EGT2: IIA
ENGINEERING TRIPOS PART IIA

Thursday 05 May 2022 2 to 3.40

Module 3E3

MODELLING RISK

Answer both questions in Section A (Question 1 and Question 2).

Answer only one question from Section B (either Question 3 or Question 4).

Show your calculation steps for all questions other than Question 1. This includes drawing decision trees, where relevant. All names of individuals, institutions, and places in questions are fictitious.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Table A: Standardised Normal Probability Table (1 page)
Table B: Standardised Normal Loss Table (1 page)
Table C: L_q (Length of queue) Values for multi-server queue (1 page)

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

Section A

1 Choose only one alternative when answering each of the multiple-choice questions below.

(a) Glenshee is a ski resort in Scotland. One of their triple chair lifts unloads 1350 skiers per hour at the top of the slope. (A triple chair lift can carry three passengers per chair.) The ride from the bottom to the top takes 8 minutes. How many skiers are riding on the lift at any one time? [4%]

A. 60
B. 168.75
C. 180
D. 450
E. None of the above

(b) A doctor's office has 3 physicians. Traditionally, each physician would hold a "patient list," meaning that patients on that list would always be seen by the same physician. For example, if Patient A is on the list of Physician 1, then Patient A will always see Physician 1 when they visit the doctor's office, regardless of their condition (e.g., regardless of whether they are there for a regular check-up or for treatment of illness). Recently, waiting times have been going up and based on their knowledge of queuing theory, the doctors decide to change how they see patients. Now, patients will see whichever physician is next available, rather than waiting to see their assigned physician. Which of the following is likely to occur as a result of this change?

I. The utilization of each physician will increase.
II. The coefficient of variation of patient inter-arrival times will increase.
III. The average number of patients "in service," i.e., who are being seen by a physician at any point in time, will increase. [4%]

A. I only
B. II only
C. III only
D. I and II only
E. I and III only
F. II and III only
G. I, II, and III (i.e., all of them)
H. None of the above (i.e., none of them)

(c) Which of the following principles are important to keep in mind when establishing a forecasting process within your organization? [4%]

I. Convergence: Allowing individuals within your organization to discuss first before submitting their forecasts, to ensure that they have as much relevant information as possible.
II. Incentives: Ensuring that individuals are incentivised to accurately report their true forecast.
III. Diversity: Trying to ensure you ask a diverse set of individuals who have potentially relevant information that can be used in forming their forecast.

A. I only
B. II only
C. III only
D. I and II only
E. I and III only
F. II and III only
G. I, II, and III (i.e., all of them)
H. None of the above (i.e., none of them)

(d) Demand for antigen test kits (flow tests) has increased significantly due to the Omicron variant. The manufacturer of the test kits, a company named Let-it-Flow, needs to rapidly increase throughput in its test cassette production (this is the piece where you deposit your sample). Currently, test cassettes are made by hand in a four-step process:

In Step 1, the lower plastic base of the cassette is placed in position and sprayed with an adhesive. In Step 2, the plastic base is layered with a sample pad, conjugation pad, test/control lines, and an absorption pad. In Step 3, the top plastic cover is clicked onto the base on 4 sides (one at a time) to arrive at the final product. In Step 4, the test cassette is visually inspected for quality. One in 100 test cassettes are found to be faulty and discarded. The steps take 10 seconds, 25 seconds, 15 seconds, and 10 seconds, respectively, for a total of 1 minute per test cassette.

Let-it-Flow is approached by Jim Cook who offers to provide them with a machine that combines steps 2 and 3 into a single step, yet performs them in just 30 seconds, saving 10 seconds per test cassette! Quality is expected to remain the same as before. Should Let-it-Flow take Jim Cook up on this offer? [4%]

A. Yes
B. No
C. The answer cannot be determined based on the given information

(e) Traditionally, Heppers only ordered course textbooks once per term from Caledonia University Press, to arrive two weeks before the term starts. They are considering the following business model innovation: if a student does not find the book they want to purchase on the shelves, she can leave a request at Heppers. The Heppers store manager will then inform her when the book becomes available from a second order placed with Caledonia University Press, which is expected to arrive two days before term starts. A student opting for this option is also given a small discount. How does the optimal order quantity placed by Heppers from Caledonia University Press (for the initial shipment arriving two weeks before term starts) differ under this business model when compared to the traditional model?

A. It is larger
B. It is smaller
C. It is the same
D. The answer cannot be determined based on the given information

(f)

