Version FE/2

EGT2
ENGINEERING TRIPOS PART IIA

Thursday 6 May 2021 1.30 to 3.10

Module 3E3

MODELLING RISK

Answer not more than ***two*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet and at the top of each answer sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Attachment: 3E3 Modelling Risk data sheet (4 pages).
You are allowed access to the electronic version of the Engineering Data Books.

10 minutes reading time is allowed for this paper at the start of the exam.
The time taken for scanning/uploading answers is 15 minutes.
Your script is to be uploaded as a single consolidated pdf containing all answers.

1 (a) A queuing system with 3 servers is observed for a long period of time and data are collected on the proportion of time the system is in each of the states. Assume an infinite calling population. Capacity is limited, so whenever there is an arrival when 4 customers are present in the system, the arriving customer balks and goes elsewhere for service. Each state, denoted by n ($0 \leq n \leq 4$), represents the number of customers present in the system. Let the steady-state probability for state n be π_n.

(i) Assuming an arrival rate of λ and a service rate of μ, draw the state diagram. [5%]

(ii) What is the probability that all servers are idle? [5%]

(iii) What is the probability that an arriving customer will not have to wait? [5%]

(iv) What is the probability that more than one customer will have to wait in the queue? [5%]

(v) What is the probability that an arriving customer will be lost? [5%]

(vi) What is the expected number of customers in service? [5%]

(vii) What is the utilisation of the servers? [5%]

(viii) Find the expressions for the steady-state probabilities in terms of λ and μ. [15%]

(b) Newsgirl Ada buys newspapers from the local kiosk at 10 pence a copy and sells them at 35 pence a copy. She buys all her papers at once early in the morning and she sells on each day until she is pretty sure that no one else will buy her papers, or she runs out of copies. Ada recycles any unsold papers at 5 pence each.

(i) Find the unit underage and overage costs. [5%]

(ii) Ada knows that the demand is normally distributed with mean 100 and variance 100. How many papers should Ada buy each day? [5%]

(iii) What is the expected fill rate with the quantity you found in (ii)? [10%]

(iv) Now assume that the demand has a normal distribution with mean μ and variance 100. What is the minimum value of μ such that Ada's daily expected profit is at least £40? [10%]

(c) Explain the simple exponential smoothing model and the triple exponential smoothing model in your own words. Explain when you would prefer to use the triple exponential smoothing model over the simple exponential smoothing model. [10%]

(d) Which forecasting method would be appropriate in each of the following scenarios?

(i) A hotel chain is attempting to predict next year's demand for motel rooms based on a history of demand observations. [5%]

(ii) A paint company has developed a new type of outdoor paint. The company wishes to forecast sales based on new housing starts. [5%]

2 (a) Briefly explain CAPM and the meaning of β under CAPM. [10%]

(b) Consider two stocks in a market where the risk free rate is 5.5% and the market risk premium is 7.5%.

Stock	β	expected return
Y	1.5	17%
Z	0.8	10.5%

Are these stocks correctly priced? For what values of risk-free rate would the stocks be correctly priced? [20%]

(c) Cut'nTrim manufactures lawnmowers using an engine purchased from an outside vendor. Their demand for engines in each of the following four weeks is d_1, d_2, d_3, and d_4, respectively. All demand must be satisfied. Purchases are made at the beginning of each week, and delivery is instantaneous. Purchases can only be made for an integral number of weeks. Relevant costs are:

£P = Purchase Cost/engine purchased

£F = Fixed Cost/order placed (regardless of order size)

£H = Inventory Holding Cost/engine/week (charged only on end-of-week inventory)

Because of the short time horizon, there is assumed to be no discounting of purchases over time. Cut'nTrim currently has no engines in inventory, and any remaining engines at the end of the fourth week have no value. No backlogs or lost sales are allowed.

Cut'nTrim wants to determine its purchasing schedule x_t for the next four weeks to minimise its total costs (purchase, ordering, and inventory holding).

(i) Suppose that you are given values for all of the above parameters, and you solve the dynamic program and find an optimal solution $x^* = (x_1^*, x_2^*, x_3^*, x_4^*)$. If the price P is 10% higher than you thought, must you re-solve the dynamic program with the new price data in order to find the new optimal solution, or will x^* still be optimal? Explain your reasoning. [15%]

(ii) Suppose that you are given values for all of the above parameters, and you solve the dynamic program and find an optimal solution $x^* = (x_1^*, x_2^*, x_3^*, x_4^*)$. If demand in each week is 10% higher than you thought, must you re-solve the dynamic program with the new demand data in order to find the new optimal solution, or will x^* still be optimal? Explain your reasoning. [15%]

 (cont.

