Version FE/1

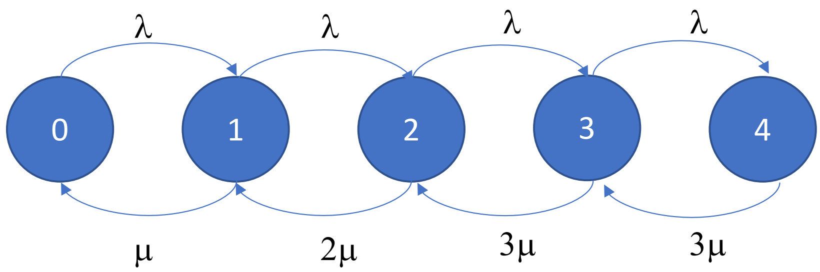
EGT2: IIA
ENGINEERING TRIPOS PART IIA

Module 3E3

MODELLING RISK - CRIB

1 (a) Please see below.

(i) Please see the figure.



(ii) π_0

(iii) $\pi_0 + \pi_1 + \pi_2$

(iv) 0

(v) π_4

(vi) $\pi_1 + 2 * \pi_2 + 3 * \pi_3 + 4 * \pi_4$

(vii) $1/3 * \pi_1 + 2/3 * \pi_2 + \pi_3 + \pi_4$

(viii) The expressions for the steady-state probabilities in terms of λ and μ are as follows:

$$
\begin{aligned}
\lambda \pi_0 = \mu \pi_1 \quad &\Rightarrow \quad \pi_1 = \lambda/\mu = \rho \pi_0 \\
\lambda \pi_0 + 2\mu \pi_2 = (\lambda + \mu)\pi_1 \quad &\Rightarrow \quad \pi_2 = \rho^2/2\pi_0 \\
\lambda \pi_1 + 3\mu \pi_3 = (\lambda + 2\mu)\pi_2 \quad &\Rightarrow \quad \pi_3 = \rho^3/6\pi_0 \\
\lambda \pi_2 + 3\mu \pi_4 = (\lambda + 3\mu)\pi_3 \quad &\Rightarrow \quad \pi_4 = \rho^4/6\pi_0 \\
\pi_0 + \pi_1 + \pi_2 + \pi_3 + \pi_4 = 1 \quad &\Rightarrow \quad \pi_0 = 1/(1 + \rho + \rho^2/2 + \rho^3/6 + \rho^4/6)
\end{aligned}
$$

(b) Please see below.

(i) The unit underage cost is $c_u = 35 - 10 = 25$ and the unit overage cost is $c_o = 10 - 5 = 5$.

(ii)

$$
\begin{aligned}
F(Q) &= c_u/(c_u + c_o) = 25/30 = 0.83 \\
z &= 0.965 \\
Q &= 100 + 0.965 * 10 = 109.65 \approx 110
\end{aligned}
$$

(iii)

$$\begin{aligned} z = 0.965 \quad &\Rightarrow \quad L(z) = 0.088 \\ 1 - \sigma L(z)/\mu \quad &= \quad 1 - (10)(0.088)/100 = 99.1\% \end{aligned}$$

(iv) Let the selling price be p, salvage value be s, and the buying cost be c. Note that sales and leftovers always sum up to Q. Thus $E[leftover] = Q - E[sales]$

$$\begin{aligned} E[\pi] &= pE[sales] - cQ + s(Q - E[sales]) \\ E[\pi] &= p[\mu - \sigma L(z)] - cQ + s(Q - \mu + \sigma L(z)) \\ Q &= \mu + z\sigma \\ E[\pi] &= (p-c)\mu + \sigma z(s-c) + \sigma L(z)(s-p) \\ &= 25\mu + 10(0.965)(-5) + 10(0.088)(-30) \geq 4000 \end{aligned}$$

Thus, $\mu \geq 162.98$.

(c) Simple exponential smoothing:

- Exponential smoothing forecasts contain information on all previous demands, each demand is given a weight that is decreasing exponentially back in time.
- Smoothing constant: $0 < \alpha < 1$
- The general formula for exponential smoothing is:

$$S_t = \alpha x_t + \alpha(1-\alpha)x_{t-1} + \alpha(1-\alpha)^2 x_{t-2} + \alpha(1-\alpha)^3 x_{t-3} + \cdots$$

- S_t is based on all (available) data up to period t to forecast x_{t+1}
- It copes OK with step changes in demand
- It does not cope well with linear trends or seasonality

Triple exponential smoothing:

- Triple exponential smoothing takes seasonality factors and trends in demand into account by using three smoothing equations and the forecasting equation.
- Basic idea is to introduce a trend estimate and a seasonality estimate
- Need to choose three smoothing rates, α, β, and γ
- Also called Winter's Linear and Seasonal ES model
- Forecasts are only good for a short term

When demand exhibits seasonality and trend simple exponential smoothing will not be able to predict the demand effectively, and triple exponential smoothing should be used.

