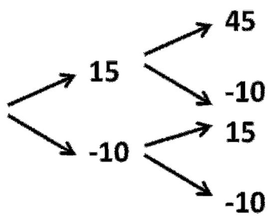
3E3 Modelling Risk -- 2011 Exam -- Cribs --- Final Version

1. (a)

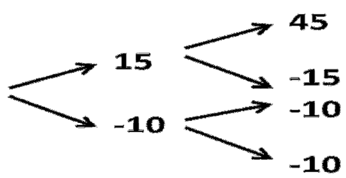
The value without any flexibility is calculated as follow. If you do not start the plant, you would simply value the firm at £0. If you do start the plant: If the price increases to £600, you earn £75M − £50M − £10M = £15M. If it decreases to £400, you earn £45M − £10M − £50M = −£15M. Therefore, your expected revenues are £0. The following year, you earn +£45M, −£15M, +£15M, or −£45M. This again comes to an expected £0M.

(i) If the retail price is above £500, you would operate. For example, if the price is retail £600, your marginal revenues are £150, 000·(£600−£100)−£50M = £25M. Subtract £10M in sunk rent cost, and you end up with revenues of £15M. If the retail price is £400, you earn £45M, which is not enough to cover the £50M fixed operating cost, so you are better off not operating and just paying the rent of £10M.
You earn +£15M or −£10M in the first year. The expected value is £2.5M, which discounts to £2.3M. The final year, you earn +£45M, −£10M, +£15M, or −£10M, which is an expected value of £10M and a discounted value of £8.3M. Therefore, this firm is worth +£10.5M.
Total expected profits= expected profit from Y1+expected profit for Y2 are shown below:



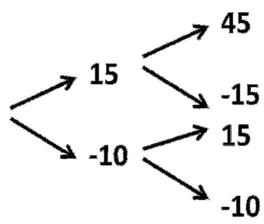
E=10.537 **E_1=2.272** **E_2=8.264**

(ii) If the retail price increases to £600, your best decision is to operate the plant. You will earn £15 million in the first year, and either gain £45 million or lose £15 million the second year. Your net is £15/(1 + 10%) ≈ £13.6 million plus (0.5 · £45 + 0.5 ·(−£15))/(1 + 10%)2 ≈ £12.4 million. The total is £26 million in expected present value.
If the retail price falls to £400, you commit to shuttering the plant. Your net is a loss of £10 million in each of two years. In present value, this is −£9.1 million followed by −£8.3 million. Your total is a loss of £17.4 million in expected present value. Both price paths are equally likely, so the plant is worth about 0.5 · (−£17.4)+0.5 · £26 ≈ £4.3 million. Total expected profits= expected profit from Y1+expected profit for Y2 are shown below:



E=4.338 **E_1=2.272** **E_2=-2.066**

(iii) The analysis is similar to (ii) expect for the case where the price in year 2 is £600 where you can now operate the plant.



E=9.504 **E_1=2.272** **E_2=7.231**

(b)

i) Following the method of the replicating portfolio: say we buy x stocks and y bonds then :
if the stock goes up: 30x+1.1B(0)y=10 (the value of the option in this case)
if the stock goes down: 20x+1.1B(0)y=0
from which we get x=1 and B(0)y=-18.18
So, the price of option now is 25x+B(0)y=6.82

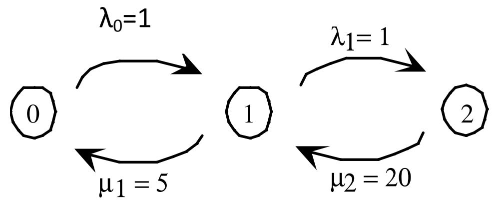
ii) in this case:
if the stock goes up: 30x+1.1B(0)y=4
if the stock goes down: 20x+1.1B(0)y=0
which gives x=0.4 and B(0)y= -7.27 , so C(0)= 25x+B(0)y=2.73

2 (a)

(i) Let state *n* = number of machines broken down.. Since the third machine is shut off when the second machine breaks down, we have *n* = 0, 1, or 2.

The failure rate of each machine is ½=0.5. When in state 0, we have two machines so 2x0.5=1 and when in state 1, again two machines, so 2x0.5=1. Similarly, the repair rate is 5 when in state 1 and 20 when in state 2.

The rate diagram is shown below.



