EGT2
ENGINEERING TRIPOS PART IIA

Monday 9 May 2022 2 to 3.40

Module 3F2

SYSTEMS AND CONTROL

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed

Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 Two masses m_1 and m_2 are connected by a linear viscous damper with constant $c > 0$ in series with an actuator A as shown in Fig. 1. The relative velocity across the attachment points of the actuator is modelled by the ideal equation

$$\dot{w} - \dot{z}_2 = u$$

where u is the control input. The damper and actuator may be assumed to have negligible mass in comparison with the two masses.

(a) Write down the dynamic equations satisfied by the system. Hence show that the state-space equations $\dot{\underline{x}} = A\underline{x} + Bu$ hold where:

$$A = \begin{bmatrix} -\frac{c}{m_1} & \frac{c}{m_1} & 0 \\ \frac{c}{m_2} & -\frac{c}{m_2} & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{c}{m_1} \\ -\frac{c}{m_2} \\ 1 \end{bmatrix}$$

and $\underline{x} = [\dot{z}_1, \dot{z}_2, w]^T$. [20%]

(b) Let $m_1 = m_2 = 1$.

(i) Find the controllability matrix of the system and hence show that the system is uncontrollable. [20%]

(ii) Find two unit vectors which span the reachability space. [20%]

(c) For general m_1 and m_2 show that

$$v^T = [m_1, m_2, 0]$$

is a left eigenvector of A. Hence, or otherwise, show that the system is uncontrollable for general m_1 and m_2. [20%]

(d) What can be said about the quantity $m_1\ddot{z}_1 + m_2\ddot{z}_2$? Give a physical explanation for the uncontrollability of the system. [20%]

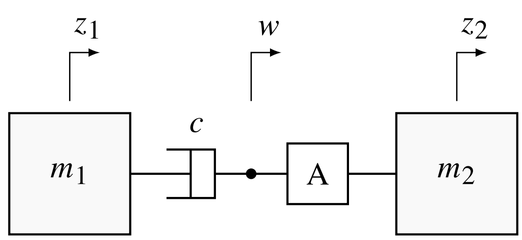

Fig. 1

2 An active control study is being undertaken of the Lorenz model of atmospheric convection:

$$\begin{aligned}\dot{x} &= a(y-x)\\ \dot{y} &= x(c-z)-y+u\\ \dot{z} &= xy-bz\end{aligned}$$

where x is a convection rate, y and z are temperature variations, u is a control input, and a, b, c are positive parameters.

(a) Assume that the input is held constant with $u = u_0$.

(i) Find equations that determine the states at equilibrium: $x = x_0$, $y = y_0$ and $z = z_0$. [15%]

(ii) Assuming $c = 1$ deduce that $x_0^3 = bu_0$ and write down equations for y_0 and z_0. [10%]

(b) Let $b = 4$, $c = 1$ and $u_0 = 2$.

(i) Write $u = u_0 + \delta u$, $x = x_0 + \delta x$ etc., and derive the linearised equations of motion about the equilibrium point. [20%]

(ii) Show that the transfer function relating δx to δu is given by

$$G(s) = \frac{s+4}{h(s)}$$

where $h(s) = [(s+1)(s+4)+4]\,(a^{-1}s+1)+4$. [20%]

(c) Let $a = 1$ and take $b = 4$, $c = 1$ and $u_0 = 2$ as in Part (b).

(i) Sketch the root-locus diagram for $G(s)$ in Part (b)(ii). [Hint: one of the poles of $G(s)$ is at $s = -3$.] [25%]

(ii) Briefly describe the behaviour of the control system near the equilibrium point as a function of the proportional feedback gain. [10%]

3 The circuit shown in Fig. 2 satisfies the (Kirchoff) equations

$$\begin{cases} u - v_c - L\frac{di_L}{dt} & = & 0 \\ i_L + \frac{1}{R}(u - v_c) & = & C\frac{dv_c}{dt} + \frac{v_c}{R} \end{cases}$$

The parameters of the circuit are the positive constants R, L, and C.

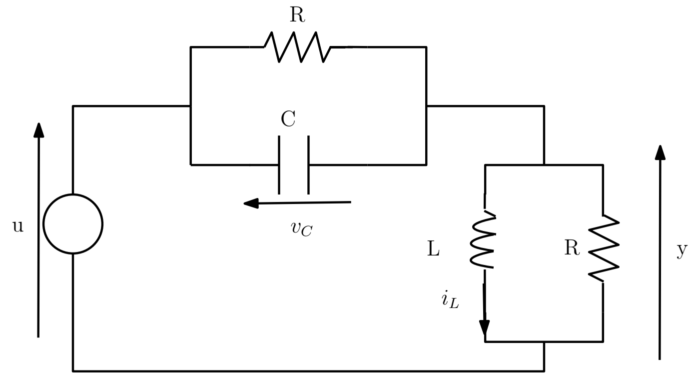

Fig. 2

(a) Write a state-space model of the circuit with the voltage u as input and the voltage y as output. [15%]

(b) Determine the transfer function of the circuit. [15%]

(c) Use the circuit equations (not the observability matrix) to show that the circuit is observable for all values of the parameters. [30 %]

(d) Assuming that $L = C = 1$, determine the value of R that makes the circuit uncontrollable. (Suggestion: determine this value from the transfer function rather than from the controllability matrix). [20 %]

(e) Determine a minimal state-space model of the uncontrollable circuit found in (d). [20 %]

4 A system is modeled as $\dot{x} = 3x + u$, $y = x + w$ where w is the measurement noise. The input $u = k\hat{x} + r$ is based on an estimated state $\hat{x}$, and r is a reference signal.

(a) Design a state observer with observer gain h to provide the estimate $\hat{x}$ and draw a block diagram of the control system. [30%]

(b) Find the closed-loop transfer functions from $\bar{r}$ and $\bar{w}$ to $\bar{y}$ in terms of k and h. [10%]

(c) Find the transfer function of the controller, that is from $\bar{y}$ and $\bar{r}$ to $\bar{u}$. [10%]

(d) Use your design to explain the meaning and significance of the separation principle. Explain the trade-offs to keep in mind when selecting the gains k and h. [30%]

(e) Propose a simplified design of the observer when the system is instead $\dot{x} = -3x + u$. Under which circumstances would you recommend this simplified design ? [20%]

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