Version HJ/3

EGT2: IIA
ENGINEERING TRIPOS PART IIA

Thursday 1 May 2014 2 to 3:30

Module 3E3

MODELLING RISK

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your number **not** your name on the cover sheet.*

STATIONARY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3E3 Modelling Risk data sheet (3 pages).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Analysis of arrivals to a single pump gas station has shown that the time between arrivals can be depicted by an exponential distribution with a mean of 10 minutes. Service times were also observed to be exponentially distributed, with a mean time of 6 minutes.

(i) What is the steady-state mean number of customers at the station? [15%]

(ii) What is the steady-state mean number of customers that are waiting? [15%]

(iii) Using this example, define and illustrate the meaning of Little's law. [15%]

(b) A queueing system has three servers with expected service times of 30 minutes, 20 minutes, and 15 minutes. Each service time has an exponential distribution. Each server has been busy with a current customer for 10 minutes. Determine the expected remaining time until the next service completion. [15%]

(c) Define and briefly explain the Capital Asset Pricing Model (CAPM) and the capital market line. [10%]

(d) Define the least square method and derive the mathematical formulas for the y-intercept and the slope of the simple linear regression equation. [15%]

(e) In Fig. 1, there are two histograms for two designs of the same project. Histogram A is represented by the columns with black filling and Histogram B is represented by the columns with broken border lines. Compare the risk profiles of designs A and B. [15%]

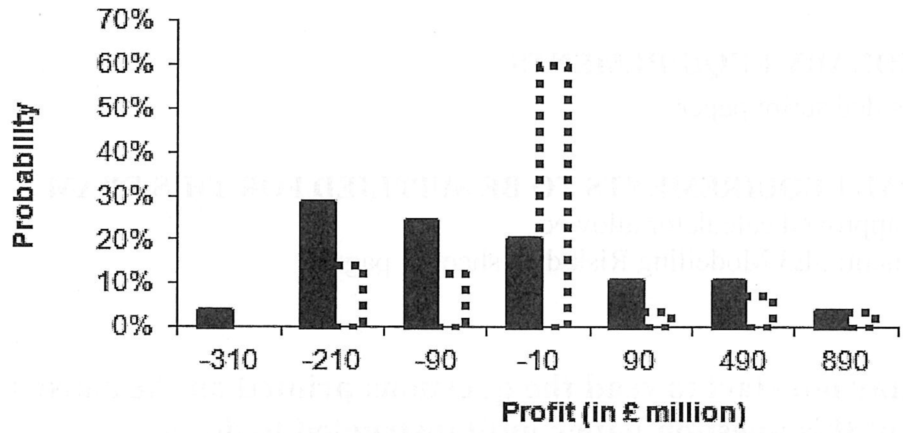

Fig. 1

2 (a) A small firm has to complete a contract for a large customer or pay high financial penalties for failing to deliver. Unfortunately, the stocks of one particular component used to assemble the product have been exhausted. The firm has a number of possible local suppliers of this component. The quality of the component is variable. Based on past data, the firm has found that the quality of 55% of components is Acceptable, 30% are Poor, and the remainder are Defective. Assume that if sample components from a supplier are Acceptable (or Poor or Defective), then all components from the same supplier are Acceptable (or Poor or Defective). The firm needs 10,000 units of the component, which cost £1 each. Poor components can be improved to Acceptable at a cost of 25p per unit while Defective components can be improved to Acceptable at a cost of £1 per unit.

(i) Based on a decision tree approach and the expected value criterion, evaluate the likely financial consequences for the firm if they can only choose one supplier and purchase 10,000 units of the component from them. [10%]

Because of the short timescale for completion of the contract, the firm decides to visit each supplier in turn. A small sample of components will be inspected and if the components look Acceptable, 10,000 units will be purchased from that supplier. If the first supplier's sample is not acceptable, the firm can decide if they inspect a sample from the second supplier, and so on. If however, the firm gets to the third and final supplier, it will have to purchase all 10,000 units from this supplier regardless of quality.

(ii) Based on a decision tree approach and the expected value criterion, evaluate the likely financial consequences for the firm if they can inspect at most three suppliers as described above. [20%]

(iii) Based on a quantitative approach (for example, recursive analysis), evaluate the likely financial consequences for the firm if they can inspect an infinite number of suppliers. [18%]

(iv) Assume that the base scenario is that the firm can only inspect one supplier as in (i). Compare the above three scenarios in (i), (ii) and (iii) by linking the number of inspections with the concept of value of information. [5%]

(b) Consider a Markov chain.