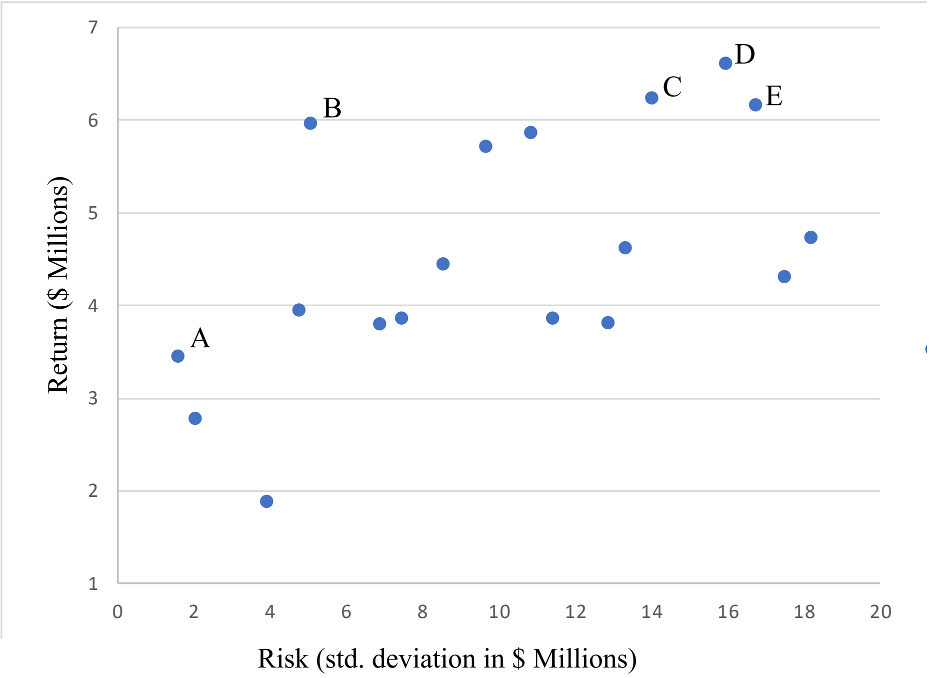

Fig. 1

Each point on the Fig. 1 above represents a portfolio of oil & gas projects. Some are labelled with letters. Which of these portfolios would you definitely not invest in? (Labels A through E are placed to the right of the corresponding portfolio). [4%]

A. Portfolio A
B. Portfolio B
C. Portfolio C
D. Portfolio D
E. Portfolio E
F. The answer cannot be determined from the information given

(g) Suppose a multiple-choice question (in a different exam) is worth 4 marks when answered correctly and has 5 potential answers to choose from (labelled A, B, C, D, E). Choosing a wrong answer results in a mark of -1 (minus 1). Skipping the question results in a mark of 0. Sally has eliminated option C as definitely wrong and places equal probability on each of the remaining options being correct. Sally is risk-neutral. What should Sally do? [4%]

A. Pick one of the options A, B, C, D, E with equal probability
B. Pick one of the options A, B, D, E with equal probability
C. Pick one of the options A, B, C, D with equal probability
D. Skip the question.
E. Be indifferent between picking one of A, B, D, E and skipping the question
F. The answer cannot be determined from the information given

(h) Jack is trying to answer the same question as Sally, which was described in Question 1(g). Jack has also eliminated option C as definitely wrong and places equal probability on each of the remaining options being correct. However, Jack is risk-averse. What should Jack do? [4%]

A. Pick one of the options A, B, C, D, E with equal probability
B. Pick one of the options A, B, D, E with equal probability
C. Pick one of the options A, B, C, D with equal probability
D. Skip the question
E. Be indifferent between picking one of A, B, D, E and skipping the question
F. The answer cannot be determined from the information given

2 Cambridge Credit Union:

After completing the 3E3 Course, confident in your skills managing risks, you decided to set up your own bank called the Cambridge Credit Union (CCU). One of your colleagues has set up a credit rating agency to rival Fitch and called it Abercrombie. Abercrombie provides its Basic Credit assessment for free but charges for a detailed assessment. A customer has approached CCU for a £100,000 1-year loan at a 11% interest.

(a) What is the primary decision faced by CCU? [2%]

(b) What should CCU do to maximise its expected monetary value (EMV)? [5%]

At a cost of £1500, Abercrombie can conduct a more thorough investigation of the customer's credit record. This service is called AbercrombiePlus. Past performance indicates that in cases where the customers did not default on their loans, the chance of receiving a favourable credit worthiness report via AbercrombiePlus was 80%. In cases where the customers did default, the chance of getting a favourable credit worthiness rating was 30%.

(c) Is the AbercrombiePlus service worth its cost? [8%]

(d) Suppose AbercrombiePlus had 100% forecast accuracy. What is the maximum Abercrombie could charge for this service? [6%]

Section B: Answer only one of Question 3 and Question 4.

3 Healthcare Management Conference at Matau:

Miyagi and Winston (Miyagi's PhD student) arrived in Matau for a Healthcare Management conference. Airport security was always a frustrating and painful experience for Winston. He started a consulting firm that helps different firms manage their customer queues better.

Coincidentally, his firm has been hired by the Matau Airport to help with the queues at the airport security. This project was initially pitched to Miyagi, but he passed it on to Winston as Miyagi wanted to use his time for better things than working on queuing systems while in Matau.

First, Winston collected some data. He found that passengers arrive at the security area with a rate of 0.9 passengers/minute. The average time of service at the security lane is 2 minutes. The coefficient of variation (CV) for arrival and service times are assumed to be equal to 1. For simplicity, assume that each passenger waits in the queue before the passengers in front of them completes the entire service process at the security lane.

Matau airport has given a target for Winston such that, on average, no more than 8 people should be waiting in queue.