(d) (i) Since the hotel chain is using historical demand observations, a time series method is the appropriate type of forecasting method for the firm.

(ii) Since the paint company does not have any historical data for their product and would like to base their forecast on new housing starts, they can use a causal model for their prediction.

2 (a) For a given level of risk contribution (the β of a potential investment), the CAPM gives the minimum expected return (price) that would make such an investment worth adding to the current market portfolio.

That is, if the market portfolio M is efficient, then the expected rate of return r_i of any asset i satisfies

$$r_i - r_f = \beta_i(r_M - r_f),$$

where r_f is the risk free rate and r_M is the expected return of the portfolio. Here, β_i measures the risk that asset i contributes to a diversified portfolio.

(b) One way is to find the SML/CAPM line $y = 5.5 + 7.5 * \beta$ and notice that for $\beta = 1.5$, the CAPM would predict a return of 16.5% < 17% so stock Y is currently undervalued. On the other hand, for stock Z the CAPM would predict a return of 11.5>10.5% so stock Z is overvalued. The reward-to-risk ratio for the two stocks has to be the same: (17- R_f)/ 1.5 = (10.5-R_f)0.8 from which we get R_f= 3.07%.

(c) Please see below.

(i) x^* still be optimal. The change in purchasing cost would not affect the optimal solution since the company would pay regardless of which week they purchase engines. You can consider the purchasing cost as a sunk cost in this case, and can find the optimal solution that balances the fixed cost and the inventory holding cost.

(ii) x^* cannot be optimal. Since the demand in each week is increasing, $(x_1^*, x_2^*, x_3^*, x_4^*)$ should be increasing as well. In addition, increasing x^* by 10% cannot be optimal either, since increasing the demand will break the previous balance between the fixed cost and the inventory holding cost found by x^*.

(d) Please see below.

(i) The set of states of the Markov chain is $S = \{0, 1, 2, 3\}$ representing the number of marbles in the first box. The transition matrix is as follows:

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1/3 & 0 & 2/3 & 0 \\ 0 & 2/3 & 0 & 1/3 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

(ii) This is an irreducible chain as there is a path from every state to any other state. It is not aperiodic as $p_{0,0}^{(n)} = 0$ when n is odd.

(iii) This Markov chain is not aperiodic, so it does not necessarily have a steady-state distribution. However, we can show that there is a unique solutionto the following equations:

$$\begin{aligned}
\Pi_0 &= \frac{\Pi_1}{3} \Rightarrow \Pi_1 = 3\Pi_0 \\
\Pi_1 &= \Pi_0 + \frac{2*\Pi_2}{3} \Rightarrow \Pi_2 = 3\Pi_0 \\
\Pi_2 &= \frac{2*\Pi_1}{3} + \Pi_3 \Rightarrow \Pi_3 = \Pi_0 \\
\Pi_3 &= \frac{\Pi_2}{3} \\
1 &= \Pi_0 + \Pi_1 + \Pi_2 + \Pi_3,
\end{aligned}$$

which is

$$\begin{aligned}
\Pi_0 &= \Pi_3 = \frac{1}{8} \\
\Pi_1 &= \Pi_2 = \frac{3}{8}
\end{aligned}$$

(iv)

$$H_{00} = \frac{1}{\Pi_0} = 8.$$

3 (a) Please see below.

(i) EMV = 400,000 * 0.25 - 100,000 * 0.75 = £25,000. The optimal decision is to always invest.

(ii) EVPI = (400,000 * 0.25 - 0 *0.25) - 25,000 = £75,000.

