Version MS/3

EGT2: IIA
ENGINEERING TRIPOS PART IIA

Thursday 4 May 2023 14.00 to 15.40

Module 3E3

MODELLING RISK

Answer not more than ***two*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator is allowed
Attachment: 3E3 Modelling Risk Data Sheet (3 pages)

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 A major microwave retailer in the country is concerned about the cost of running its various outlets. To get a handle on costs, it has gathered data for the last month of operations from a sample of 60 outlets. The data shows that the number of microwaves sold among all 60 outlets ranges from 50-160. The outlets vary in size and hours of operation, but for each outlet, the retailer has obtained the cost in pounds for running the outlet that month (these costs include the cost of merchandise sold) and the number of microwaves that were sold. An Excel spreadsheet regression model is shown below in Tables 1-2 for the relationship between the number of microwaves sold and the operation cost.

Table 1 Summary Output

Regression Statistics	
Multiple R	0.754455
R Square	0.569203
Adjusted R Square	0.561775
Standard Error	17735.48
Observation	60

Table 2 ANOVA

	df	*SS*	*MS*	*F*	*Significance Factor*
Regression	1	2.41E+10	2.41E+10	76.63412	3.38E-12
Residual	58	1.82E+10	3.15E+08		
Total	59	4.23E+10			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 99%*	*Upper 99%*
Intercept	853.29	7987.67	0.10	0.91	-15135.82	16842.4	-20420.24	22126.82
Units sold	627.64	71.67	8.75	3.38E-12	484.12	771.16	436.69	818.60

(a) The retailer is concerned about costs at outlets with low sales. Based on the fit of this model as given, what would be the 95% confidence/prediction interval for the operation cost of an outlet that sells 52 microwaves in a month. [30%]

(b) One outlet reports that its operation cost was £90,000 and it sold 70 microwaves last month. Do you think that the operation cost in these outlets is within a normal range compared with all outlets of the retailer? Explain your answer. [40%]

(c) Do you think that the slope of this regression equation is significantly different from zero? Explain your answer. What would be a possible implication on the regression model if your answer was no? [30%]

2 Mark Smith has this idea for a freshly brewed coffee twist. A few years back, he opened a café, CoffeeBreak. The café was a huge success and now operates a chain across the city. Mark glazes the coffee beans by adding sugar during the roasting process. The additional sugar provides the beans with a distinct flavour. However, coffee is one of the commodities whose price has experienced a significant increase along with the huge volatility of the world markets. It is therefore crucial for Mark to improve the performance of CoffeeBreak's process, which is shown below in Figure 1:

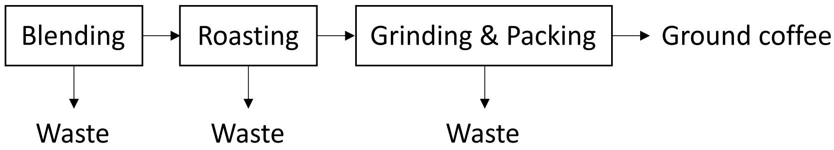

Figure 1

The process consists of three steps: blending of different coffee beans followed by the roasting of the blend, and finally grinding and packing. CoffeeBreak plans to have three extremely reliable continuous processing machines for the three steps. To ensure the freshness of coffee, at no point in the process can one build work-in-process inventory. The performance of the individual processing machines is described below in Table 3:

Table 3

	Step1	Step 2	Step 3
	Blending	**Roasting**	**Grinding & Packing**
Capacity	90 tonnes/hr	60 tonnes/hr	50 tonnes/hr
Waste	90%	80%	85%

Mark has hired you to help him with innovating these processes.

Note: For questions on plant capacity, assume that the bottleneck works at full capacity.

(a) Which step is the bottleneck and why? [10%]

(b) What is the capacity of this plant (in terms of tonnes/hr) [10%]

(c) If 70 tonnes/hr of coffee beans are introduced to the blending stage, what would be the utilisation of each of the three stages? [20%]

To mitigate the waste losses, several new technologies can re-process the losses from the grinding and packing stage. Consider the following two competing process technologies:

Technology A can take all the losses from the grinding and packing stage and convert all the waste to ground coffee (the final product). This technology is illustrated below in Figure 2.

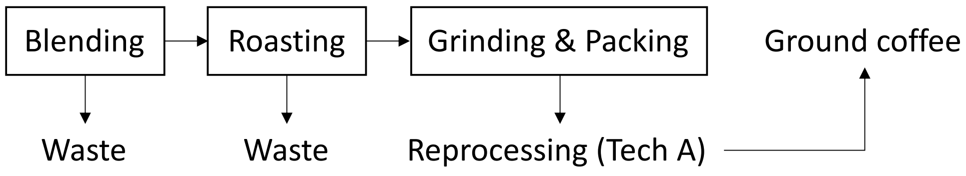

Figure 2

Technology B on the other hand, takes the losses from the grinding and packing stage and re-introduces them back into that stage, as illustrated below in Figure 3.