(a) Without calculating the number of lanes needed, should Winston recommend pooled security (where all passengers queue up for the security area and proceed to the lane that is available when it is their turn) or dedicated lanes (where passengers queue up for particular lanes)? Why? When might that not be the case? [3%]

(b) Suppose the airport were to opt for pooled security. How many security lanes should Winston recommend for a pooled system to satisfy the target set by the client? [6%]

(c) Suppose the airport opts for two dedicated security lanes in a way that split the arriving passengers equally between those two lanes. With this arrangement, what is the average number of people waiting in queue for security at the airport? [6%]

Next, consider two security lanes that are dedicated:

(d) While leaving Matau, Miyagi is not happy with the queuing delay at the airport and offers the airport some free consulting. It turns out that 65% of the passengers at the airport are frequent flyer gold or above status members with at least one of the airlines flying in or out of Matau. These passengers are also very efficient travellers and require an average service time of 1.5 minutes. The other 35% require an average service time of 2.93 minutes. Miyagi's suggestion is to split the two dedicated security lanes such that one serves all the frequent flyers and all other passengers are served by the other. How will such a system perform compared to the one in part C for each customer segment and as a whole? Do you have any further suggestions for how the airport should improve this queueing system? (as before, assume all relevant CVs to be 1) [9%]

Sitting in the Airport lounge, waiting for their flight back to Caledonia, Winston and Miyagi thought it would be fun to work on a consulting project together. It so happened that there was a project at Allenbrooke's Hospital about improving the cleaning and sterilizing capacity for a surgical suite. However, they will be very jet-lagged upon arrival and could really use your expertise in process analysis. Here are the details of the project:

Surgical instruments, when they are not in use, are stored in "trays" that are wrapped and kept sterile until they are needed. Once a tray of instruments has been used in surgery and returned to the supplies area, the instruments must be 1) positioned for washing, 2) washed and dried, 3) inspected for damage and packed back in the tray, and 4) sterilized. The four-step process is shown in Fig. 2 below:

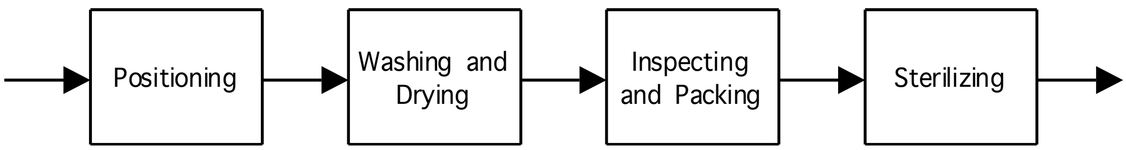

Fig. 2

Positioning is done by three people working in parallel and a person can do one tray in 8 minutes. The washing and drying is done by 10 automated machines. Each machine washes and dries a tray of instruments in 60 minutes. No labour is required except to place the instruments in the machine and take them out, which we will ignore for this problem.

Inspecting and packing can be broken into two cases. Everything is fine (80% of the trays) and some instrument is damaged (20% of the trays). When everything is fine the person can work at a rate of 6.67 trays/hour (or 9 minutes/tray). If any instrument is damaged the service rate of one person to (1) repair or find a replacement and (2) inspect and pack is 2.73 trays/hour (or 22 minutes/tray). Two people work at this stage, and both people do both steps. Union contracts restrict us to use full-time labour for this task and they cannot be employed for any other task.

The sterilizing step is like a baking or kiln operation that cannot be interrupted to add more trays. The sterilizing step takes 2.5 hours and one "oven" can handle up to 24 trays at once. There is one oven available for this step.

(e) What is the capacity of this system in trays/hour? [6%]

(f) An expansion of the Surgical Suite will require that trays be handled at the rate of 12/hour. What changes/additions need to be made to meet this demand? [9%]

(g) Given the increase in the required handling rate of trays to 12/hour, the Chief Operating Officer (COO) of the hospital has asked the administration team to come up with cost savings measures. One suggestion to cut costs is to reduce the damage rate from 20% to something below 5% (assume the reduction in the damage rate is costless). An accountant, Mr. Dean J. Conen, has computed that the savings are minor and equal to about 2 minutes / tray. He argues that even though nearly 100 trays per day will be handled (assuming roughly 8-hour workdays), the savings correspond to only about 3 or so hours of one person's time multiplied by that person's wage rate. Do you agree or disagree with Dean's calculation and estimate of the cost savings? Justify your response briefly. [8%]

4 Distribution Management at PolyCo:

PolyCo is a leading South African chemical distribution and trading company. Their primary product is the chemical polyamide 6 (30% glass fibre reinforced), which is used by their customers for injection moulding purposes. PolyCo has five distribution centres in South Africa, which supply specific regions of the country, each of these five distribution centres act as strategic business units (SBUs) and are responsible for their individual profits and losses. Due to a data leak, the firm only has past demand data for Region 1 and only at the annual level for the past 13 years. They have run a regression with that data, which led to the results in Fig. 3, below:

Regression Statistics	
Multiple R	0.47
R Square	0.2209
Standard Error	40000
Observations	13

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*[Confidence Interval]*	
Intercept	19,998	22,458	0.89	0.39	[-24,019,	64,015]
Year	23	18	1.28	0.22	[-12.28,	58.28]
Advertising	0.02972	0.035664	0.83	0.42	[-0.0402,	0.0996]

Fig. 3

The dependent variable Y is the demand, in tons of polyamide 6, in a given year. The explanatory variable Year is the year in YYYY format and the explanatory variable Advertising is the budget for a given year in the South African currency, Rand. The advertising budget for 2022 is 1,800,000 Rand. Since the regions in the country are divided equally between the distribution centres, the firm assumes that each of them has the same average yearly demand and the same standard deviation. For simplicity, you may round the expected annual demand to the nearest 1000 tons and the monthly demand to the nearest ton. Furthermore, assume the following about monthly demand: its mean is 1/12 of the annual demand and its variance is 1/12 of the annual demand.

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The price offered for polyamide 6 by the South African market is 1000 Rands per ton of polyamide 6. The chemical polyamide 6 is perishable, and once delivered by PolyCo to the distribution centre, it can only be used by their customers for injection moulding within one month of the date of delivery. If it is not sold by the distribution centre within one month of delivery, then the degraded chemical is sold for 300 Rands per ton in the open commodity market.

