Version GV/1

EGT2
ENGINEERING TRIPOS PART IIA

Monday 10 May 2021 1.30 to 3.10

Module 3F2

SYSTEMS AND CONTROL

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet and at the top of each answer sheet.*

STATIONERY REQUIREMENTS
Write on single-sided paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
You are allowed access to the electronic version of the Engineering Data Books.

10 minutes reading time is allowed for this paper at the start of the exam.

The time taken for scanning/uploading answers is 15 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers.

1 (a) (i) Explain, in one sentence, what is meant by *observability* of a linear dynamic system. [10%]

(ii) For the system

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

explain how an *observer* can be used together with feedback of the estimated states to achieve arbitrary placement of the closed loop poles, and derive the state-space equations of the closed loop system. When might it be appropriate to use an observer in this way? [20%]

(b) Let

$$A = \begin{bmatrix} -2 & 4 & 1 \\ 0 & 1 & 0 \\ -2 & 3 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 10 & -13 & -5 \end{bmatrix}$$

(i) Find the eigenvalues of A, as well as the eigenvectors corresponding to any unstable eigenvalues. [30%]

(ii) Determine whether the system is observable. [30%]

(iii) Explain why it is impossible to design a stable feedback system comprising an observer and state feedback in this case. [10%]

2 Consider the following dynamics

$$\ddot{x} + 4\dot{x} + 3x + x^3 = u \qquad (1)$$

which represent a mass with a linear damper and a nonlinear spring.

(a) Explain why it is possible to linearise the system about an arbitrary value of x and derive a linearised state-space model parameterised in terms of x_e, the equilibrium value of x. [15%]

(b) Sketch state-space trajectories for the system of (1) for the cases $u = 0$ and $u = 4$. [70%]

(c) Under what circumstances is the use of a model linearised around the origin justified in terms of understanding the overall behaviour of this system? [15%]

3 A control system is designed to maintain a satellite in a geostationary orbit. The satellite is modelled as a point mass moving at constant angular speed Ω on a planar circular orbit of radius R_0 in the gravitational field. The linearised equations of motion are

$$\ddot{r} - 2\Omega R_0\dot{\theta} - 3\Omega^2 r = u_r$$
$$\ddot{\theta} + 2\frac{\Omega}{R_0}\theta = u_\theta$$

where (r, θ) denotes the deviation from the circular orbit in polar coordinates and (u_r, u_θ) denote the radial and tangential input thrusts, respectively.

(a) Write the linearised equations of motion in state-space form, using the state vector $\underline{x} = [r, \dot{r}, \theta, \dot{\theta}]^T$. [10%]

(b) Show that the state-space model is controllable. [30%]

(c) Assume that only one of the four state variables can be measured. Provide one choice that makes the system observable, and one other choice that makes the system unobservable. Justify your answer in each case, avoiding any rank calculation if possible. [30%]

(d) Consider the situation in which only the radial thrust can be used for control ($u_\theta = 0$). Design a state-feedback for u_r that ensures asymptotic stability of the radial position. Describe the resulting steady-state motion of the satellite for an arbitrary initial condition of the linearised model. [30%]

4 (a) What is the single key principle behind the construction of a root locus diagram? [15%]

(b) Consider the system with transfer function

$$G(s) = \frac{s+3}{s^2 - s - 2}$$

which is to be controlled in a negative feedback configuration with a controller $K(s)$.

(i) Sketch the root locus diagram for a proportional controller $K(s) = k$, finding any breakaway points. For what range of k, if any, is the feedback system stable with real poles? [45%]

(ii) Now include in a controller an actuator modelled as a first order lag, so $K(s) = \frac{k}{1+s\tau}$. What constraint on τ would guarantee that the feedback system is stable for sufficiently large k? Sketch the root locus diagram, as k varies, for such a τ (you are not required to calculate any breakway points). [40%]

END OF PAPER

1) b) i) 1, 1+j, 1−j, [7,5,1]'
ii) Not observable

2) –

3) –

4) b) i) k> 13.33
ii) 0<tau<0.25