7m$ $g = 9.8\ m/s^2$ Coef $= 0.61$

$T_o = 273 + 5 = 278\ K$

$T_i = 273 + 20 = 293\ K$

(i) <u>Ventilation Rate</u>

$$Q = A^* \,\text{Coef} \sqrt{2gH\left(\frac{273}{T_o} - \frac{273}{T_i}\right)}$$

$$A^* = \frac{A_1 A_2}{\sqrt{A_1^2 + A_2^2}} = \frac{0.32}{\sqrt{0.8}} = \underline{0.36\ m^2}$$

$$Q = 0.36 \times 0.61 \times \sqrt{2 \times 9.8 \times 7 \times \left(\frac{273}{278} - \frac{273}{293}\right)}$$

$$\Rightarrow \underline{\underline{0.57\ m^3/s}}$$

(ii) Air changes per hour $Q = ACH \cdot V/3600$

$$ACH = \frac{3600 \times 0.57}{1000\ m^3} = \underline{\underline{2.06}}$$

Ventilation Heat loss

$$Q_{vent} = Q \times C_p \times \rho \times (T_i - T_o)$$

$$= 0.57 \times 1 \times 1.2 \times (21 - 5)$$

$$= \underline{11\ KW}$$

3 (b)

(iii) Maximum Number of people w/ 10 l/s per person.

Total l/s in room = $Q \times 1000$

$= 573$ l/s

No. of people $= \frac{573}{10} = \underline{\underline{57 \text{ people}}}$.

(iv) → Slightly increase the effective area of the openings (increase A_1 & A_2)

→ Raise the height of the "stack"

IV

(a) ~~Rad~~ $A_{rad} = 0.8 \times 1.0 = 0.8\ m^2$

$A_{surfaces} = 50\ m^2$

$T_{rad}\ (T_1) = 75°C$ $\qquad$ $T_{surface}\ (T_2) = 21°C$

$\epsilon_{rad}\ (\epsilon_1) = 0.97$ $\qquad$ $\epsilon_{surface}\ (\epsilon_2) = 0.9$

longwave radiation in Enclosure

$$Q_{12} = h_r \cdot A_1 \cdot (T_1 - T_2)$$

where

$$h_r = \frac{4 \sigma T_{12}^3}{\frac{1-\epsilon_1}{\epsilon_1} + \frac{1}{F_{12}} + \frac{1-\epsilon_2}{\epsilon_2}\frac{A_1}{A_2}}$$

$\sigma = 5.67 \times 10^{-8}\ W m^2 K^4$ (stefan-boltzmann constant)

$T_{12} = \frac{T_1 + T_2}{2}$; $F_{12} = 1$ (view factor)

$$h_r = 7.26\ W/m^2 K$$

$$Q_{12} = 7.26 \times 0.8 \times (75 - 21) = 314\ W$$

IV (b) $A_{room} = 20 m^2$

$N_{people} = 2$ Ventilation is $5 \, l/s$ per person

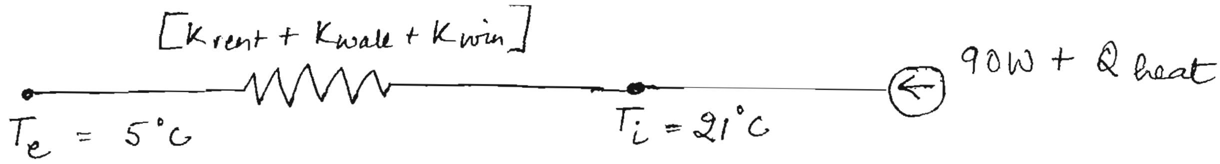
$$K_{vent} = \frac{2 \times 5}{1000} \times \rho_{air} \times C_p$$

4×5

$$= \frac{10}{1000} \times 1.2 \times 1000 = 12 \, W/K$$

$$K_{wall} = A_{wall} \times U_{wall} = 15 \times 0.35 = 5.25 \, W/K$$

$$K_{window} = A_{win} \times U_{win} = 5 \times 1.5 = 7.5 \, W/K$$



$$T_i = 21 = \frac{[K_{vent} + K_{wall} + K_{win}] \times 5 + 90 + Q_{heat}}{[K_{vent} + K_{wall} + K_{win}]}$$

$\underline{Q_{heat} = 306 \, W}$

Yes its adequate!

IV (c)

$T_{wall} = 40\,°C$ $\quad T_{rad} = 75\,°C$ (from (a))

$\epsilon_{wall} = 0.9$ $\quad \epsilon_{rad} = 0.97$

$\epsilon_{foil} = 0.04$

$$\left.\begin{aligned} h_r &= 4\epsilon_{12}\sigma T_{12}^3 \\ 1/\epsilon_{12} &= 1/\epsilon_1 + 1/\epsilon_2 - 1 \\ T_{12} &= \frac{T_1 + T_2}{2} \end{aligned}\right] \text{data book}$$

~~No foil~~

No Foil

$1/\epsilon_{12} = 1.14$ $\quad \epsilon_{12} = 0.87$

$T_{12} = 330\ K$

$h_r = 7.16\ W/m^2K$

$Q_{NO\ FOIL} = 200\ W$

With Foil

$1/\epsilon_{12} = 25$ $\quad \epsilon_{12} = 0.04$

$T_{12} = 330.5\ K$

$h_r = 0.33\ W/m^2$

$Q_{FOIL} = 9\ W$