Each of the distribution centres places a bulk order one month in advance with the main manufacturing facility, located in Johannesburg. The manufacturing facility then sets up the raw material orders with their suppliers and produces the chemical as close to the end of the month as possible. The chemical is then stored in a special refrigerated cell to maintain its integrity and chemical composition and delivered to the distribution centre on the first of each month. The unit cost of manufacturing the chemical in bulk at PolyCo is 600 Rands per ton, inclusive of transportation costs, which is the cost charged to each distribution centre as well.

In addition to the bulk orders, the main manufacturing facilities can also deliver emergency or rush orders to the distribution centres. PolyCo has a contract with Cheetah – a third party logistics services provider – for delivering these emergency orders. Cheetah currently charges the distribution centres 150 Rands per ton for delivering rush orders. In addition to the 600 Rands per ton unit cost, the main manufacturing facility also charges a rushed-order cost of 50 Rands per ton to the distribution centre for rush orders. In doing so, the main manufacturing facility aims to encourage the distribution centres to order everything they need per month in the bulk order. It believes that the distribution centres will do a better job at forecasting demand, if forced to order in bulk.

(a) What is the expected demand in Region 1 for the year 2022? What is a 95% confidence interval for this expected demand? [7%]

(b) How well do the explanatory variables explain annual demand in Region 1? [8%]

(c) What are some problems with the variables chosen and the predictions made? What information would you need to understand the extent of these concerns? [9%]

(d) How many units should the distribution centres order from the Johannesburg facility each month? [6%]

(e) What are the expected profits of each distribution centre for the year 2022? What are the total profits of the PolyCo group from the polyamide 6 chemical for the year 2022? [7%]

(cont.

(f) Can you identify any forms of waste in this operating model of PolyCo from the perspective of the distribution centres, or the main facility, or the end customer? What changes would you recommend, and what would be the impact of the changes recommended on PolyCo's profits? (Note: this answer should comprise both qualitative arguments and a quantitative solution.) [10%]

END OF PAPER

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Attachments

Table A: Standardised Normal Probability Table

Z	0.00	-0.01	-0.02	-0.03	-0.04	-0.05	-0.06	-0.07	-0.08	-0.09
-2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
-2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
-2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
-2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
-2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
-2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
-2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
-2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
-1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
-1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
-1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
-1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
-1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
-1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
-1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08691	0.08534	0.08379	0.08226
-1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
-1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
-1.0	0.15866	0.15625	0.15386	0.15151	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
-0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
-0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
-0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
-0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
-0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
-0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
-0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
-0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
-0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43251	0.42858	0.42465
0.0	0.50000	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414
z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736