(ii) The balance equations are

State 0: P0=5P1

State 1: P0+20P2=6P1

State 2: P1=20P2

Using the first and third balance equations, along with $P_0 + P_1 + P_2 = 1$, we obtain

(P0,P1,P2)= (100/121, 20/121, 1/121).

(iii) E[number of operators available] = 2 P_0 + 2 P_1 + 0 P_2 = 520 / 263 = 1.977.

(b) This is an M/M/1 queue with λ = 20 and μ = 30.

E [# of airplanes at the end of the thunderstorm]

= E [# of airplanes before thunderstorm] + E[# arrived planes during the thunderstorm]

$$= L_q + \lambda t = \lambda(W_q + t) = 20\left(\frac{20}{30(30-20)} + 0.5\right) = 11.33.$$

3 (a)

(i) High R Square. Significant t-stat for SIZE and TOWN (or to say that 95% confidence intervals for SIZE and TOWN does not contain 0). The t-stat for AGE is also reasonably good.

(ii) The t-stat for BDRM is quite weak and the 95% confidence interval for BDRM obviously contains 0. You could also point out that the t-stat for the intercept is very bad and the

confidence interval for the intercept is very wide. It is okay if you do not mention this as we often attach much less significance to the intercept estimate.
The confidence interval for AGE barely includes 0. It is fine if you view this as a sign of a good model or a not-so-good model, depending how "good" you want the model to be. But you have to be consistent. It should be pointed out that BDRM and SIZE tend to have high correlation by common sense and intuition. There is most likely a problem of multicollinearity. The least you can do to improve the model is to remove BDRM as an independent variable and run the regression again. Better yet, you should compute the correlation between SIZE and BDRM and confirm that they are indeed very highly correlated.

(iii) Based on the regression model, the fair market value of the house is: 180.833 + 3.07*175 +
(-3.571) * 57 + 45.421*3 + 0 = 650.799 (unit: £1,000) = £650,799. The asking price is lower than the fair market value.

(iv) Extra 300 square meters living space is estimated to be worth 3.07 * 30 = 92.1 (unit: £1000) = £92,000. The 99% confidence interval to supplement your estimate is: 30*(3.07-2.58*0.83, 3.07+2.58*0.83) = 30*(0.93, 5.21) = (27.858, 156.342) = (£27,858, £156,342).

(b) (i)

Alternatives	Flood	No Flood	Exp. value
Buy Insurance	247,800	247,800	247,800
Do not buy ins.	50,000	250,000	**248,000**
Prior probabilities	0.01	0.99	

Optimal decision: do not buy insurance

(ii) with perfect info= 0.01*247,800+0.99*250,000= 249,978. So, EVPI=249,978-248,000=1,978

(iii)

Alternatives	Flood	No Flood	Exp. value
Buy Insurance	497,795	497,795	**497,795**
Do not buy ins.	223,607	500	497,236
Prior probabilities	0.01	0.99	

Optimal decision: buy insurance

4 (a)

(i) 4 ↔ 3 ← 5 ↔ 6 → 1 ↔ 2

(ii) {3,4} recurrent, {1,2} recurrent, {5,6} transient

(b)

(i) The transition matrix is :

$$\mathbf{P} = \begin{Vmatrix} 0.7 & 0.2 & 0.1 \\ 0.4 & 0.3 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{Vmatrix}$$

The answer is π_1 which is obtained by solving the following system: $\pi = \pi P$

$\pi_1 = 0.7\,\pi_1 + 0.4\,\pi_2 +$ **$0.2\,\pi_3$**

$\pi_2 = 0.2\,\pi_1 + 0.3\,\pi_2 + 0.4\,\pi_3$

$\pi_3 = 0.1\,\pi_1 + 0.3\,\pi_2 + 0.4\,\pi_3$

$\pi_1 + \pi_2 + \pi_3 = 1$

the solution gives $\pi_1 = 30/59$

ii) We need to solve the following system:
$k_1=0$
$k_2=1+0.3k_2+0.3k_3$
$k_3=1+0.4k_2+0.4k_3$
Which gives $k_2=3$ and $k_3=11/3$

(c)

i) The practice of taking advantage of a price difference between two markets by buying (short-selling) the asset from one market and selling it to the other. Options are priced in a way that makes such risk-free profit opportunities impossible.

ii) For a given level or risk contribution (the beta of a potential investment) the CAPM gives the minimum expected return (price) that would make such an investment worth adding to the current market portfolio.