EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 1 May 2019 9.30 to 11.10

Module 3F4

DATA TRANSMISSION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Let $x(t) = \sum_{k=-\infty}^{\infty} X_k p(t-k)$ be a transmitted PAM signal, where $p(t)$ is the rectangular pulse

$$p(t) = \begin{cases} 1, & 0 \leq t < 1, \\ 0, & \text{otherwise.} \end{cases}$$

The $\{X_k\}$ are information symbols chosen from a constellation. The received signal is $y(t) = x(t) + n(t)$, where $n(t)$ is white Gaussian noise with zero-mean and power spectral density $N_0/2$. The received signal is passed through a filter with impulse response $p(-t)$. The output of the filter is $r(t) = \sum_{k=-\infty}^{\infty} X_k g(t-k) + \tilde{n}(t)$, where

$$g(t) = p(t) \star p(-t) \quad \text{and} \quad \tilde{n}(t) = n(t) \star p(-t).$$

(a) Sketch $g(t)$. [15%]

(b) Show that the filter output sampled at times $m = 1, 2, \ldots$ is $r(m) = X_m + \tilde{n}(m)$ for $m \geq 1$. [5%]

(c) Due to imperfect synchronization, the filter output $r(t)$ is sampled at time instants $\{1-\Delta, 2-\Delta, \ldots, \}$, where $0 < \Delta < 1$ is the timing error. Obtain a compact expression for the sampled filter output $r(m-\Delta)$, for $m \geq 1$. [15%]

(d) If you had to recover the symbol X_m from any *one* of the sampled filter outputs, which one of the sampled outputs $\{r(1-\Delta), r(2-\Delta), \ldots\}$ would you use? Justify your answer, and state any assumptions you make. [15%]

(e) Suppose the PAM constellation is $\{-A, A\}$, with each symbol being equally likely. Also assume that $\Delta < \frac{1}{2}$. Suppose that we detect X_m from $r(m-\Delta)$ (without any equalisation). Compute the exact probability of detection error, expressing your answer in terms of Δ and the ratio $\frac{A^2}{N_0}$ using the Q-function [25%]

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2} du.$$

(f) Assuming $\Delta = 0.1$, design a three-tap FIR zero-forcing equaliser to recover the transmitted symbols from the sequence $\{r(m-\Delta)\}_{-\infty<m<\infty}$. Write an expression for the output of the equaliser, indicating the residual interference and additive noise. [25%]

2 (a) Consider a single TCP-Reno connection with window size at time t denoted by $W(t)$, and the packet loss probability by $q(t)$.

(i) If the loss probability $q(t)$ is doubled from 0.05 to 0.1, what is the factor by which the average transmission rate decreases? Assume that the other parameters of the connection remain unchanged. [10%]

(ii) Briefly describe how $W(t)$ is increased in the slow start phase versus the congestion avoidance phase. [10%]

(iii) Briefly describe the two ways in which $W(t)$ can decrease when congestion is detected. [10%]

(b) Consider the set of M waveforms $s_1(t), \ldots, s_M(t)$ defined within the time interval $t \in (0, 1]$ as follows. For $i = 1, \ldots, M$,

$$s_i(t) = \begin{cases} \sqrt{E\,M}, & \text{for } \frac{(i-1)}{M} < t \leq \frac{i}{M} \\ 0, & \text{otherwise,} \end{cases}$$

where E is the energy of each waveform.

The transmitter wishes to transmit one of M equally likely messages using the set of waveforms. Denote the transmitted signal by $x(t)$, where $x(t)$ is one of the waveforms $s_1(t), s_2(t), \ldots, s_M(t)$. The received signal is $y(t) = x(t) + n(t)$, where $n(t)$ is white Gaussian noise with zero mean and power spectral density $\frac{N_0}{2}$.

(i) Determine an orthonormal basis for the M waveforms $\{s_1(t), \ldots, s_M(t)\}$, and use the basis to express the waveforms as vectors $\underline{s}_1, \ldots, \underline{s}_M$. [10%]

(ii) Assuming the messages are equally likely, specify a receiver to optimally detect the transmitted message from $y(t)$. Your answer should describe any operations on $y(t)$ and a detection rule to determine which message was transmitted. [20%]

(iii) Obtain an upper bound for the probability of error for the detection rule in (b)(ii) above. The bound must be expressed in terms of $\frac{E_b}{N_0}$ and the Q-function, where E_b is the average transmitted energy per bit and [25%]

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2} du.$$

(iv) How does the error probability bound computed in (b)(iii) scale with increasing M? State any assumptions required on $\frac{E_b}{N_0}$, and briefly explain the reason for the scaling behaviour. [15%]

Hint: You may use the bound $Q(x) \leq e^{-x^2/2}$, for $x \geq 0$.

3 (a) Consider an 8-ary phase shift keying (PSK) constellation consisting of 8 points uniformly spaced around a circle of radius A. This constellation is used for signalling over the discrete-time AWGN channel

$$Y = X + N,$$

where N is a complex random variable with real and imaginary parts each i.i.d. Gaussian $\sim \mathcal{N}(0, \frac{N_0}{2})$.

(i) Sketch the constellation and indicate the decision regions for the optimal detector. [10%]

(ii) Show that the probability of detection error P_e can be bounded as

$$P_e \leq 2Q\left(\sqrt{\frac{6E_b}{N_0}}\sin(\pi/8)\right),$$

where E_b is the average energy per bit of the constellation, and [25%]

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-u^2/2} du.$$

(b) Suppose that due to channel uncertainty, the received output is

$$Y = \sqrt{s}\, X + N,$$

where $s \geq 0$ is real-valued, and is called the 'fading' coefficient. The input symbol X is drawn from the same 8-PSK constellation used in part (a), and N is a complex random variable with real and imaginary parts each i.i.d. Gaussian $\sim \mathcal{N}(0, \frac{N_0}{2})$.