Table B: Standardised Normal Loss Table

z	0.00	-0.01	-0.02	-0.03	-0.04	-0.05	-0.06	-0.07	-0.08	-0.09
-2.7	2.70106	2.71103	2.72099	2.73096	2.74093	2.75090	2.76087	2.77084	2.78081	2.79079
-2.6	2.60146	2.61142	2.62137	2.63133	2.64129	2.65125	2.66121	2.67117	2.68113	2.69110
-2.5	2.50200	2.51194	2.52188	2.53183	2.54177	2.55171	2.56166	2.57161	2.58156	2.59151
-2.4	2.40272	2.41264	2.42256	2.43248	2.44241	2.45234	2.46227	2.47220	2.48213	2.49207
-2.3	2.30366	2.31356	2.32345	2.33335	2.34325	2.35316	2.36307	2.37298	2.38289	2.39280
-2.2	2.20489	2.21475	2.22462	2.23449	2.24436	2.25423	2.26411	2.27400	2.28388	2.29377
-2.1	2.10647	2.11629	2.12612	2.13595	2.14579	2.15563	2.16547	2.17532	2.18517	2.19503
-2.0	2.00849	2.01827	2.02805	2.03783	2.04762	2.05742	2.06722	2.07702	2.08683	2.09665
-1.9	1.91105	1.92077	1.93049	1.94022	1.94996	1.95970	1.96945	1.97920	1.98896	1.99872
-1.8	1.81428	1.82392	1.83357	1.84323	1.85290	1.86257	1.87226	1.88195	1.89164	1.90134
-1.7	1.71829	1.72785	1.73742	1.74699	1.75658	1.76617	1.77578	1.78539	1.79501	1.80464
-1.6	1.62324	1.63270	1.64217	1.65165	1.66114	1.67064	1.68015	1.68967	1.69920	1.70874
-1.5	1.52931	1.53865	1.54800	1.55736	1.56674	1.57612	1.58552	1.59494	1.60436	1.61380
-1.4	1.43667	1.44587	1.45508	1.46431	1.47356	1.48281	1.49208	1.50137	1.51067	1.51998
-1.3	1.34553	1.35457	1.36363	1.37270	1.38179	1.39090	1.40002	1.40916	1.41831	1.42748
-1.2	1.25610	1.26496	1.27384	1.28274	1.29165	1.30059	1.30954	1.31851	1.32750	1.33650
-1.1	1.16862	1.17727	1.18595	1.19465	1.20336	1.21210	1.22086	1.22964	1.23844	1.24726
-1.0	1.08332	1.09174	1.10019	1.10866	1.11716	1.12568	1.13422	1.14279	1.15138	1.15999
-0.9	1.00043	1.00860	1.01680	1.02503	1.03328	1.04156	1.04986	1.05819	1.06654	1.07491
-0.8	0.92021	0.92810	0.93603	0.94398	0.95196	0.95997	0.96801	0.97607	0.98417	0.99229
-0.7	0.84288	0.85048	0.85810	0.86576	0.87345	0.88117	0.88892	0.89669	0.90450	0.91234
-0.6	0.76867	0.77595	0.78325	0.79059	0.79797	0.80537	0.81281	0.82028	0.82778	0.83531
-0.5	0.69780	0.70473	0.71170	0.71870	0.72573	0.73281	0.73991	0.74705	0.75422	0.76143
-0.4	0.63044	0.63701	0.64362	0.65027	0.65695	0.66367	0.67042	0.67721	0.68404	0.69090
-0.3	0.56676	0.57296	0.57920	0.58547	0.59178	0.59813	0.60452	0.61094	0.61740	0.62390
-0.2	0.50689	0.51271	0.51856	0.52445	0.53038	0.53634	0.54235	0.54840	0.55448	0.56060
-0.1	0.45094	0.45635	0.46181	0.46731	0.47285	0.47842	0.48404	0.48969	0.49539	0.50112
0.0	0.39894	0.40396	0.40902	0.41412	0.41926	0.42444	0.42966	0.43492	0.44022	0.44556
z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.39894	0.39396	0.38902	0.38412	0.37926	0.37444	0.36966	0.36492	0.36022	0.35556
0.1	0.35094	0.34635	0.34181	0.33731	0.33285	0.32842	0.32404	0.31969	0.31539	0.31112
0.2	0.30689	0.30271	0.29856	0.29445	0.29038	0.28634	0.28235	0.27840	0.27448	0.27060
0.3	0.26676	0.26296	0.25920	0.25547	0.25178	0.24813	0.24452	0.24094	0.23740	0.23390
0.4	0.23044	0.22701	0.22362	0.22027	0.21695	0.21367	0.21042	0.20721	0.20404	0.20090
0.5	0.19780	0.19473	0.19170	0.18870	0.18573	0.18281	0.17991	0.17705	0.17422	0.17143
0.6	0.16867	0.16595	0.16325	0.16059	0.15797	0.15537	0.15281	0.15028	0.14778	0.14531
0.7	0.14288	0.14048	0.13810	0.13576	0.13345	0.13117	0.12892	0.12669	0.12450	0.12234
0.8	0.12021	0.11810	0.11603	0.11398	0.11196	0.10997	0.10801	0.10607	0.10417	0.10229
0.9	0.10043	0.09860	0.09680	0.09503	0.09328	0.09156	0.08986	0.08819	0.08654	0.08491
1.0	0.08332	0.08174	0.08019	0.07866	0.07716	0.07568	0.07422	0.07279	0.07138	0.06999
1.1	0.06862	0.06727	0.06595	0.06465	0.06336	0.06210	0.06086	0.05964	0.05844	0.05726
1.2	0.05610	0.05496	0.05384	0.05274	0.05165	0.05059	0.04954	0.04851	0.04750	0.04650
1.3	0.04553	0.04457	0.04363	0.04270	0.04179	0.04090	0.04002	0.03916	0.03831	0.03748
1.4	0.03667	0.03587	0.03508	0.03431	0.03356	0.03281	0.03208	0.03137	0.03067	0.02998
1.5	0.02931	0.02865	0.02800	0.02736	0.02674	0.02612	0.02552	0.02494	0.02436	0.02380
1.6	0.02324	0.02270	0.02217	0.02165	0.02114	0.02064	0.02015	0.01967	0.01920	0.01874
1.7	0.01829	0.01785	0.01742	0.01699	0.01658	0.01617	0.01578	0.01539	0.01501	0.01464
1.8	0.01428	0.01392	0.01357	0.01323	0.01290	0.01257	0.01226	0.01195	0.01164	0.01134
1.9	0.01105	0.01077	0.01049	0.01022	0.00996	0.00970	0.00945	0.00920	0.00896	0.00872
2.0	0.00849	0.00827	0.00805	0.00783	0.00762	0.00742	0.00722	0.00702	0.00683	0.00665
2.1	0.00647	0.00629	0.00612	0.00595	0.00579	0.00563	0.00547	0.00532	0.00517	0.00503
2.2	0.00489	0.00475	0.00462	0.00449	0.00436	0.00423	0.00411	0.00400	0.00388	0.00377
2.3	0.00366	0.00356	0.00345	0.00335	0.00325	0.00316	0.00307	0.00298	0.00289	0.00280
2.4	0.00272	0.00264	0.00256	0.00248	0.00241	0.00234	0.00227	0.00220	0.00213	0.00207
2.5	0.00200	0.00194	0.00188	0.00183	0.00177	0.00171	0.00166	0.00161	0.00156	0.00151
2.6	0.00146	0.00142	0.00137	0.00133	0.00129	0.00125	0.00121	0.00117	0.00113	0.00110

Table C
L_q Values for the Multi-server Queue

Values of L_q for s servers, with mean utilisation rate ρ, assuming Poisson arrivals and exponential service times (known as a *M/M/s* queue).

