EGT2
ENGINEERING TRIPOS PART IIA

Tuesday 1 May 2018 9.30 to 11.10

Module 3F4

DATA TRANSMISSION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Consider the four waveforms shown in Fig. 1 below.

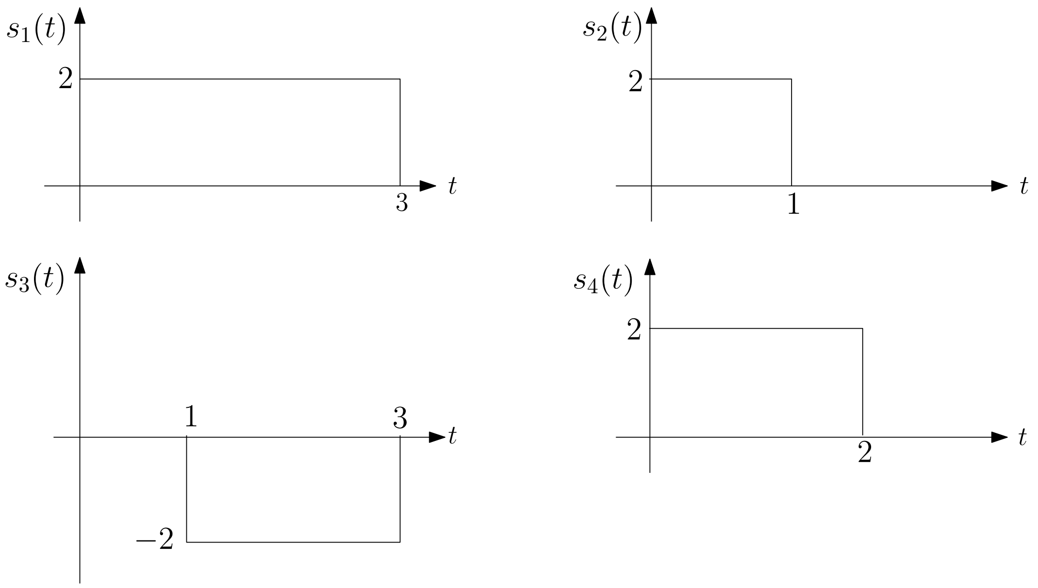


Fig. 1

(i) Determine a set of orthonormal basis functions for the four waveforms above. [10%]

(ii) Express each waveform in Fig. 1 in terms of the basis functions determined in part (a).(i). [15%]

(iii) A transmitter wishes to transmit one of four messages, using the waveforms in Fig. 1. Denote the transmitted signal by $x(t)$, where $x(t)$ is either $s_1(t), s_2(t), s_3(t)$, or $s_4(t)$. The received signal is $y(t) = x(t) + n(t)$, where $n(t)$ is white Gaussian noise. Assuming the messages are equally likely, specify a receiver to optimally detect the transmitted message from $y(t)$. Your answer should describe any operations on $y(t)$ and a decision rule to determine which of the four messages was transmitted. [25%]

(b) Let $x(t) = \sum_{k=0}^{\infty} X_k p(t - kT)$ be a transmitted PAM signal, where $p(t)$ is a pulse waveform, and the $\{X_k\}$ are information symbols chosen from a constellation. The received signal $y(t)$ is passed through a low-pass filter with impulse response $q(t)$. The filter output, denoted by $r(t)$, is sampled at times $\{nT\}$ to obtain the sequence $\{r(nT)\}$, for $n = 0, 1, \ldots$.

Assume a noiseless channel, so that $y(t) = x(t)$. Let $g(t) = p(t) \star q(t)$, and let $G(f)$ denote the Fourier transform of $g(t)$.

(i) Show that the filter output is given by $r(t) = \sum_{k=0}^{\infty} X_k g(t - kT)$. [10%]

 (cont.

(ii) State the condition $G(f)$ has to satisfy so that the sampled filter output has no intersymbol interference. [10%]

(iii) Let $p(t), q(t)$ have Fourier transforms $P(f), Q(f)$, respectively, shown in Fig. 2 below. (Only the positive frequencies are plotted. Both $p(t), q(t)$ are real-valued and even.) Sketch $G(f)$, the Fourier transform of $g(t)$. Your sketch should include both positive and negative frequencies. [10%]

(iv) With $G(f)$ as in part (b)(iii), determine the sampled filter output $r(nT)$, for $n = 0, 1, \ldots$. [20%]

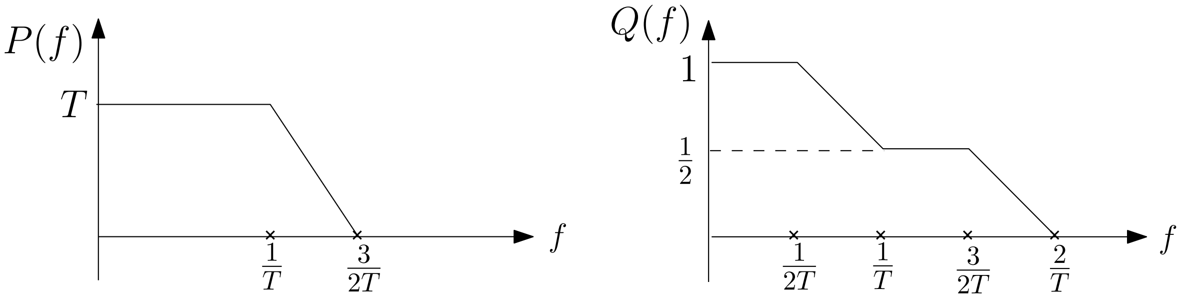


Fig. 2

2 (a) Consider the 6-point QAM constellation shown in Fig. 3 below, with neighbouring symbols in the vertical and horizontal directions spaced d apart. The constellation is used for signalling over the discrete-time AWGN channel $Y = X + N$, where the noise N is a complex random variable whose real and imaginary parts are each i.i.d. Gaussian $\sim \mathcal{N}(0, \frac{N_0}{2})$. Assume that each of the constellation symbols is equally likely to be transmitted.

