Version GV/2

EGT2
ENGINEERING TRIPOS PART IIA

Tuesday 2 May 2023 9.30 to 11.10

**Module 3F2**

**SYSTEMS AND CONTROL**

*Answer not more than* ***three*** *questions.*

*All questions carry the same number of marks.*

*The* ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number* ***not*** *your name on the cover sheet.*

**STATIONERY REQUIREMENTS**
Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**
CUED approved calculator allowed.
Engineering Data Book

**10 minutes reading time is allowed for this paper at the start of the exam.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

**You may not remove any stationery from the Examination Room.**

1 Consider the system with the following dynamics:

$$\ddot{x} + 3\dot{x} - 3(\dot{x}^2 + x^2)\dot{x} + x = u \qquad (1)$$

which represents a mass with a nonlinear damper and a linear spring.

(a) Express the system in state-space form. [15%]

(b) Explain why it is possible to linearise the system about an arbitrary value of $x$ (but not $\dot{x}$) and derive a linearised state-space model parameterised in terms of $x_e$, the equilibrium value of $x$. [15%]

(c) Sketch state-space trajectories for the system of equation (1) for the case $u = 0$, taking care to characterize the linearised behavior in the neighbourhood of the origin and to find the slope of the trajectory as it crosses the axes at a variety of points. [40%]

Is the origin stable? If so, estimate the region in which all trajectories converge to the origin. [15%]

(d) Under what circumstances is the use of a model linearised around the origin justified in terms of understanding the overall behaviour of this system? [15%]

2 (a) What is the key principle behind the construction of a root locus diagram? [5%]

(b) Consider the system with transfer function

$$G(s) = \frac{1}{s^3}$$

which is to be controlled in a negative feedback configuration with a controller $K(s)$.

(i) Consider the controller

$$K(s) = k\frac{s+1}{s+10}$$

What is the name of this type of controller? [5%]

Sketch the root locus diagram for $G(s)$ with this controller for $k > 0$, finding any breakaway points. [30%]

Are there any values of $k$ for which the closed loop system is stable? [10%]

(ii) Now consider the controller

$$K(s) = k\frac{(s+1)^2}{(s+10)^2}$$

Sketch the root locus diagram for $G(s)$ with this new controller, again for $k > 0$. You are not required to calculate any breakway points, but you should consider whether the root locus crosses the imaginary axis. [30%]

For what range of $k$ is the closed loop system stable? [10%]

Estimate the value of $k$ for which the time constant of the dominant (i.e. slowest) closed loop poles is smallest (i.e. they are fastest). [10%]



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3 A state feedback controller is being designed for position control of a DC motor with transfer function

$$Y(s) = \frac{1}{s(s+1)}\, U(s)$$

(a) Write down a state-space model of the plant. [20%]

(b) Explain (preferably without calculation) why your state-space model is both controllable and observable. [10%]

(c) Design a state feedback controller to place the poles of the closed-loop system at $s = -1$. [20%]

(d) Interpret the state feedback control as the output of a controller with input $-y$ and output $u$. Calculate the transfer function of this controller and draw a conventional block diagram of the feedback system with a load disturbance signal $d$ added to the input $u$. [10%]

(e) Assume a constant (but unknown) disturbance $d(t) = d_0$ added at the input. Explain how the state-space model can be augmented by one extra state variable in such a way that any state feedback controller that stabilises the closed-loop system will also ensure zero steady-state error. Can the poles of this feedback system be freely assigned? Briefly justify your answer with no extra calculation. [20%]

(f) Determine the transfer function of the augmented controller (with input $-y$) designed in part (e). Do you recognise this as a classical controller structure? Provide both one advantage and one limitation of your state feedback design over classical design methods using Bode and Nyquist plots. [20%]

4 (a) State a standard test for observability of the linear system

$$\dot{x} = Ax + Bu, \quad y = Cx$$

[10%]

(b) Explain briefly why this test does not depend on the matrix $B$. [20%]

(c) A cart-mounted inverted pendulum makes an angle $\theta$ with the vertical, and the cart moves in a straight line with velocity $v$. The cart is driven by a force $f$. For small angles the linearised equations of motion are

$$\begin{cases} \ddot{\theta} &= \theta + v + f \\ \dot{v} &= \theta - v - f \end{cases}$$

Define a suitable state vector, keeping the state dimension as small as possible, and write these equations in standard state-space form. [15%]

(d) Is the system observable if only $\dot{\theta}$ (and the force $f$) are measured? Justify your answer. [20%]

(e) Consider the output $y = \theta$. Design a state observer, locating all the observer poles at $s = -1$. [20%]

(f) If measurements of both $\theta$ and $v$ are available, discuss why it would be desirable to use both measurements for the estimation of the state, despite the result of part (e). [15%]

**END OF PAPER**

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