1) a) i) The ability to infer the value of the state given measurements of the input and output.

Feedback H corrects state estimate.

ii) Useful is not all states are measured OR measurements of states are noisy.

$$\dot{\hat{x}} = A\hat{x} + Bu + H(C\hat{x} - Cx) \qquad u = -Kx$$

$$\Rightarrow \begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & BK \\ -HC & A+HC-BK \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

b) i)
$$\begin{vmatrix} \lambda+2 & -4 & -1 \\ 0 & \lambda-1 & 0 \\ 2 & -3 & \lambda \end{vmatrix} = (\lambda+2)(\lambda-1)\lambda - 1 \cdot -2(\lambda-1) = 0$$

$$\lambda^3 + \lambda^2 - 2\lambda + 2\lambda - 2 = 0$$

$\lambda = 1$ is soln by inspection

$$\Rightarrow (\lambda-1)(\lambda^2 + 2\lambda + 2) = 0$$

$$((\lambda+1)^2 + 1)$$

$$\Rightarrow \lambda = 1, \; -1 \pm j$$

for e.v.
$$\begin{bmatrix} 3 & -4 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

$$3x - 4y = 1$$
$$2x - 3y = -1$$
$$-8y + 9y = 5 \quad \Rightarrow y = 5 \quad \Rightarrow x = 7 \quad \Rightarrow v = \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 10 & -13 & -5 \\ -10 & 12 & 10 \\ 0 & 2 & -10 \end{bmatrix}$$

row 1 + row 2 + $\frac{\text{row 3}}{2}$ = 0

$\Rightarrow$ not observable

iii) $Q \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix} = 0 \Rightarrow$ unstable subspace is unobservable.

2 a) For any x_e there exists a corresponding u_e to maintain equil^m $\cdot u_e = 3x_e + x_e^3$

$x_1 = x$
$x_2 = \dot{x}$

$\dot{x}_1 = x_2$

$\dot{x}_2 = u - 4x_2 - 3x_1 - x_1^3$

$\frac{\partial f_2}{\partial x_1} = -3 - 3x_e^2$

$\Rightarrow \dot{x}_2 = u - 4x_2 - 3 - 3x_e^2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3-3x_e^2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$\underline{u = 0} \Rightarrow x_e = 0$ $\quad \lambda^2 + 4\lambda + 3 \quad (\lambda+1)(\lambda+3)$

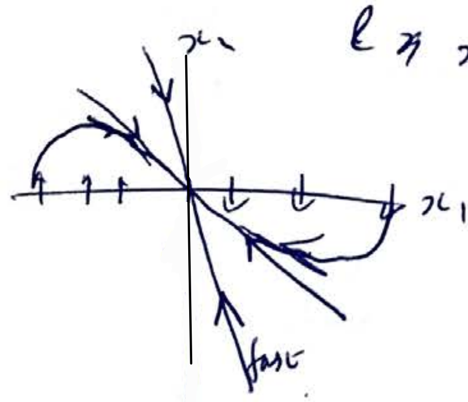
$\lambda = -1, -3$

eigenvectors: $\begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \begin{matrix} x_1 \\ x_2 \end{matrix} = \begin{bmatrix} -x_1 \\ -x_2 \end{bmatrix}$

$x_2 = -x_1$ $\quad \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

& $x_2 = -3x_1$ $\quad \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

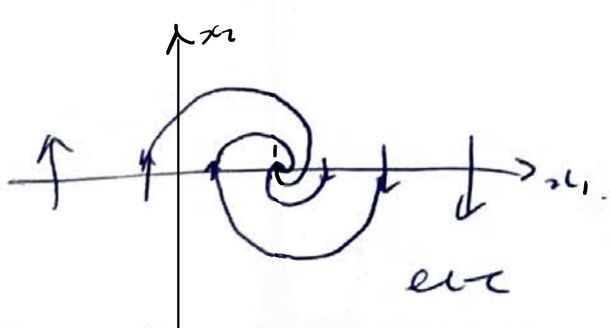
$\Rightarrow$



$x_2 = 0 \Rightarrow \dot{x}_1 = 0$
$\dot{x}_2 = -3x_1$

$u_e = 4 \Rightarrow x_e = 1$ $\quad \lambda^2 + 4\lambda + 6 \quad (\lambda+2)^2 + 2 = 0$

$\lambda = -2 \pm \sqrt{2}j$



c) justified as long as u is small,

(a)

$$\frac{d}{dt}\begin{bmatrix} r \\ \dot r \\ \theta \\ \dot\theta \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3\Omega^2 & 0 & 0 & 2\Omega R_0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{2\Omega}{R_0} & 0 \end{bmatrix}\begin{bmatrix} r \\ \dot r \\ \theta \\ \dot\theta \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} u_r \\ u_\theta \end{bmatrix}$$

(b)

$$\underbrace{AB = \begin{bmatrix} 1 & 0 \\ 0 & 2\Omega R_0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}}_{\text{full rank matrix}}$$

(c) $y = r$ makes the system observable:

$$\dot y = \dot r$$

$$\ddot y = 2\Omega R_0 \dot\theta \ (+ 3\Omega^2 y + u_r)$$

$$\dddot y = -4\Omega^2 \theta \ (+ u_\theta + 3\Omega^2 \dot y + \dot u_r)$$

$\Rightarrow$ $\underline{x}$ in terms of y and its derivatives

~~Note: no other choice~~ works.

