Version NT/5

Numerical/Relevant Answers

Question 1

Part (a):

Answer is C: 180

Part (b):

Answer is H

Part (c)

Answer is F

Part (d)

Answer is B

Part (e)

Answer is B

Part (f)

Answer is E

Part (g)

Answer is B

Part (h)

Answer is F

Version NT/5

Question 2:

Part (a)

The primary decision is whether to give out the loan or not.

Part (b)

Denote D=Default and ND=No Default and let P(ND)=x. Also denote all monetary values in thousands for simplicity (e.g., 1.5 instead of £1500, 100 instead of £100000).

Then, the payoff to the firm if a loan is given out is $111x$ and payoff without a loan is 100.

$111x>100$ when $x>100/111$.

The decision can be characterised as: Give loan when $x>100/111$ for an EMV of $111x$ and do not give out the loan otherwise for an EMV of 100.

Part (c)

Expected value of sample information (EVSI) is positive if we give a loan after a positive report and don't give a loan after a negative report. Otherwise, EVSI=0.

For the case when EVSI is positive, we can draw the tree of Figure A, with P(-) and P(+) denoting the probability of a negative and positive report.

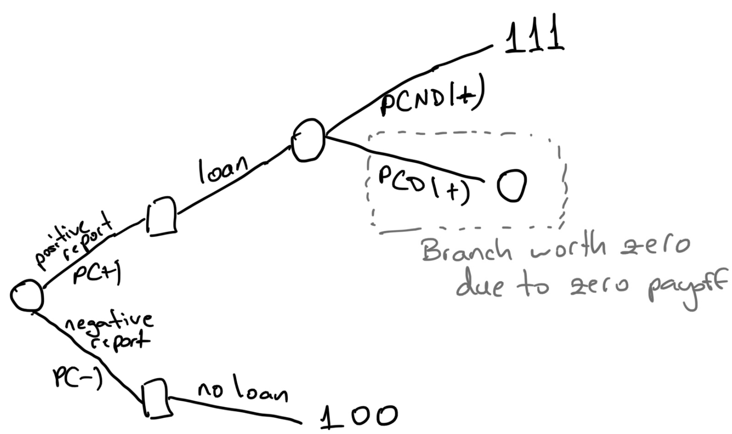

Fig A: Tree with Sample Information, when EVSI is Positive

Note:

$P(+)=P(+|ND)P(ND)+P(+|D)P(D)=(0.8)x + (0.3)(1-x)=0.3+0.5x$

And $P(-) = 1-P(+) = 0.7-0.5x$

Substituting P(+) and P(-) below gives the value of the tree of Figure A as

$$\begin{aligned} Tree\ Value\ (Fig.\ A) &= 100P(-) + (111)\frac{P(+|ND)P(ND)}{P(+)}P(+) \\ &= 100P(-) + (111)P(+|ND)P(ND) \\ &= (100)(0.7 - 0.5x) + (111)(0.8)(x) \\ &= 70 + 38.8x \end{aligned}$$

From part (b), EMV=111*x* if *x*>100/111 and 100 otherwise. Thus,

1) When *x*>111/100, the information is worth its cost of 1.5 if 70+38.8*x*-111*x*>1.5, which simplifies to *68.5-72.2x>0.*
2) When *x<=111/100*, the information is worth its cost of 1.5 if 70+38.8*x*-10>1.5, which simplifies to *38.8x-31.5>0.*

When conditions 1 or 2 are not satisfied, the information is not worth its cost.

Part (d)

With perfect information, the probability of a positive report is equal to the probability of no default (*x*). This gives us the tree in figure B.

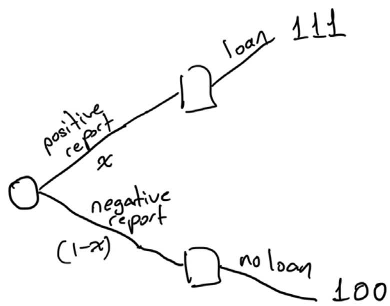

Fig B: Tree with Perfect Information.

Tree Value (Fig. B)=111*x* + (100) (1-*x*) = 100+11*x*

We need to subtract the appropriate EMV (111x or 100) from part (b) to get the expected value of perfect information (EVPI):

If *x*>100/111, EVPI=100+11*x*-111*x*= (1-*x*)100
If *x*<=100/111, EVPI=100+11*x*-100=11*x*

Version NT/5

Question 3:

Part (a)

Winston should recommend pooled security, as pooled queues always dominate dedicated queues in terms of waiting times and the number in the queue. An exception could be the loss in efficiency if people need to travel a long distance from the head of the pooled queue to the respective security lane.

Part (b)

To satisfy the target, we start with 2 lanes and increase the number of lanes until the target is met.