(d) There are two boxes that always contain three marbles in total. At each step, one of the marbles is chosen at random and moved to the other box.

(i) Express this problem as a Markov chain. What is the transition matrix for this Markov chain? [10%]

(ii) Show that this Markov chain is irreducible but not aperiodic. [10%]

(iii) Find the stationary distribution. [10%]

(iv) Assume that box 1 starts empty. Find the expected number of steps it takes until box 1 will be empty again. [10%]

3 (a) Investor Eren earns an average of £400,000 from a successful investment and loses an average of £100,000 from a bad one. Of all his investments, 25% turn out to be successful and 75% are flops. For £40,000 a more experienced investor can advise him on whether an investment is going to be successful or not. If an investment is going to be successful, there is a 90% chance that the experienced investor will predict the investment to be successful. If the investment is going to be a flop, there is an 80% chance that the experienced investor will predict the investment to be a flop.

(i) Determine the expected monetary value for investor Eren without the consulting service of the experienced investor. [15%]

(ii) Determine the Expected Value of Perfect Information. [15%]

(iii) Determine the Expected Value of Sample Information. [20%]

(b) The Economic Order Quantity (EOQ) model has been criticised due to its 'unrealistic' assumptions. Outline ways in which the various assumptions of EOQ are not matched by reality. To what extent is the validity of the EOQ model affected by these issues? [20%]

(c) The master brewer at Hubbard Brewing Co. (HBC) would like to predict the annual sales of beers that are brewed at HBC. To this end, the master brewer has assembled data and run a multiple linear regression model of beer sales based on the following five factors:

- Hops: the ounces of hops used in the brewing of a keg of beer.
- Malt: the pounds of malt used in the brewing of a keg of beer.
- Annual Advertising: the annual advertising expenditures for the beer in pounds.
- Bitterness: the bitterness rating of the beer. Bitterness is rated on a scale of 1 to 10 with a 10 being the most bitter.
- Initial Investment: the initial investment in brewing equipment in £million.

The master brewer gathered data on these five factors for 50 different beer brands brewed by HBC. The master brewer then ran a regression model to predict the annual sales of beers (in £thousands) using the previous five factors as the independent variables. The table below shows the key ingredients of the regression output.

SUMMARY OUTPUT

Regression Statistics	
R Square	0.8790
Standard Error	558.9364
Observations	50

	Coefficients	*Standard Error*	*Lower 95%*	*Upper 95%*
Intercept	13,707.1386	1.368.4086	11,025.05736	16,389.22064
Hops	37.3444	42.4970	-45.95012	120.63812
Malt	1,319.2696	161.0762	1,003.56104	1,634.97896
Annual Advertising	0.0493	0.0048	0.0392	0.0588
Bitterness	-63.1678	83.1086	-226.06164	99.72564
Initial Investment	53.2305	133.1423	-207.72832	314.18832

Suppose that the standard linear regression assumptions hold.

(i) What is the regression equation produced by the master brewer's regression model? [10%]

(ii) Evaluate the validity of the master brewer's regression model. In particular, which regression coefficients are significant? How might the master brewer change the model to arrive at a more valid model? [10%]

(iii) Table below contains data for four proposed new beers being developed for possible production at HBC. Assuming the coefficients for the statistically significant variables stay the same (which is of course not necessarily true), what is your prediction of the annual sales of these four new beers based on the new regression model? [10%]

Proposed New Beer	Hops (ounces per keg)	Malt (pounds per keg)	Annual Advertising (£)	Bitterness	Initial Investment (in £million)
Great Ale	12.0	8.0	150,000	6	1.2
Fine Brau	7.0	6.0	155,000	2	1.3
HBC Porter	11.0	8.0	180,000	3	2.1
HBC Stout	9.0	7.0	150,000	6	1.0

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EGT2: IIA
ENGINEERING TRIPOS PART IIA
6 May 2021, Module 3E3, Questions 1-3.

SPECIAL DATA SHEET

Standard errors

$$STEM = \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}}, \quad STEP = \sqrt{\frac{p(1-p)}{n}} \approx \sqrt{\frac{q(1-q)}{n}},$$

$$STEDM = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}.$$

Covariance, Correlation and Regression

Consider data pairs (X_1, Y_1), (X_2, Y_2), …, (X_n, Y_n).

Let m_X and m_Y denote the respective means of the X and Y data.

Let s_X and s_Y denote the respective standard deviations of the X and Y data.