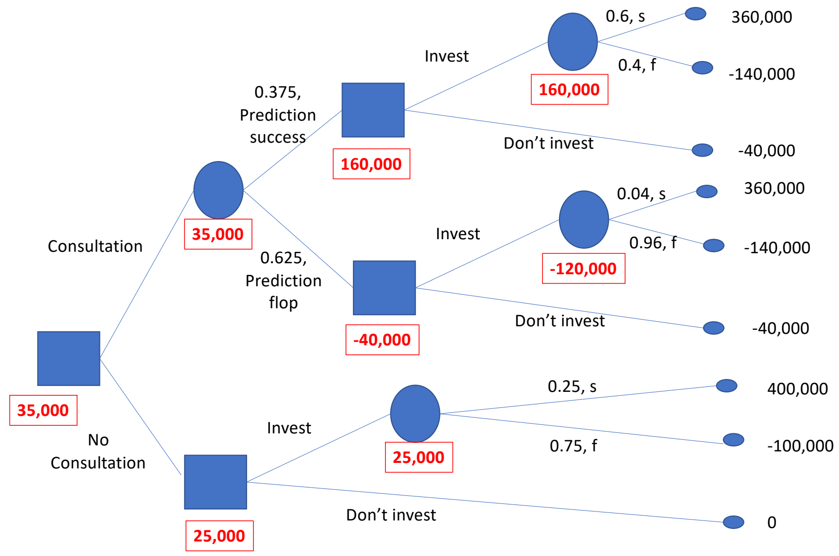
(iii) It is necessary to calculate some conditional probabilities. Let s and f be the random variables that the consultant will predict that the investment to be successful and a flop, respectively. Let S and F be the random variables that investments are successful and flop, respectively.
Then the data shows that $P(S) = 0.25$, $P(F) = 0.75$, $P(s|S) = 0.9$, and $P(f|F) = 0.8$. By the total probability law, we have

$$P(s) = P(s|S)P(S) + P(s|F)P(F) = 0.375,$$

and $P(f) = 1 - P(s) = 0.625$. By Bayes' Theorem, we have

$$P(S|s) = P(s|S)P(S) = 0.6, \qquad P(S|f) = P(f|S)P(S) = 0.04,$$

$P(F|s) = 1 - P(S|s) = 0.4$, and $P(F|f) = 1 - P(S|f) = 0.96$.
The EVSI = 35,000 - 25,000 = £10,000 (see the figure below for details).



(b) The EOQ model has been criticised due to its 'unrealistic' assumptions:

- Assumptions of relatively stable and known demand. The EOQ model is based on the assumption of constant demand over the year, which is hardly the case in reality. Demand is rarely constant, nor is it known a year in advance.

- Holding costs are estimates. How to determine appropriate opportunity cost of holding inventory, for example? (i.e., the bank rate or return on capital?). This results in the model tendering to favour large batch sizes, in particular if the holding cost is based on interest rates only (i.e. omits warehousing, obsolescence, handling and quality cost).
- Assumes only one part purchased, independent of all others. Does not consider interactions/synergies between parts sharing the same transportation equipment
- Assumes all inventory arrives in one delivery. Does not consider any supply chain implications of the batches (synchronisation with suppliers).
- Assumes no part shortages. This assumption may not be correct, particularly for purchasing commodities or industries of restricted supply.
- The cost factors for placing an order for one period are very hard to determine exactly. Estimating the cost of administrative processes is in particular very hard to quantify, as often fractions of a person's work time need to be estimated. Therefore, the cost data the model is based on often draws on inaccurate assumptions.
- However, despite the 'unrealistic' assumptions the EOQ model is robust to errors in estimation of its parameters – a key advantage. Errors in annual demand, ordering costs or holding costs have to be estimated, with risk of inaccuracy. However, because of the square root in the model, it mathematically mitigates this risk resulting in the model being less sensitive to deviations from estimated cost factors. Wrong estimates only move the EOQ marginally away from the optimal position.
- The EOQ model is also adaptable. For example, where demand or lead time is not constant, application of the perpetual inventory model with reorder point can help minimise the costs of variability. The model can also be adapted for quantity discounts, or situations of product interdependence. These modifications help overcome departures from EOQ's underlying assumptions.

(c) Please see below.

(i) The regression equation produced by the regression model is
Predicted sales = 13,707.14 + 37.34 * hops + 1,319.27 * malt + 0.05 * advertising - 63.17 * bitterness + 53.23 * investment.

(ii) From the table it follows that the intervals corresponding to the variables hops, bitterness, and initial investment contain zero, and so, those coefficients are not statistically significant. However, since the R^2 value is 0.88, we can safely say that the model is valid. Thus, we can eliminate the coefficients which are not statistically significant to obtain a more valid model.

(iii) Using

Predicted sales = 13,707.14 + 1,319.27 * malt + 0.05 * advertising,

we can calculate the predicted sales of each new beer as follows:

$$\begin{aligned}
\text{Predicted Sales (Great Ale)} &= 13,707.14 + 1,319.27 * 8 + 0.05 * 150,000 \\
&= 31,761.3 \\
\text{Predicted Sales (Fine Brau)} &= 13,707.14 + 1,319.27 * 6 + 0.05 * 155,000 \\
&= 29,372.76 \\
\text{Predicted Sales (HBC Porter)} &= 13,707.14 + 1,319.27 * 8 + 0.05 * 180,000 \\
&= 33,261.3 \\
\text{Predicted Sales (HBC Stout)} &= 13,707.14 + 1,319.27 * 7 + 0.05 * 150,000 \\
&= 30,442.03
\end{aligned}$$

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