* ~~θ (resp r) cannot be reconstructed from~~ $\dot\theta$ (resp $\dot r$)
* ~~Differentiating θ only provides $\dot\theta$~~.

Also $y = \theta$ makes system observable

But not $\dot\theta$ since θ cannot be reconstructed from $\dot\theta$.

d)

$$u_r = -2\Omega R_0 \dot\theta - 3\Omega^2 r - k_p r - k_d \dot r$$

leads to $\ddot r + k_p r + k_d \dot r = 0$

which is stable if $k_p > 0$, $k_d > 0$

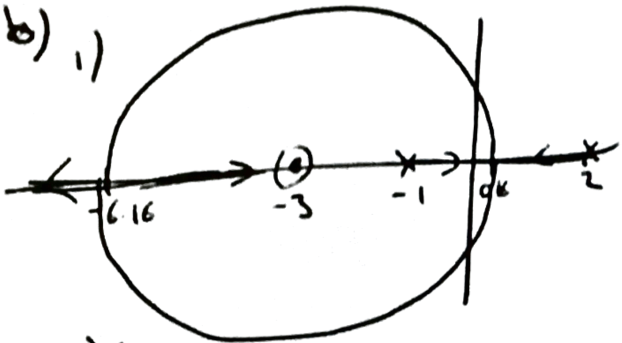
The resulting motion is still on the geostationary orbit but with an angular oscillation around the nominal orbit.

4) $G(s) = \dfrac{s+3}{(s-2)(s+1)}$

a) $1 + K.L(s) = 0$ etc

principle for s, is K then real and positive

b) i)



$\dfrac{dG}{ds} \propto (s+3)(2s-1) - (s-2)(s+1)$

$= 2s^2 + 5s - 3 - s^2 + s + 2$

$= s^2 + 6s - 1$

roots are $-6.16, 0.16$

$K = \dfrac{5.16 \times 8.16}{3.16} = 13.33$

(for large breakaway)

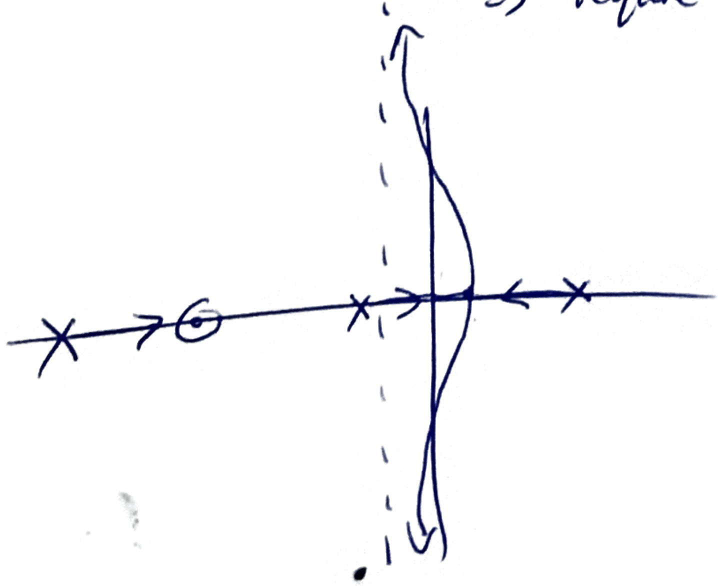
$\Rightarrow$ stable for $K > 13.33$

2 real poles

ii) asymptotes are $\dfrac{2 - 1 + 0 + 3 - \frac{1}{\tau_i}}{4 - 2}$

$\Rightarrow$ require $4 - \frac{1}{\tau_i} < 0$

$\tau_i < 0.25$ sec



Q1 was a popular question and generally well answered. In part b (iii), very few candidates identified that the unstable vector (calculated in part b (i)) was in the null space of the observability matrix (calculated in part b (ii)).

Q2 was a question on linearization and the plotting of state-space trajectories, and was generally well answered with a large number of complete or close to complete to answers. Many candidates gave overly long and general answers to part c) though, missing the fact that it simply required u to be small for this system.

Q3 was a straightforward question and generally well answered. In part c), most candidates embarked a long calculation to choose the poles of the feedback system. Only few candidates observed that stabilising only the radial position had a straightforward solution.

Q4 was a popular question on root locus, and was matched by a large number of close to complete answers. The worst answered part was a) - the "single" key principle behind the construction of a root locus diagram is the angle condition (ie choose an a s and check if it is a closed loop pole for some k).