λ_p = 0.9/min
Denote service time by τ. $(1/\mu) = \tau = 2$ mins . then $\rho = \lambda_p \tau/s = 0.9$.
From table C, $L_q = 7.6737 < 8$.
Hence, just two security lanes are enough.

Part (c)

$\lambda = 0.45$/min $\tau = 2$ mins $\rho = \lambda\, \tau = 0.9$.
$W_q = \tau\, \rho/(1-\rho)\, (CV_A^2+CV_S^2)/2 = 18$ minutes
$Lq = \lambda\, W_q = 0.45X18 = 8.1$ people per queue. Since we have 2 queues, we have 16.2 people waiting on average.

Part (d)

Customer Type	Frequent Flyer	Regular
Arrival rate λ	0.9*0.65=0.585	0.9*0.65=0.315
Service rate μ	1/1.5=0.667	1/2/93=0.3413
s	1	1
$\rho=\lambda/(\mu*s)$	0.8775	0.9230
Lq (from table C)	6.45	10.58
$Wq=Lq/\lambda$ (By Little's Law)	11.02564	33.5873
W= Wq + Service Time	12.52564	36.5173

Performance Metrics:

Average time spent in the queue by a passenger: (0.65)(11.02564)+(0.35)(33.5873) =18.9222

Average time total time spent by a passenger: (0.65)(12.52564)+(0.35)(36.5173) =20.9227

Average number of passengers waiting in the queue: 6.45+10.58=17.03

Overall, this system is slightly worse off than part C. However, in this system, the frequent travellers are significantly better off.

Further Suggestion: The faster travel lane can be used as a way to provide value added service. The airlines can be charged for this service provided by the airport. The extra revenue can be used to improve service for regular passengers and thereby promote equity.

Part (e)

Resource pool	Calculation	Capacity (trays/hour)
Positioning	Capacity = 3 × (1/8 min) × (60 min/hour)	22.5 trays/hour
Washing and Drying	Capacity = 10 × (1/60 min) × (60 min/hour)	10 trays/hour
Inspecting and Packing	Service time = 80%(9 min) + 20% (22 min) = 7.2 + 4.4 = 11.6 min Capacity =2 × (1/11.6min) × (60 min/hour)	10.3 trays/hour
Sterilizing	Capacity = (24 trays/batch)/(2.5 hours/ batch)	9.6 trays/hour

Sterilizing is the bottleneck. Therefore, the capacity of the system is 9.6 trays/hour.

Part (f)

To achieve a system capacity of 12 trays/hour, we need to increase the capacity of the second, third, and fourth steps because their current capacities are lower than 12 trays/hour.

- For the second step, we need to purchase at least two more washing and drying machines to reach a capacity of 12 trays/hour.
- For the third step, we need a third person, which will bring the capacity to 3(1/11.6 min) × (60 min/hour) = 15.5 trays/hour.
- For the fourth step, we will need to buy another sterilizing oven which would double the rate to 19.2 trays/hour.

Some possible improvements over this plan are possible: a) Only two people are needed in the first step so moving one person from step 1 to step 3 may be possible depending on the skill requirements. b) We may need a third additional washer/dryer since the rate for this resource pool is exactly the required rate. c) It may be possible to buy a smaller sterilizer as a capacity of 19.2 trays/hour is more than necessary.

Part (g)

Reducing the damage rate from 20% to 5% would change the service time to 95%(9 min) + 5% (22 min) = 9.65 min in step 3 and this would increase the capacity to 2 (1/9.65 min) × (60 min/hour) = 12.4 trays/hour with the current two people. Thus, the current two people could keep up with a demand rate of 12 trays/hour.

Therefore, if the damage rate could be reduced, the savings would be the wages of an entire person (remember that we had to hire an additional worker to keep up with the demand rate of 12 trays/hour if the damage rate had stayed the same, see part (b)).

Dean has merely calculated the time saved and not recognized that there will be idle time for the third person required to handle the 12/hour rate in the previous setting. Thus, the savings would be 8 hours of one person's time multiplied by that person's wage rate (recall that union contracts prohibit us from hiring part-time employees for this task).

Question 4

Part (a)

Y=19,998+23*2022+.02972*1800000=120,000. The expected annual demand in region 1 is 120,000 tons. Standard error for the model is given as 40,000. Then the 95% confidence interval is given by 120,000 +/- 2*40,000 tons, i.e. [40000,200000].

Part (b)

We can see that the confidence intervals for the variables Year and Advertising include zero. This means that neither variable is significant. Hence, the variables are not that good at explaining annual demand.

We can also see that the r-square is .2209. In other words, the model explains 22.09% of the variability in demand. This is relatively low relative to forecasting models we built in class. A mitigating factor could be how volatile demand inherently is for the product at hand.

Part (c)

Note: Elaborating on two problems is sufficient. Three are provided below. Other reasonable answers can be accepted as correct.

