Version GV/2

EGT2
ENGINEERING TRIPOS PART IIA

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2 May 2014 2 to 3.30

Module 3F2

SYSTEMS AND CONTROL

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A linear system is given in state-space form as:

$$\begin{aligned} \dot{\underline{x}} &= A\underline{x} + [B_1 \; B_2] \begin{bmatrix} u \\ d \end{bmatrix} \\ \underline{y} &= C\underline{x} + Dd \end{aligned}$$

where u is a scalar control input, d is a scalar disturbance, $\underline{x}$ is the state vector and $\underline{y}$ is the output vector.

(a) Show that the transfer function matrix from u to $\underline{y}$ is $C(sI-A)^{-1}B_1$, and find the transfer function matrix from d to $\underline{y}$. [25%]

(b) State the definition of the *matrix exponential* function e^M, where M is a square matrix. Verify that $\underline{x}(t) = e^{At}\underline{x}(0)$ satisfies the equation $\dot{\underline{x}}(t) = A\underline{x}(t)$. If $u(t) = 0$, write down the expression for the state $\underline{x}(t)$ which results from an initial condition $\underline{x}(0)$ and disturbance signal $d(t)$. [25%]

(c) A particular system has a linearised model:

$$A = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ -0.25 & 0 & -2 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -1 \\ 2 \\ -5 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \end{bmatrix}.$$

Find the eigenvalues of A. [25%]

If proportional feedback is used (namely $u(t) = ky(t)$), show that the closed loop is not asymptotically stable for any choice of the gain k. [25%]

2 In a position control system for a microscope stage, the transfer function from the amplifier input voltage to the position is given by

$$G(s) = \frac{1}{(s+1)(s^2+6s+10)}$$

The position of the stage is to be controlled by a feedback system.

(a) Find the breakaway points and sketch the root-locus diagram for this transfer function when the controller is a *proportional gain*. Determine the gain required to obtain two closed-loop poles located at the breakaway point at -2. [30%]

Where is the third closed-loop pole located with this value of proportional gain? [20%]

(b) It is decided to use a *proportional and integral* (PI) controller with transfer function

$$K(s) = k\left(1 + \frac{1}{s}\right)$$

Sketch the root-locus diagram for the return-ratio $G(s)K(s)$ (assuming $k > 0$). Show any root-locus asymptotes on your sketch. (Accurate location of breakaway points is *not* required.) Estimate the maximum value of k for which the feedback system is stable. [40%]

(c) What would be the benefit of using the PI controller in this application, compared with a proportional gain only? [10%]

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3 The speed v of a train is assumed to satisfy the differential equation

$$M\frac{dv}{dt} = -kv^3 + f$$

where f is the traction force produced by the engine, which is assumed to depend on the train's throttle position u according to

$$\frac{df}{dt} = -f + F\frac{u}{1+u}.$$

M, k and F are constants and $u \geq 0$.

(a) Find the values of u and f required to establish equilibrium at a speed $v = v_e$. What is the range of possible values of v_e? [30%]

(b) Obtain a state-space model of the train and engine, linearised at the equilibrium conditions found in (a). [30%]

(c) Verify that the linear state-space model found in (b) is controllable from the throttle position u. Design a state-feedback controller for the throttle position that results in both the closed-loop poles being placed at -2. [40%]

4 Consider a linear system in standard state-space form:

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}, \qquad \underline{y} = C\underline{x} + D\underline{u}$$

(a) Define what it means for such a system to be *observable*, and state how its observability can be checked. [20%]

(b) Show that if the standard rank test for observability is satisfied, then the system is observable. [30%]

(c) Let

$$A = \begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 5 & -5 & -3 \end{bmatrix}$$

(i) Find the eigenvalues and eigenvectors of A. [25%]

(ii) Investigate the observability of the system, and characterise any unobservable states [25%]

END OF PAPER

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