(i) Assuming the receiver knows the value of the fading coefficient s, give a bound on the probability of error of the optimal detector in terms of s, the ratio $\frac{E_b}{N_0}$, and the Q-function. [10%]

(ii) The probability density function of the fading coefficient is given by $f(s) = e^{-s},\ s \geq 0$. Using this, compute a bound for the expected probability of detection error, where the expectation is computed with the distribution of s. [15%]
Hint: Use the bound $Q(x) \leq \frac{1}{2}e^{-x^2/2}$ for $x \geq 0$.

(iii) Compare the error probability bound computed in (b)(ii) with the one for the AWGN channel in (a)(ii), commenting on how they decrease as $\frac{E_b}{N_0}$ increases. [10%]

(c) Consider the 8-point QAM constellation shown in Fig. 1. The distance between the nearest neighbour points in this QAM constellation is the same as the distance between adjacent points in the 8-PSK constellation in part (a).

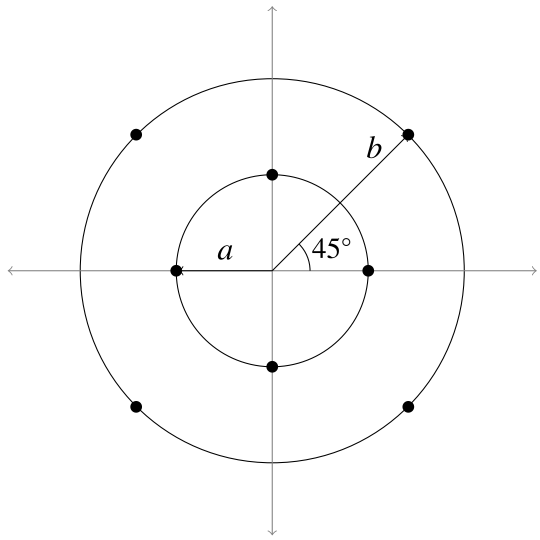

Fig. 1

(i) Determine the radii a and b of the inner and outer circles in terms of A, the radius of the 8-PSK constellation. (Note that the constellation points at $(a, 0)$, $(0, a)$ and $(\frac{b}{\sqrt{2}}, \frac{b}{\sqrt{2}})$ are equidistant from one another.) [20%]

(ii) Compute the average power of the QAM constellation in Fig. 1 in terms of A. [10%]

4 (a) Consider the *directed* network shown in Figure 2 below. The direction of each link is indicated by the arrow, and the cost along the directed link is indicated by the number. For example, the cost from B to D is 3, while the cost from D to B is 2.

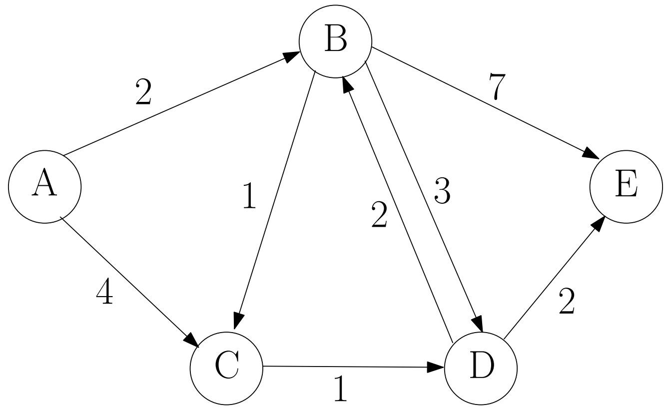

Fig. 2

(i) Use the Bellman-Ford algorithm to determine the minimum-cost path to node *E* from each of the other nodes. Use a table to specify each step of the computation process. [25%]

(ii) Briefly describe the main differences between the Bellman-Ford and Djikstra algorithms to determine minimum-cost paths in a network. [15%]

(b) Consider the convolutional code shown in Figure 3. *Two* input bits at a time are shifted into the register to produce three code bits. For example, if the shift register currently contains (from left to right) 01 11 and the next input bits are 10, the new content of the shift register will be 10 01.

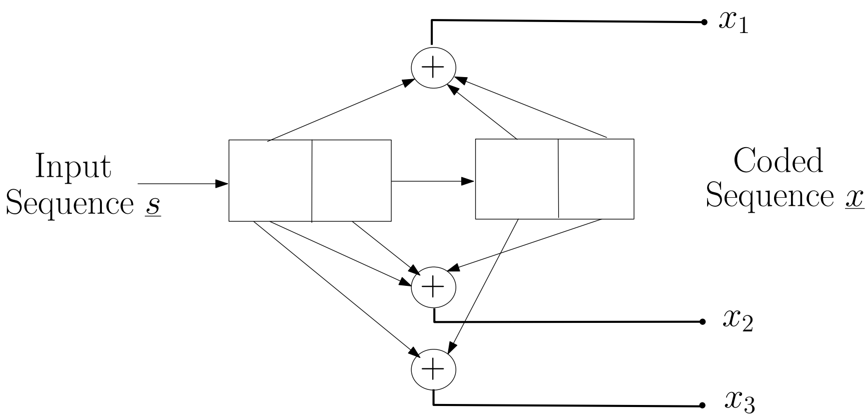

Fig. 3

(i) What is the rate of the code? [5%]

(ii) Draw the state diagram of the code, labelling each branch with the input and output bits corresponding to the transition along that branch. [25%]

(iii) Define the free distance of a convolutional code, and briefly explain its significance. [15%]

(iv) Find the free distance of the code in Figure 3, and justify the procedure you use. [15%]

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