Utilisation rate (ρ)*	***Number of servers*** (*s*)				
	1	**2**	**3**	**4**	**5**
.10	.0111	.0020	.0004	.0001	.0000
.20	.0500	.0167	.0062	.0024	.0010
.30	.1286	.0593	.0300	.0159	.0086
.35	.1885	.0977	.0552	.0325	.0196
.40	.2667	.1524	.0941	.0605	.0398
.45	.3682	.2285	.1522	.1052	.0743
.50	.5000	.3333	.2368	.1739	.1304
.55	.6722	.4771	.3583	.2772	.2185
.60	.9000	.6750	.5321	.4306	.3542
.62	1.0116	.7743	.6213	.5109	.4269
.64	1.1378	.8880	.7246	.6051	.5130
.66	1.2812	1.0188	.8446	.7158	.6152
.68	1.4450	1.1698	.9847	.8461	.7367
.70	1.6333	1.3451	1.1488	1.0002	.8816
.72	1.8514	1.5500	1.3423	1.1834	1.0553
.74	2.1062	1.7914	1.5721	1.4025	1.2646
.76	2.4067	2.0785	1.8472	1.6668	1.5187
.78	2.7655	2.4237	2.1803	1.9887	1.8302
.80	3.2000	2.8444	2.5888	2.3857	2.2165
.82	3.7356	3.3661	3.0979	2.8832	2.7029
.84	4.4100	4.0265	3.7456	3.5190	3.3273
.86	5.2829	4.8852	4.5914	4.3526	4.1493
.88	6.4533	6.0414	5.7345	5.3834	5.2682
.90	8.1000	7.6737	7.3535	7.0898	6.8624
.92	10.5800	10.1392	9.8056	9.5290	9.2893
.94	14.7267	14.2712	13.9240	13.6344	13.3821
.96	23.0400	22.5698	22.2088	21.9060	21.6408
.98	48.0200	47.5350	47.1602	46.8439	46.5656
.99	98.0101	97.5176	97.1357	96.8127	96.4274

* When one is faced with a value of ρ that lies between two values in the table, one may have to interpolate between the two values of *Lq* associated with the two successive values of ρ. For example, consider $s = 2$ with $\rho = 0.75$. This utilization rate lies in between the two table values of 0.74 and 0.76. The L_q for this M/M/2 queue can be approximated by th L_q llowing linear interpolation:

$$L_q = 1.7914 + \frac{(2.0785 - 1.7914)}{(0.76 - 0.74)}(0.75 - 0.74) = 1.7914 + 0.1435 = 1.9349$$

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Numerical/Relevant Answers

Question 1

Part (a):

Answer is C: 180

Part (b):

Answer is H

Part (c)

Answer is F

Part (d)

Answer is B

Part (e)

Answer is B

Part (f)

Answer is E

Part (g)

Answer is B

Part (h)

Answer is F

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Question 2:

Part (a)

The primary decision is whether to give out the loan or not.

Part (b)

Denote D=Default and ND=No Default and let P(ND)=x. Also denote all monetary values in thousands for simplicity (e.g., 1.5 instead of £1500, 100 instead of £100000).

Then, the payoff to the firm if a loan is given out is 111x and payoff without a loan is 100.

$111x>100$ when $x>100/111$.

The decision can be characterised as: Give loan when $x>100/111$ for an EMV of 111x and do not give out the loan otherwise for an EMV of 100.

Part (c)

Expected value of sample information (EVSI) is positive if we give a loan after a positive report and don't give a loan after a negative report. Otherwise, EVSI=0.

For the case when EVSI is positive, we can draw the tree of Figure A, with P(-) and P(+) denoting the probability of a negative and positive report.

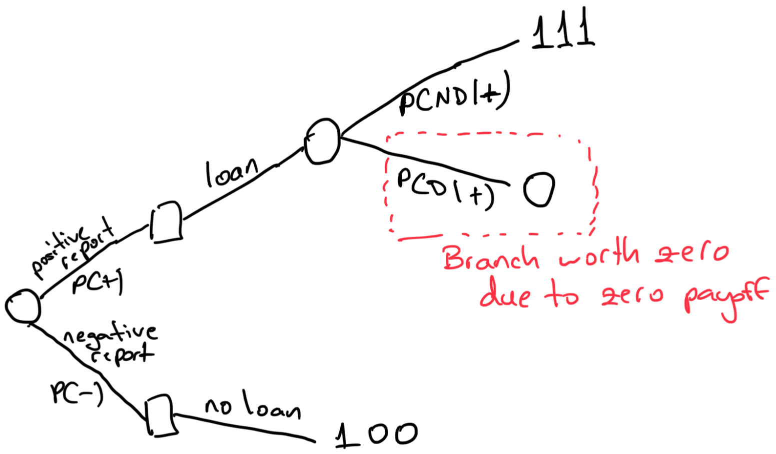

Fig A: Tree with Sample Information, when EVSI is Positive

Note:

$P(+)=P(+|ND)P(ND)+P(+|D)P(D)=(0.8)x + (0.3)(1-x)=0.3+0.5x$

And $P(-) = 1-P(+) = 0.7-0.5x$

Substituting P(+) and P(-) below gives the value of the tree of Figure A as

$$\begin{aligned} Tree\ Value\ (Fig.\ A) &= 100P(-) + (111)\frac{P(+|ND)P(ND)}{P(+)}P(+) \\ &= 100P(-) + (111)P(+|ND)P(ND) \\ &= (100)(0.7 - 0.5x) + (111)(0.8)(x) \\ &= 70 + 38.8x \end{aligned}$$

From part (b), EMV=111*x* if *x*>100/111 and 100 otherwise. Thus,

1) When *x*>111/100, the information is worth its cost of 1.5 if 70+38.8*x*-111*x*>1.5, which simplifies to *68.5-72.2x>0.*
2) When *x*<=*111/100*, the information is worth its cost of 1.5 if 70+38.8*x*-10>1.5, which simplifies to *38.8x-31.5>0.*

When conditions 1 or 2 are not satisfied, the information is not worth its cost.