Covariance between X and Y is given by

$$\text{cov}(X, Y) = \frac{\sum_{i=1}^{n} (X_i - m_X)(Y_i - m_Y)}{n} = \frac{\sum_{i=1}^{n} X_i Y_i}{n} - m_X m_Y$$

The correlation coefficient between X and Y is given by

$$\text{correl}(X, Y) = r = \frac{\text{cov}(X, Y)}{s_X s_Y}.$$

The line of best fit is given by

$$Y - m_Y = \frac{r s_Y}{s_X}(X - m_X).$$

Variance of a portfolio

Consider three random variables x, y and z with means m_x, m_y, and m_z, respectively; variances Var(x), Var(y), and Var(z), respectively; and covariance between x and y, for example, given by the formula above. Given any numbers α_x, α_y, α_z, let $v = \alpha_x x + \alpha_y y + \alpha_z z$. Then the variance of v is given by

$$\begin{aligned}\text{Var}(v) &= \alpha_x{}^2 \text{Var}(x) + \alpha_y{}^2 \text{Var}(y) + \alpha_z{}^2 \text{Var}(z) \\ &\quad + 2\left(\alpha_x \alpha_y \text{cov}(x, y) + \alpha_y \alpha_z \text{cov}(y, z) + \alpha_x \alpha_z \text{cov}(x, z)\right)\end{aligned}$$

Time Series Forecasting (Winters' multiplicative smoothing method)

$$E_t = \alpha \frac{X_t}{S_{t-c}} + (1-\alpha)(E_{t-1} + T_{t-1})$$
$$T_t = \beta(E_t - E_{t-1}) + (1-\beta)T_{t-1}$$
$$S_t = \gamma \frac{X_t}{E_t} + (1-\gamma)S_{t-c}$$

$$F_{t+k} = (E_t + kT_t)S_{t+k-c}$$

Markov Chains (calculate probabilities for first passage time and expected first passage times)

$$f_{ij}(1) = P_{ij}$$
$$\vdots$$
$$f_{ij}(n) = P_{ij}^{(n)} - f_{ij}(1)P_{jj}^{(n-1)} - \ldots - f_{ij}(n-1)P_{jj}^{(1)}.$$

$$E(H_{ij}) = 1 + \sum_{k \neq j} E(H_{kj})P_{ik}, \forall i.$$

Queueing Theory (Poisson distribution, exponential distribution, performance metrics for the M/M/s queue, the M/M/1 queue is a special case of the M/M/s queue)

$$P(X = k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}, \quad k = 0,1,...$$

$$P(X \leq t) = 1 - e^{-\mu t}, \quad \forall t \geq 0.$$

$$p_0 = \frac{1}{\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!}\left(\frac{s\mu}{s\mu - \lambda}\right)}$$

$$p_n = \begin{cases} \frac{(\lambda/\mu)^n}{n!} p_0 & if\ 0 \leq n \leq s \\ \frac{(\lambda/\mu)^n}{s! s^{n-s}} p_0 & if\ n \geq s \end{cases}$$

$$L_q = \left(\frac{(\lambda/\mu)^{s+1}}{(s-1)!(s-\lambda/\mu)^2}\right) p_0.$$

Inventory Management

(Q,R) Model:

Optimal solution :

$$Q = \sqrt{\frac{2\lambda[K + p\,n(R)]}{h}} \qquad F(R) = 1 - \frac{Qh}{p\lambda}$$

Newsvendor Model:

$F(Q^*) = c_u/(c_u+c_o)$

Standard Normal Distribution and Standard Loss Function Table

(Areas under the standard normal curve beyond z*, i.e., shaded area)

F(Z) is the probability that a variable from a standard normal distribution will be less than or equal to Z, or alternately, the service level for a quantity ordered with a z-value of Z.

L(Z) is the standard loss function, i.e. the expected number of lost sales as a fraction of the standard deviation. Hence, the lost sales = L(Z) x σ_{DEMAND}

