Version AGiF/3

EGT2
ENGINEERING TRIPOS PART IIA

Friday 29 April 2022   9.30 to 11.10

Module 3F4

DATA TRANSMISSION

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 Consider a modulation scheme with M waveforms defined in the interval $0 \leq t \leq T$. For $m = 1, \ldots, M$, the m-th waveform is given by

$$s_m(t) = \begin{cases} 0 & \frac{(m-1)T}{M} \leq t \leq \frac{mT}{M} \\ +A & \text{otherwise} \end{cases}$$

(a) Calculate the energy of each of the signals $s_1(t), \ldots, s_M(t)$ and the average energy of the signal constellation E_{s}. [10%]

(b) Specify an orthonormal basis for the signal set, specify the dimension and give the vector representation of each signal. [20%]

(c) We consider transmission over an additive Gaussian noise channel with power spectral density $\frac{N_0}{2}$.

(i) Assuming that symbols are equiprobable, derive the optimal detector. [10%]

(ii) Assume that $M = 2$. Show that the pairwise probability of error is given by

$$p_e = Q\left(\sqrt{\frac{E_{\mathrm{s}}}{N_0}}\right)$$

and compare it with the error probability of 2-PAM. [20%]

(iii) Using the previous result, provide an upper bound to the probability of error with arbitrary M. [20%]

(iv) Assume that $M = 2^k$, where k is a positive integer. Show that if $k \to \infty$, the error probability vanishes as long as

$$\frac{E_{\mathrm{b}}}{N_0} > 2 \ln 2.$$

You may use the inequality $Q(x) \leq \frac{1}{2}e^{-\frac{x^2}{2}}$. [20%]

2 (a) Consider the signal set shown in Fig. 1.

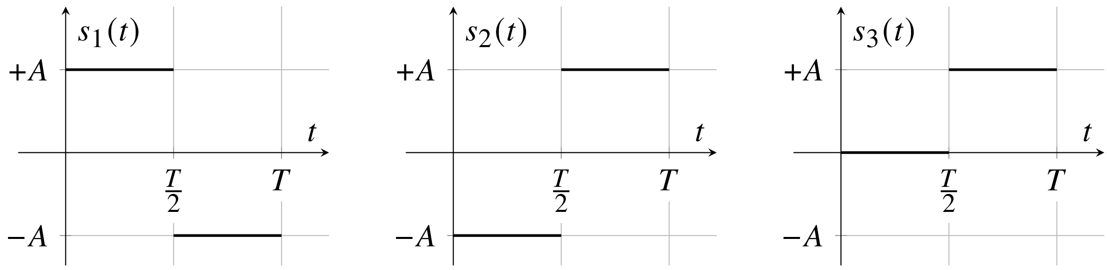

Fig. 1

(i) Determine the dimension of the signal space and find an orthonormal basis using the Gramm-Schmidt procedure. [20%]

(ii) Draw the signal space and calculate the average energy per symbol E_s. [15%]

(iii) Sketch the optimal decision regions with equiprobable symbols and calculate the error probability. Express the result in terms of the average energy of the signal set and of the Q-function, where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{u^2}{2}} du$. [35%]

(b) The 12-QAM constellation shown on Fig. 2 (left) is used for transmission over a communications channel. In absence of noise, the received constellation is shown on Fig. 2 (right).

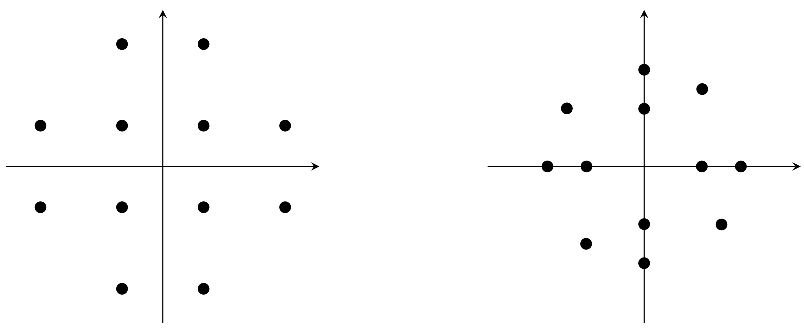

Fig. 2

(i) Discuss the input-output characteristics of the channel. [15%]

(ii) What operations would the transmitter need to do in order to compensate for the channel effects and receive the original 12-QAM constellation? Sketch the transmitted constellation. [15%]

3 (a) Consider a PAM system with pulse shape $p(t)$ where the transmitted signal is $x(t) = \sum_{k=-\infty}^{\infty} X_k p(t - kT)$, where X_k are chosen from a constellation. The received signal $y(t)$ is filtered through a low-pass filter with impulse response $q(t)$.

(i) Write down the expression for the output of the low-pass filter. [15%]

(ii) Write the time- and frequency-domain conditions that the pulse and low-pass filters need to fulfill in order not to have inter-symbol interference. [20%]

(iii) Consider a frequency-selective channel with impulse response

$$h(t) = \sum_{\ell=1}^{L} \alpha_\ell \delta(t - \tau_\ell).$$

Explain the main impairment this channel introduces. Briefly explain the main methods to deal with this channel impairment. [15%]

(b) Describe briefly the difference between distance-vector protocols and link-state protocols for routing. Give an example for each and state which routing algorithm it uses. [15%]

(c) Consider a network with 3 nodes (A, B and C). Shortest path routing is used where the weight of each link depends on its capacity and the flow on that link. The initial weight of the links from A to B and B to A, and from B to C and C to B, are 1. There is no direct link beween A and C. Due to a change in flow, the weight of the link from B to C suddenly changes to 100. Describe how the routing information is subsequently updated in the case of:

(i) A link state protocol. [15%]

(ii) A distance vector protocol (assume in this case that B receives a packet from A containing its distance vector at the same instance as the weight changes). [20%]

4 Consider the convolutional encoder in Fig. 3.

(a) What is the rate of the code? [5%]

(b) Draw the state diagram of the encoder, labelling each branch with the input and output bits corresponding to that branch. [15%]

(c) A sequence of four symbols followed by two termination zeros is transmitted over a binary symmetric channel (BSC) and the received sequence is $10, 10, 11, 01, 10, 01$. Determine a maximum likelihood code sequence and the corresponding information sequence for this received sequence. [30%]

(d) The bit and block error frequencies for a Viterbi decoder and a Bahl Cocke Jelinek Raviv (BCJR) decoder were measured through simulation for long data blocks and recorded as $f_1 = 0.000126$, $f_2 = 0.000174$, $f_3 = 0.0346$, $f_4 = 0.0375$, but we forgot to label the measurements. Determine which of the four frequencies f_1, f_2, f_3 and f_4 corresponds to which measurement, i.e., (Viterbi, block error), (Viterbi, bit error), (BCJR, block error) and (BCJR, bit error) and justify your answer. [10%]

(e) Is the encoder catastrophic? Justify your answer. [10%]

(f) Determine the free distance d_{free} of the code and specify what length input sequences generate code sequences of weight d_{free}? Explain if and how your findings are consistent with your answer to part (e). [30%]

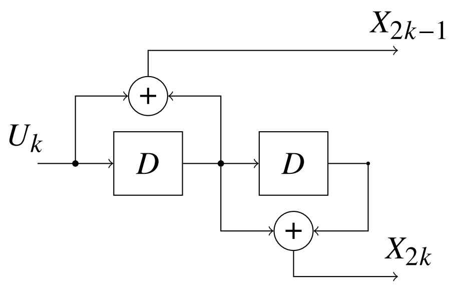

Fig. 3

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