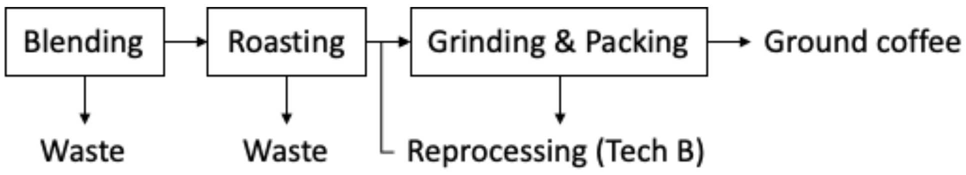

Figure 3

(d) If Technology A is adopted with a capacity of 10 tonnes/hour (for the reprocessing stage), what is the new capacity of the plant (in terms of tonnes/hour)? [30%]

(e) If Technology B is adopted with a capacity of 20 tonnes/hour, what is the new capacity of the plan (in terms of tonnes/hour)? [30%]

3 You have been appointed as the financial officer of a property firm, GothamBuild. The owner of the firm is faced with multiple choices for a property that they are currently managing:

Choice A: Make a large-scale investment to improve the flats which could produce a substantial pay-off in terms of increased revenue net of costs but will require an investment of £1,400,000. After extensive market research, it is considered that there is a 40% chance that a pay-off of £2,500,000 (high return) will be obtained, but there is a 60% chance that the pay-off will be only £800,000.

Choice B: Redecorate the flats and premises by spending £500,000, which is less costly than Choice A, but will produce a lower pay-off. The market research suggests a 30% chance of a gain of £1,000,000 (high return) but a 70% chance of pay-off being only £500,000.

Choice C: Continue the current operations without any change. This will cost nothing, but neither will it produce any pay-off. Clients will be unhappy, and it will become harder to rent the flats out when they become vacant.

Using decision trees, recommend an investment route to the owner.

(a) Draw decision tree representing the choices. [10%]

(b) Calculate the expected values for all the choices. [40%]

After you submitted your recommendations, the owner tells you that they spoke to one of their marketing advisors. The suggestion was to launch an advertising campaign to boost the guarantee of high returns for both Choice A and B. There is a 70% chance that the advertisements guarantee high returns for both these choices.

(c) In your view, how much would you be willing to pay for this advertisement campaign? Draw a decision tree and calculate the expected values for all the choices. [50%]

END OF PAPER

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SPECIAL DATA SHEET

Standard errors

$$STEM = \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}}, \quad STEP = \sqrt{\frac{p(1-p)}{n}} \approx \sqrt{\frac{q(1-q)}{n}},$$

$$STEDM = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}.$$

Covariance, correlation and Regression

Consider data pairs $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$.

Let m_X and m_Y denote the respective means of the X and Y data.

Let s_X and s_Y denote the respective standard deviations of the X and Y data.

Covariance between X and Y is given by

$$\mathrm{cov}(X, Y) = \frac{\sum_{i=1}^{n} (X_i - m_X)(Y_i - m_Y)}{n} = \frac{\sum_{i=1}^{n} X_i Y_i}{n} - m_X m_Y$$

The correlation coefficient between X and Y is given by

$$\mathrm{correl}(X, Y) = r = \frac{\mathrm{cov}(X, Y)}{s_X s_Y}.$$

The line of best fit is given by

$$Y - m_Y = \frac{r s_Y}{s_X}(X - m_X).$$

Variance of a portfolio

Consider three random variables x, y and z with means m_x, m_y, and m_z, respectively; variances Var(x), Var(y), and Var(z), respectively; and covariance between x and y, for example, given by the formula above. Given any numbers α_x, α_y, α_z, let $v = \alpha_x x + \alpha_y y + \alpha_z z$. Then the variance of v is given by

$$\mathrm{Var}(v) = \alpha_x{}^2 \mathrm{Var}(x) + \alpha_y{}^2 \mathrm{Var}(y) + \alpha_z{}^2 \mathrm{Var}(z) \\ + 2\left(\alpha_x \alpha_y \mathrm{cov}(x, y) + \alpha_y \alpha_z \mathrm{cov}(y, z) + \alpha_x \alpha_z \mathrm{cov}(x, z)\right)$$

Time Series Forecasting

$$E_t = \alpha \frac{X_t}{S_{t-c}} + (1-\alpha)(E_{t-1} + T_{t-1})$$

$$T_t = \beta(E_t - E_{t-1}) + (1-\beta)T_{t-1}$$

$$S_t = \gamma \frac{X_t}{E_t} + (1-\gamma)S_{t-c}$$

$$F_{t+k} = (E_t + kT_t)S_{t+k-c}$$

Queueing Theory (Poisson distribution, exponential distribution, performance metrics for the M/M/s queue, the M/M/1 queue is a special case of the M/M/s queue)

$$P(X = k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}, \quad k = 0,1,\ldots$$

$$P(X \le t) = 1 - e^{-\mu t}, \quad \forall t \ge 0.$$

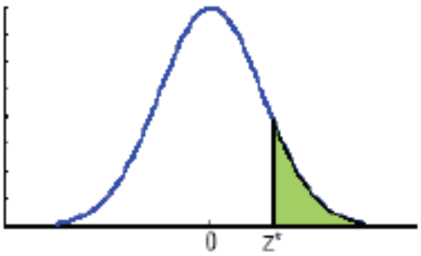
$$p_0 = \frac{1}{\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!}\left(\frac{s\mu}{s\mu - \lambda}\right)}$$

$$p_n = \begin{cases} \frac{(\lambda/\mu)^n}{n!} p_0 & \text{if } 0 \le n \le s \\ \frac{(\lambda/\mu)^n}{s! s^{n-s}} p_0 & \text{if } n \ge s \end{cases}$$

$$L_q = \left(\frac{(\lambda/\mu)^{s+1}}{(s-1)!(s-\lambda/\mu)^2}\right) p_0.$$

Standard Normal Distribution Table

(Areas under the standard normal curve beyond z*, i.e., shaded area)



z*	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010