Part (d)

With perfect information, the probability of a positive report is equal to the probability of no default (*x*). This gives us the tree in figure B.

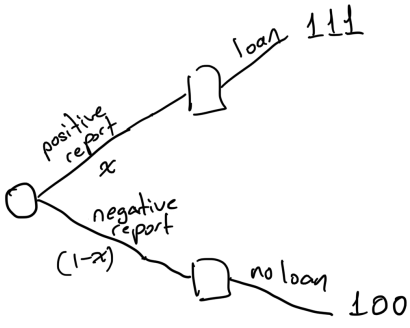

Fig B: Tree with Perfect Information.

Tree Value (Fig. B)=111*x* + (100) (1-*x*) = 100+11*x*

We need to subtract the appropriate EMV (111x or 100) from part (b) to get the expected value of perfect information (EVPI):

If *x*>100/111, EVPI=100+11*x*-111*x*= (1-*x*)100

If *x*<=100/111, EVPI=100+11*x*-100=11*x*

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Question 3:

Part (a)

Winston should recommend pooled security, as pooled queues always dominate dedicated queues in terms of waiting times and the number in the queue. An exception could be the loss in efficiency if people need to travel a long distance from the head of the pooled queue to the respective security lane.

Part (b)

To satisfy the target, we start with 2 lanes and increase the number of lanes until the target is met.

$\lambda_p = 0.9/\text{min}$
Denote service time by τ. $(1/\mu) = \tau = 2$ mins . then $\rho = \lambda_p \tau/s = 0.9$.
From table C, $L_q = 7.6737 < 8$.
Hence, just two security lanes are enough.

Part (c)

$\lambda = 0.45/\text{min}$ $\tau = 2$ mins $\rho = \lambda\,\tau = 0.9$.
$W_q = \tau\,\rho/(1-\rho)\,(CV_A^2+CV_S^2)/2 = 18$ minutes
$Lq = \lambda\, W_q = 0.45\text{X}18 = 8.1$ people per queue. Since we have 2 queues, we have 16.2 people waiting on average.

Part (d)

Customer Type	Frequent Flyer	Regular
Arrival rate λ	0.9*0.65=0.585	0.9*0.65=0.315
Service rate μ	1/1.5=0.667	1/2/93=0.3413
s	1	1
ρ=λ/(μ*s)	0.8775	0.9230
Lq (from table C)	6.45	10.58
Wq=Lq/λ (By Little's Law)	11.02564	33.5873
W= Wq + Service Time	12.52564	36.5173

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Performance Metrics:

Average time spent in the queue by a passenger: (0.65)(11.02564)+(0.35)(33.5873) =18.9222

Average time total time spent by a passenger: (0.65)(12.52564)+(0.35)(36.5173) =20.9227

Average number of passengers waiting in the queue: 6.45+10.58=17.03

Overall, this system is slightly worse off than part C. However, in this system, the frequent travellers are significantly better off.

Further Suggestion: The faster travel lane can be used as a way to provide value added service. The airlines can be charged for this service provided by the airport. The extra revenue can be used to improve service for regular passengers and thereby promote equity.

Part (e)

Resource pool	Calculation	Capacity (trays/hour)
Positioning	Capacity = 3 × (1/8 min) × (60 min/hour)	22.5 trays/hour
Washing and Drying	Capacity = 10 × (1/60 min) × (60 min/hour)	10 trays/hour
Inspecting and Packing	Service time = 80%(9 min) + 20% (22 min) = 7.2 + 4.4 = 11.6 min Capacity =2 × (1/11.6min) × (60 min/hour)	10.3 trays/hour
Sterilizing	Capacity = (24 trays/batch)/(2.5 hours/ batch)	9.6 trays/hour

Sterilizing is the bottleneck. Therefore, the capacity of the system is 9.6 trays/hour.

Part (f)

To achieve a system capacity of 12 trays/hour, we need to increase the capacity of the second, third, and fourth steps because their current capacities are lower than 12 trays/hour.

- For the second step, we need to purchase at least two more washing and drying machines to reach a capacity of 12 trays/hour.
- For the third step, we need a third person, which will bring the capacity to 3(1/11.6 min) × (60 min/hour) = 15.5 trays/hour.
- For the fourth step, we will need to buy another sterilizing oven which would double the rate to 19.2 trays/hour.

Some possible improvements over this plan are possible: a) Only two people are needed in the first step so moving one person from step 1 to step 3 may be possible depending on the skill requirements. b) We may need a third additional washer/dryer since the rate for this resource pool is exactly the required rate. c) It may be possible to buy a smaller sterilizer as a capacity of 19.2 trays/hour is more than necessary.

Part (g)

Reducing the damage rate from 20% to 5% would change the service time to 95%(9 min) + 5% (22 min) = 9.65 min in step 3 and this would increase the capacity to 2 (1/9.65 min) × (60 min/hour) = 12.4 trays/hour with the current two people. Thus, the current two people could keep up with a demand rate of 12 trays/hour.

Therefore, if the damage rate could be reduced, the savings would be the wages of an entire person (remember that we had to hire an additional worker to keep up with the demand rate of 12 trays/hour if the damage rate had stayed the same, see part (b)).

Dean has merely calculated the time saved and not recognized that there will be idle time for the third person required to handle the 12/hour rate in the previous setting. Thus, the savings would be 8 hours of one person's time multiplied by that person's wage rate (recall that union contracts prohibit us from hiring part-time employees for this task).