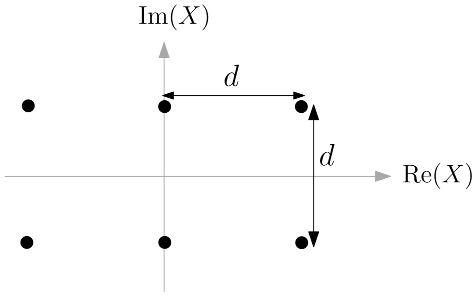


Fig. 3

(i) Calculate the average energy per symbol E_s, and the average energy per bit E_b of the constellation, in terms of d. [15%]

(ii) Sketch the decision regions for the optimal detector. [10%]

(iii) Compute an upper bound on the probability of detection error. Your upper bound should be in terms of the ratio $\frac{E_b}{N_0}$ and the Q-function, where [30%]

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-u^2/2} du.$$

(iv) Now consider a 6-PSK constellation with the same average energy per symbol as the constellation in Fig. 3. What is the distance between neighbouring points in this PSK constellation, in terms of d? [10%]

(v) Is the probability of detection error for the 6-PSK constellation in part (a).(iv) larger or smaller than for the 6-QAM constellation in Fig. 3. Briefly justify your answer without explicitly calculating the error probability for 6-PSK. [10%]

(b) Consider a binary random variable X, which takes value $+3$ with probability p and -1 with probability $(1-p)$. Suppose that you observe $Y = X + N$, where N is a Gaussian random variable $\sim \mathcal{N}(0, \sigma^2)$. Determine the optimal decision rule to decode X from Y. Your answer must be expressed in terms of a threshold T such that the decoded symbol is $+3$ if $Y > T$, and is -1 otherwise. [25%]

3 A rate $\frac{1}{2}$ binary convolutional code is shown in Fig. 4 below.

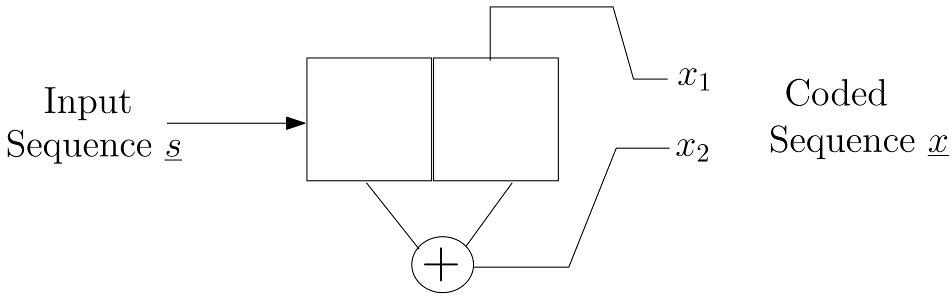


Fig. 4

(a) Draw the state diagram of the code. [15%]

(b) Use the state diagram to determine the extended transfer function for the code. [15%]

(c) Briefly explain the significance of the exponents of the three variables in the expansion of the extended transfer function, and interpret the first two terms in the expansion of the extended transfer function obtained in part (b). [20%]

(d) What is the free distance of the code? [5%]

(e) Suppose the code is used on a binary symmetric channel and the received sequence is 01 00 11 01. Determine the decoded sequence $\hat{\underline{x}}$, and the corresponding input sequence $\hat{\underline{s}}$. [35%]

(f) When is a convolutional encoder said to be catastrophic? [10%]

4 (a) An OFDM system is used over a dispersive channel, whose discrete-time equivalent can be modelled as a filter with impulse response of length $L+1$, where $L=40$. The OFDM signal has 128 subcarriers, with the spacing between subcarriers being 100 kHz.

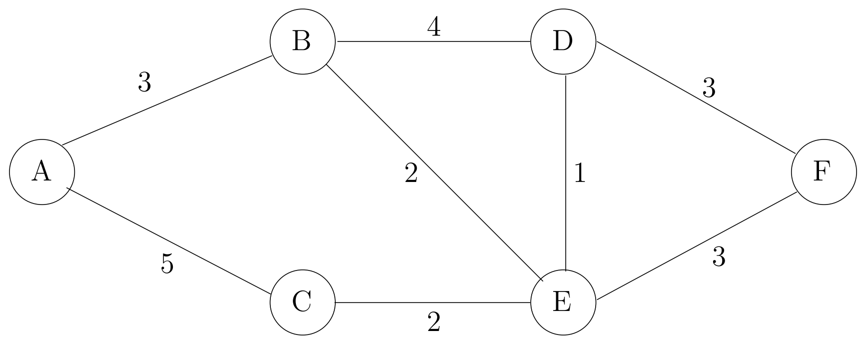
(i) What is the bandwidth of the OFDM signal? [10%]

(ii) What is the duration of the guard interval required for each OFDM symbol period? [10%]

(iii) Information bits are first encoded using a binary error-correcting code of rate $\frac{3}{4}$, and then mapped to OFDM symbols drawn from a 16-QAM constellation. Calculate the user data rate of the OFDM system. [15%]

(iv) Briefly explain the role of the cyclic prefix in an OFDM signal. [20%]

(b) Consider the network shown in the figure below, with the number on each link indicating the cost along the link. The links are all bidirectional. Use Djikstra's algorithm to determine the minimum-cost path from node A to each of the other nodes. Use a table to specify each step of the computation process. [45%]



END OF PAPER