Problem 1:
It seems from the positive coefficient for Year that the sales are increasing every year. As the firm grows, the amount it spends on advertising may also grow. That would make the two variables (Year and Advertising) highly correlated. That, in turn, leads to a multicollinearity problem. A multicollinearity problem could explain the insignificance of the two explanatory variables.

One would need to see the correlation table to have a better understanding of the extent of this problem. If they are indeed highly correlated I would run regressions with each of the explanatory variables separately to select the best model.

Problem 2:
By definition, the data are from previous years and we are predicting demand for the year 2022. That means that we are extrapolating. Extrapolation is dangerous.

To address the issue, I would want to see a regression with a hold-out sample of one year and compare the residuals of that model to those of the current model.

(cont.

Problem 3:
Note that we also do not know if the firm has previously spent more than the current year's advertising budget. Again, we may be extrapolating. To address this issue, I would need to see a table of previous years' advertising budgets.

Part (d)

We need to take into account the "reactive" capacity that costs 600 + 150 + 50 = 800 Rand per ton. The Overall cost structure is Co = c – s = 600 – 300 = 300 Rand. Cu is 1000 – (1000 – 800) – 600 = 200 Rand, where p=1000 is diminished by (1000-800) = 200 Rand to account for the guaranteed revenue to the distribution centre by using reactive capacity, if needed.

So F* = Cu/(Cu+Co) = 200/500 = 0.4 is the critical fractile. From Table A, z* = -0.25. Expected monthly demand (given assumptions listed in the question) is 120,000/12 and standard deviation is 40,000/sqrt(12). The optimal inventory level is therefore

$$Q^* = \mu + z^* \sigma = \frac{120000}{12} - 0.25\frac{40000}{\sqrt{12}} = 7113 \text{ tons.}$$

Each distribution centre should thus order 7113 tons per month.

Part (e)

Let's call the orders fulfilled by the initial order "primary sales" and note that additional demand can be met with reactive orders while any excess inventory is sold at salvage value. L_n values are given by Table B.

$$\text{E[reactive orders]} = \text{E[lost sales from primary]} = \sigma L_n(z^*) = \frac{40000}{\sqrt{12}} \times L_n(-0.25)$$

= 11547 x 0.53634 = 6193

E[primary sales] = E[demand] – E[reactive orders] = 10000 – 6193 = 3807

E[Salvage] = amount ordered – E[primary sales] = 7113 – 3807 = 3306

E[profit] = p x E[Demand] + s x E[salvage] – c x Q* - 800 x E[reactive orders]
= 1000 x 10 000 + 300 x 3306 – 600 x 7113 – 800 x 6193 Rand
= 1,769,600 Rand

So each of the 5 distribution centres make 1.7696 Million Rand for a total of 8,848,000 (8.848 Million) Rand per month.

But the main centre makes 50 x 6193 = 309,650 Rand per distribution centre during the transfer associated with reactive orders, so the total profit for the whole group is:

 (TURN OVER

8,848,000 + 5*309,650 = 10,396,250 Rand per month

Part (f)

Qualitatively, the 50 Rand/ton transfer for reactive capacity leads to waste. The artificially high reactive ordering costs cause the distribution centres to order more initially and rely less on reactive capacity. Lowering the reactive capacity cost will reduce the amount ordered initially, and eliminating this waste should increase overall profits.

Co stays the same: Co = c – s = 600 – 300 = 300 Rand. Removing the 50 Rand reactive capacity cost means Cu = 750-600 = 150 Rand, instead of 200 Rand.

Then, F* = Cu/Cu+Co = 150 / 450 = 0.33, so the z* = -0.43. The new inventory order level is

$Q^* = \mu + z^* \sigma$ = 10000 – 0.43 x 11547 = 5035 tons.

We can now recompute expected profits with this new order level:

E[reactive orders] = E[lost sales from primary] = $\sigma L_n(z^*)$ = 11 547 x $L_n(-0.43)$

= 11 547 x 0.65026 = 7 509 tons

E[primary sales] = E[demand] – E[reactive orders] = 10 000 – 7 509 = 2 491 tons

E[salvage] = amount ordered – E[primary sales] = 5 035 – 2 491 = 2 544 tons

E[profit] = p x E[Demand] + s x E[salvage] – c x Q* - 750 x E[reactive orders]

= 1000 x 10 000 + 300 x 2 544 – 600 x 5 035 – 750 x 7 509 Rand

= 2 110 450 Rand

So each of the 5 distribution centres make 2 110 450 for a total of

10,552,250 Rand per month.

The main centre no longer contributes further revenues from the receipt of money from the distribution centres.

So the improvement from eliminating this waste increases expected profits by 10,552,250 – 10,396,250 = 156,000 Rand per month, or 1,872,000 Rand per year.