Question 4

Part (a)

Y=19,998+23*2022+.02972*1800000=120,000. The expected annual demand in region 1 is 120,000 tons. Standard error for the model is given as 40,000. Then the 95% confidence interval is given by 120,000 +/- 2*40,000 tons, i.e. [40000,200000].

Part (b)

We can see that the confidence intervals for the variables Year and Advertising include zero. This means that neither variable is significant. Hence, the variables are not that good at explaining annual demand.

We can also see that the r-square is .2209. In other words, the model explains 22.09% of the variability in demand. This is relatively low relative to forecasting models we built in class. A mitigating factor could be how volatile demand inherently is for the product at hand.

Part (c)

Note: Elaborating on two problems is sufficient. Three are provided below. Other reasonable answers can be accepted as correct.

Problem 1:
It seems from the positive coefficient for Year that the sales are increasing every year. As the firm grows, the amount it spends on advertising may also grow. That would make the two variables (Year and Advertising) highly correlated. That, in turn, leads to a multicollinearity problem. A multicollinearity problem could explain the insignificance of the two explanatory variables.

One would need to see the correlation table to have a better understanding of the extent of this problem. If they are indeed highly correlated I would run regressions with each of the explanatory variables separately to select the best model.

Problem 2:
By definition, the data are from previous years and we are predicting demand for the year 2022. That means that we are extrapolating. Extrapolation is dangerous.

To address the issue, I would want to see a regression with a hold-out sample of one year and compare the residuals of that model to those of the current model.

(cont.

Problem 3:
Note that we also do not know if the firm has previously spent more than the current year's advertising budget. Again, we may be extrapolating. To address this issue, I would need to see a table of previous years' advertising budgets.

Part (d)

We need to take into account the "reactive" capacity that costs 600 + 150 + 50 = 800 Rand per ton. The Overall cost structure is Co = c – s = 600 – 300 = 300 Rand. Cu is 1000 – (1000 – 800) – 600 = 200 Rand, where p=1000 is diminished by (1000-800) = 200 Rand to account for the guaranteed revenue to the distribution centre by using reactive capacity, if needed.

So F* = Cu/(Cu+Co) = 200/500 = 0.4 is the critical fractile. From Table A, z* = -0.25. Expected monthly demand (given assumptions listed in the question) is 120,000/12 and standard deviation is 40,000/sqrt(12). The optimal inventory level is therefore

$$Q^* = \mu + z^* \sigma = \frac{120000}{12} - 0.25\frac{40000}{\sqrt{12}} = 7113 \text{ tons.}$$

Each distribution centre should thus order 7113 tons per month.

Part (e)

Let's call the orders fulfilled by the initial order "primary sales" and note that additional demand can be met with reactive orders while any excess inventory is sold at salvage value. L_n values are given by Table B.

E[reactive orders] = E[lost sales from primary] = $\sigma L_n(z^*) = \frac{40000}{\sqrt{12}} \times L_n(-0.25)$

= 11547 x 0.53634 = 6193

E[primary sales] = E[demand] – E[reactive orders] = 10000 – 6193 = 3807

E[Salvage] = amount ordered – E[primary sales] = 7113 – 3807 = 3306

E[profit] = p x E[Demand] + s x E[salvage] – c x Q* - 800 x E[reactive orders]
= 1000 x 10 000 + 300 x 3306 – 600 x 7113 – 800 x 6193 Rand
= 1,769,600 Rand

So each of the 5 distribution centres make 1.7696 Million Rand for a total of 8,848,000 (8.848 Million) Rand per month.

But the main centre makes 50 x 6193 = 309,650 Rand per distribution centre during the transfer associated with reactive orders, so the total profit for the whole group is:

8,848,000 + 5*309,650 = 10,396,250 Rand per month

Part (f)

Qualitatively, the 50 Rand/ton transfer for reactive capacity leads to waste. The artificially high reactive ordering costs cause the distribution centres to order more initially and rely less on reactive capacity. Lowering the reactive capacity cost will reduce the amount ordered initially, and eliminating this waste should increase overall profits.

Co stays the same: Co = c − s = 600 − 300 = 300 Rand. Removing the 50 Rand reactive capacity cost means Cu = 750-600 = 150 Rand, instead of 200 Rand.

Then, F* = Cu/Cu+Co = 150 / 450 = 0.33, so the z* = -0.43. The new inventory order level is

$Q^* = \mu + z^* \sigma$ = 10000 – 0.43 x 11547 = 5035 tons.

We can now recompute expected profits with this new order level:

E[reactive orders] = E[lost sales from primary] = $\sigma L_n(z^*)$ = 11 547 x L_n(-0.43)

= 11 547 x 0.65026 = 7 509 tons

E[primary sales] = E[demand] – E[reactive orders] = 10 000 – 7 509 = 2 491 tons

E[salvage] = amount ordered – E[primary sales] = 5 035 – 2 491 = 2 544 tons

E[profit] = p x E[Demand] + s x E[salvage] – c x Q* - 750 x E[reactive orders]

= 1000 x 10 000 + 300 x 2 544 – 600 x 5 035 – 750 x 7 509 Rand

= 2 110 450 Rand

So each of the 5 distribution centres make 2 110 450 for a total of

10,552,250 Rand per month.

The main centre no longer contributes further revenues from the receipt of money from the distribution centres.

So the improvement from eliminating this waste increases expected profits by 10,552,250 – 10,396,250 = 156,000 Rand per month, or 1,872,000 Rand per year.