1 a) $$s\underline{x}(s) = A\underline{x}(s) + B_1\underline{u}(s) + B_2 d(s)$$

$$\Rightarrow (sI-A)\underline{x}(s) = B_1 u(s) + B_2 d(s)$$

$$\Rightarrow \underline{x}(s) = (sI-A)^{-1}B_1 u(s) + (sI-A)^{-1}B_2 d(s)$$

$$\underline{y} = C\underline{x} + D$$

$$= C(sI-A)^{-1}B u(s) + \underbrace{\left(C(sI-A)^{-1}B_2 + D\right)}_{\text{TF } \underline{d} \to \underline{y}} d(s)$$

b) $$e^M \stackrel{\text{def}}{=} I + M + \frac{M^2}{2!} + \frac{M^3}{3!} + \dots$$

$$\frac{d}{dt} e^{At} = 0 + A + \frac{2A^2 t}{2!} + \frac{3A^3 t^2}{3!}$$

$$= A\left(I + At + \frac{A^2 t^2}{2!} + \dots\right)$$

$$= A e^{At}$$

$$\Rightarrow \dot{x} = A e^{At} x(0) = A\underline{x}$$

$$\underline{x}(t) = e^{At}\underline{x}(0) + \int_0^t e^{A(t-\tau)} B u(\tau)\, d\tau$$

c) $$\det(\lambda I - A) = \det\begin{pmatrix} \lambda+1 & 0 & -1 & 0 \\ 0 & \lambda & -1 & 0 \\ +0.25 & 0 & \lambda+2 & 0 \\ 1 & -1 & 0 & \lambda \end{pmatrix} = (\lambda+1)\lambda(\lambda+2)\lambda + \lambda \cdot 0.25\lambda$$

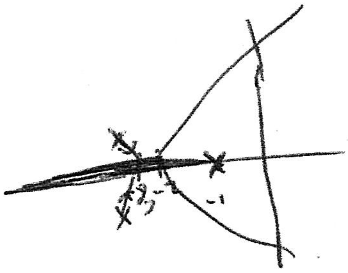
$$= \lambda^2\left(\lambda^2 + 3\lambda + \frac{9}{4}\right)$$

roots at $0, 0, -1.5$

$$BKC = \begin{bmatrix} 0 & 0 & -k & 0 \\ 0 & 0 & 2k & 0 \\ 0 & 0 & -5k & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Hence $(A + BKC)$ and A have same non-zero entries, so λ^2 is still a factor of the characteristic equation

2) a)



Breakaway points

$$\frac{d}{ds}E(s) = 0 \Rightarrow s^2+6s+10 + (s+1)(2s+6) = 0$$

$$\Rightarrow 3s^2 + 14s + 16 = 0$$

$$\Rightarrow s = -2, -8/3$$

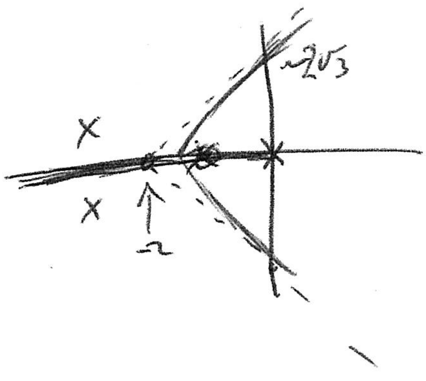
$$K = |-2 - -1| . |-1 - -3+j| . |-1 - -3-j| = 1.\sqrt{2}.\sqrt{2} = \underline{\underline{2}}$$

$$(s+1)(s^2+6s+10) + k = s^3 + 7s^2 + 16s + 12$$

$$= (s+2)(s+2)(s+3)$$

$$\Rightarrow \underline{\underline{s=-3}}$$

b)



$$K = 2\sqrt{3} . \sqrt{(-3)^2 + (2\sqrt{3}-1)^2}\sqrt{3^2 + (2\sqrt{3}+1)^2} = \underline{\underline{47}}$$

c) zero steady state position error

3) a)

$$f_e = k v_e^3$$

$$F \frac{u_e}{1+u_e} = f_e = k v_e^3$$

$$\frac{F}{k} u_e = (1+u_e) v_e^3$$

$$u_e(F/k - v_e^3) = v_e^3 \Rightarrow u_e = \frac{v_e^3}{F/k - v_e^3}$$

only possible for $v_e \le \sqrt[3]{F/k}$

b)

$$M \dot{\delta v} = -k(v_e + \delta v)^3 + f_e + \delta f$$

$$= -3k v_e^2 \delta v + \delta f + \sim$$

$$T \dot{\delta f} = -\delta f + F \frac{\delta u}{(1+u_e)^2}$$

$$x = \begin{matrix} \delta v \\ \delta f \end{matrix}$$

$$\dot{x} = \begin{bmatrix} \frac{-3k v_e^2}{M} & 1/M \\ 0 & -1/T \end{bmatrix} \delta x + \begin{bmatrix} 0 \\ \frac{F}{(1+u_e)^2} \end{bmatrix} \delta u$$

$$\delta u = -[k_1 \; k_2] x$$

$$\Rightarrow A \Rightarrow \begin{bmatrix} \frac{-3kv_e^2}{M} & \frac{1}{M} \\ -\frac{k_1 F}{(1+u_e)^2} & -\frac{1}{T} - \frac{k_2 F}{(1+u_e)^2} \end{bmatrix}$$

$$\frac{-3kv_e^2}{M} - \frac{1}{T} - \frac{k_2 F}{(1+u_e)^2} = -4 \Rightarrow k_2 = \frac{4 - \frac{3kv_e^2}{M} - 1/T}{F/(1+u_e)^2}$$

$$\frac{-3kv_e^2}{M} \cdot \left(-4 + \frac{3kv_e^2}{M}\right) + \frac{k_1 F}{M(1+u_e^2)} = 4 \Rightarrow k_1$$

4 a) A system is observable if we can deduce the state $x(t)$ from measurements of $u(t)$ and $y(t)$ over some interval

b)
$$\begin{bmatrix} y \\ \dot{y} \\ \vdots \\ y^{(n-1)} \end{bmatrix} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ CBu(t) \\ CABu + CB\dot{u} \\ \vdots \end{bmatrix}$$

$\uparrow$ known Q $\uparrow$ known

can solve for $x(t)$ iff Q is ~~full~~ rank n

c) $\det(\lambda I - A) = \det\begin{pmatrix} \lambda+1 & 1 & -1 \\ 1 & \lambda+1 & 0 \\ 0 & 0 & \lambda-1 \end{pmatrix} = (\lambda+1)^2(\lambda-1) - (\lambda-1)$

$= (\lambda^2 + 2\lambda)(\lambda-1)$

$\lambda = 0, 1, -2$

$w_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, $w_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $w_2 = \begin{matrix} x \\ y \\ z \end{matrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

$-x - y + z = x \quad \Rightarrow z = 2x + y$

$-x - y = y \quad \Rightarrow y = -\frac{x}{2}$

$z = z$

$$A = \underbrace{\begin{bmatrix} 1 & 1 & 2 \\ -1 & 1 & -1 \\ 0 & 0 & 3 \end{bmatrix}}^{W} \begin{bmatrix} 0 & & \\ & -2 & \\ & & 1 \end{bmatrix} W^{-1}$$

$C \to CW = [10 \quad 0 \quad 24]$

$\Rightarrow$ not observable $\left(Q = \begin{bmatrix} 10 & 0 & 24 \\ 0 & 0 & 24 \\ 0 & 0 & 24 \end{bmatrix} \right)$

2nd state in transformed co-ordinates is unobservable