(i) Define the concept of aperiodicty. [5%]

(ii) Let $p_{ij}(n)$ be the n-step transition probability from state i to state j. Then prove that state i is aperiodic if and only if there exists N such that $p_{ii}(N) > 0$ and $p_{ii}(N+1) > 0$. [12%]

(iii) Define the Markovian property mathematically and intuitively. [5%]

(c) In the context of Monte Carlo simulation, describe the merits and drawbacks of sensitivity analysis. [10%]

(d) Consider the exponential smoothing forecasting method.

(i) Write down the base smoothing equation and the forecasting equation. [5%]

(ii) Give a detailed mathematical argument as to why the method is called exponential smoothing. [10%]

3 (a) Mark Boots is a soft drink distributor in Boston. Each week his warehouse manager inspects his soft drink crates (each wooden crate holds 24 bottles) and classifies them as R (just rebuilt this week), G (in good working condition), F (in fair condition), or D (damaged beyond use). If a crate is damaged beyond use it is sent to the repair area, where it is out of use for a week. Mark's warehouse records indicate that this is the matrix of transition probabilities for these soft drink crates:

$$P = \begin{array}{c} \\ R \\ G \\ F \\ D \end{array} \begin{array}{c} \begin{array}{cccc} R & G & F & D \end{array} \\ \begin{pmatrix} 0 & 0.8 & 0.2 & 0 \\ 0 & 0.6 & 0.4 & 0 \\ 0 & 0 & 0.5 & 0.5 \\ 1.0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Mark's accountant informs him that it costs \$2.50 to rebuild a crate, and the company incurs a loss of \$1.85 in production efficiency each time a crate is found to be damaged beyond use. This efficiency is lost because broken crates slow down the truck-loading process.

(i) Calculate the long-run probability distribution of the status of the crates. [10%]

(ii) Use your answer in (i) to interpret the meaning of the long-run probability distribution of the status of the crates. [5%]

(iii) What is the probability of a crate being in good working condition in two weeks time if it is in good working condition this week? [10%]

(iv) Given the above information, calculate the expected weekly cost of both rebuilding the crates and loss of production efficiency. [5%]

(v) Assume that Mark wants to consider rebuilding crates whenever they are inspected and found to be in fair condition. Determine the new matrix of transition probabilities and the average weekly cost of rebuilding and loss of production efficiency under these circumstances. [20%]

(b) Consider confidence intervals for the population proportion.

(i) Explain the meaning of a 95% confidence interval. [5%]

(ii) Explain how the width of a 95% confidence interval for a population proportion changes with sample size. [5%]

(c) In 2009, the New York Yankees won 103 baseball games during the regular American League season. A multiple regression model shown in Fig. 2 was built to predict the number of victories (Win) based on the earned-run-average (ERA) and the batting average (AVG) of each team in the American League. The ERA is one measure of the effectiveness of the pitching staff and a lower number is better. The AVG is one measure of effectiveness of the hitters, and a higher number is better. Another simple regression model shown in Fig. 3 was built to predict the number of victories based on the AVG alone.

(i) Specify the multiple regression equation and explain the variables and parameters in the equation. [10%]

(ii) With a 95% confidence level, what would be the number of wins for a team whose ERA and AVG are 4.00 and 0.26, respectively? How would you present your result to the team management? [10%]

(iii) Construct an 80% confidence interval for the slope of the ERA based on the multiple regression model and explain its meaning in the language of business. [10%]

(iv) Compare the simple and multiple regression models. [10%]

(cont.

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Regression Statistics	
Multiple R	0.9172842
R Square	0.8414103
Adjusted R Square	0.8125759
Standard Error	5.2652945
Observations	14

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	1617.971981	808.986	29.1807	3.99495E-05
Residual	11	304.9565906	27.72333		
Total	13	1922.928571			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-49.531647	46.48498478	-1.06554	0.309461	-151.844408	52.781115
ERA	-24.76217	4.22391256	-5.86238	0.000109	-34.058939	-15.465401
AVG	907.2699	166.5224443	5.448334	0.000201	540.7564748	1273.7833

Fig. 2

Regression Statistics	
Multiple R	0.588154914
R Square	0.345926203
Adjusted R Square	0.291420053
Standard Error	10.23774549
Observations	14

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	665.1913792	665.1914	6.346554	0.026946944
Residual	12	1257.737192	104.8114		
Total	13	1922.928571			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-134.3514011	85.89498007	-1.56414	0.143762	-321.500485	52.7976833
AVG	811.7746957	322.2304162	2.519237	0.026947	109.6949317	1513.85446

Fig. 3

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