Z	F(Z)	L(Z)	Z	F(Z)	L(Z)	Z	F(Z)	L(Z)	Z	F(Z)	L(Z)
-3.00	0.0013	3.000	-1.48	0.0694	1.511	0.04	0.5160	0.379	1.56	0.9406	0.026
-2.96	0.0015	2.960	-1.44	0.0749	1.474	0.08	0.5319	0.360	1.60	0.9452	0.023
-2.92	0.0018	2.921	-1.40	0.0808	1.437	0.12	0.5478	0.342	1.64	0.9495	0.021
-2.88	0.0020	2.881	-1.36	0.0869	1.400	0.16	0.5636	0.324	1.68	0.9535	0.019
-2.84	0.0023	2.841	-1.32	0.0934	1.364	0.20	0.5793	0.307	1.72	0.9573	0.017
-2.80	0.0026	2.801	-1.28	0.1003	1.327	0.24	0.5948	0.290	1.76	0.9608	0.016
-2.76	0.0029	2.761	-1.24	0.1075	1.292	0.28	0.6103	0.274	1.80	0.9641	0.014
-2.72	0.0033	2.721	-1.20	0.1151	1.256	0.32	0.6255	0.259	1.84	0.9671	0.013
-2.68	0.0037	2.681	-1.16	0.1230	1.221	0.36	0.6406	0.245	1.88	0.9699	0.012
-2.64	0.0041	2.641	-1.12	0.1314	1.186	0.40	0.6554	0.230	1.92	0.9726	0.010
-2.60	0.0047	2.601	-1.08	0.1401	1.151	0.44	0.6700	0.217	1.96	0.9750	0.009
-2.56	0.0052	2.562	-1.04	0.1492	1.117	0.48	0.6844	0.204	2.00	0.9772	0.008
-2.52	0.0059	2.522	-1.00	0.1587	1.083	0.52	0.6985	0.192	2.04	0.9793	0.008
-2.48	0.0066	2.482	-0.96	0.1685	1.050	0.56	0.7123	0.180	2.08	0.9812	0.007
-2.44	0.0073	2.442	-0.92	0.1788	1.017	0.60	0.7257	0.169	2.12	0.9830	0.006
-2.40	0.0082	2.403	-0.88	0.1894	0.984	0.64	0.7389	0.158	2.16	0.9846	0.005
-2.36	0.0091	2.363	-0.84	0.2005	0.952	0.68	0.7517	0.148	2.20	0.9861	0.005
-2.32	0.0102	2.323	-0.80	0.2119	0.920	0.72	0.7642	0.138	2.24	0.9875	0.004
-2.28	0.0113	2.284	-0.76	0.2236	0.889	0.76	0.7764	0.129	2.28	0.9887	0.004
-2.24	0.0125	2.244	-0.72	0.2358	0.858	0.80	0.7881	0.120	2.32	0.9898	0.003
-2.20	0.0139	2.205	-0.68	0.2483	0.828	0.84	0.7995	0.112	2.36	0.9909	0.003
-2.16	0.0154	2.165	-0.64	0.2611	0.798	0.88	0.8106	0.104	2.40	0.9918	0.003
-2.12	0.0170	2.126	-0.60	0.2743	0.769	0.92	0.8212	0.097	2.44	0.9927	0.002
-2.08	0.0188	2.087	-0.56	0.2877	0.740	0.96	0.8315	0.090	2.48	0.9934	0.002
-2.04	0.0207	2.048	-0.52	0.3015	0.712	1.00	0.8413	0.083	2.52	0.9941	0.002
-2.00	0.0228	2.008	-0.48	0.3156	0.684	1.04	0.8508	0.077	2.56	0.9948	0.002
-1.96	0.0250	1.969	-0.44	0.3300	0.657	1.08	0.8599	0.071	2.60	0.9953	0.001
-1.92	0.0274	1.930	-0.40	0.3446	0.630	1.12	0.8686	0.066	2.64	0.9959	0.001
-1.88	0.0301	1.892	-0.36	0.3594	0.605	1.16	0.8770	0.061	2.68	0.9963	0.001
-1.84	0.0329	1.853	-0.32	0.3745	0.579	1.20	0.8849	0.056	2.72	0.9967	0.001
-1.80	0.0359	1.814	-0.28	0.3897	0.554	1.24	0.8925	0.052	2.76	0.9971	0.001
-1.76	0.0392	1.776	-0.24	0.4052	0.530	1.28	0.8997	0.047	2.80	0.9974	0.001
-1.72	0.0427	1.737	-0.20	0.4207	0.507	1.32	0.9066	0.044	2.84	0.9977	0.001
-1.68	0.0465	1.699	-0.16	0.4364	0.484	1.36	0.9131	0.040	2.88	0.9980	0.001
-1.64	0.0505	1.661	-0.12	0.4522	0.462	1.40	0.9192	0.037	2.92	0.9982	0.001
-1.60	0.0548	1.623	-0.08	0.4681	0.440	1.44	0.9251	0.034	2.96	0.9985	0.000
-1.56	0.0594	1.586	-0.04	0.4840	0.419	1.48	0.9306	0.031	3.00	0.9987	0.000
-1.52	0.0643	1.548	0.00	0.5000	0.399	1.52	0.9357	0.028			