Version AGiF/3

EGT2
ENGINEERING TRIPOS PART IIA

Friday 28 April 2023 9.30 to 11.10

Module 3F4

DATA TRANSMISSION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Write on single-sided paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Engineering Data Book.
Additional copy of Fig. 2 for Q1 is attached.

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) What is the rate of the convolutional encoder in Fig. 1 and what is its description in octal notation? [10%]

(b) How many states does the encoder have? Without drawing its full state diagram, consider the transitions from state $(1, 1, 0)$. What states do they lead to and what are the corresponding input and output symbols? [10%]

(c) A sequence of 5 symbols followed by 3 termination zeros is transmitted over a binary symmetric channel (BSC) with crossover probability $p < 1/2$ and the received sequence is $110, 010, 101, 001, 110, 001, 110, 111$. Determine a maximum likelihood code sequence and the corresponding information sequence for this received sequence. A labeled trellis is provided in Fig. 2 to help you solve this task. [30%]

An additional copy of Fig. 2 is attached to the back of this paper. It should be detached and handed in with your answers.

(d) Figure 3 shows a trellis module. The branches on the trellis are labeled with a pair $(c_0c_1c_2, p)$. $c_0c_1c_2$ correpond to the digits emitted by the encoder on that branch. $p = \alpha_i\gamma_{ij}\beta_i$ is the summary probability value produced by the forward-backward (BCJR) algorithm for each branch given an observation sequence, where these values have been suitably normalised and approximated to yield integers for easy calculation.

(i) Calculate the a-posteriori probability that the information digit U corresponding to the trellis module is 1 given the observed sequence. [20%]

(ii) Calculate the a-posteriori probability that the middle code digit emitted in this trellis module is 1 given the observed sequence. [10%]

(e) Is the encoder catastrophic? Justify your answer. [20%]

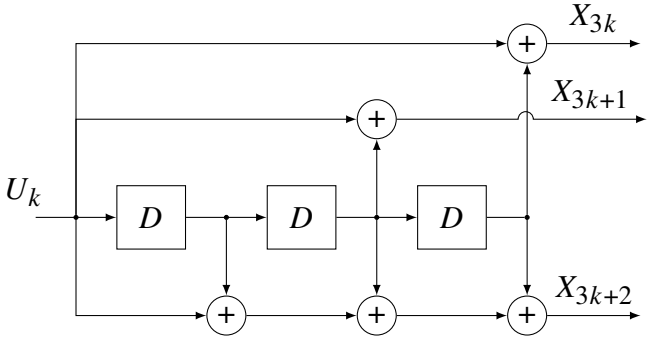


Fig. 1

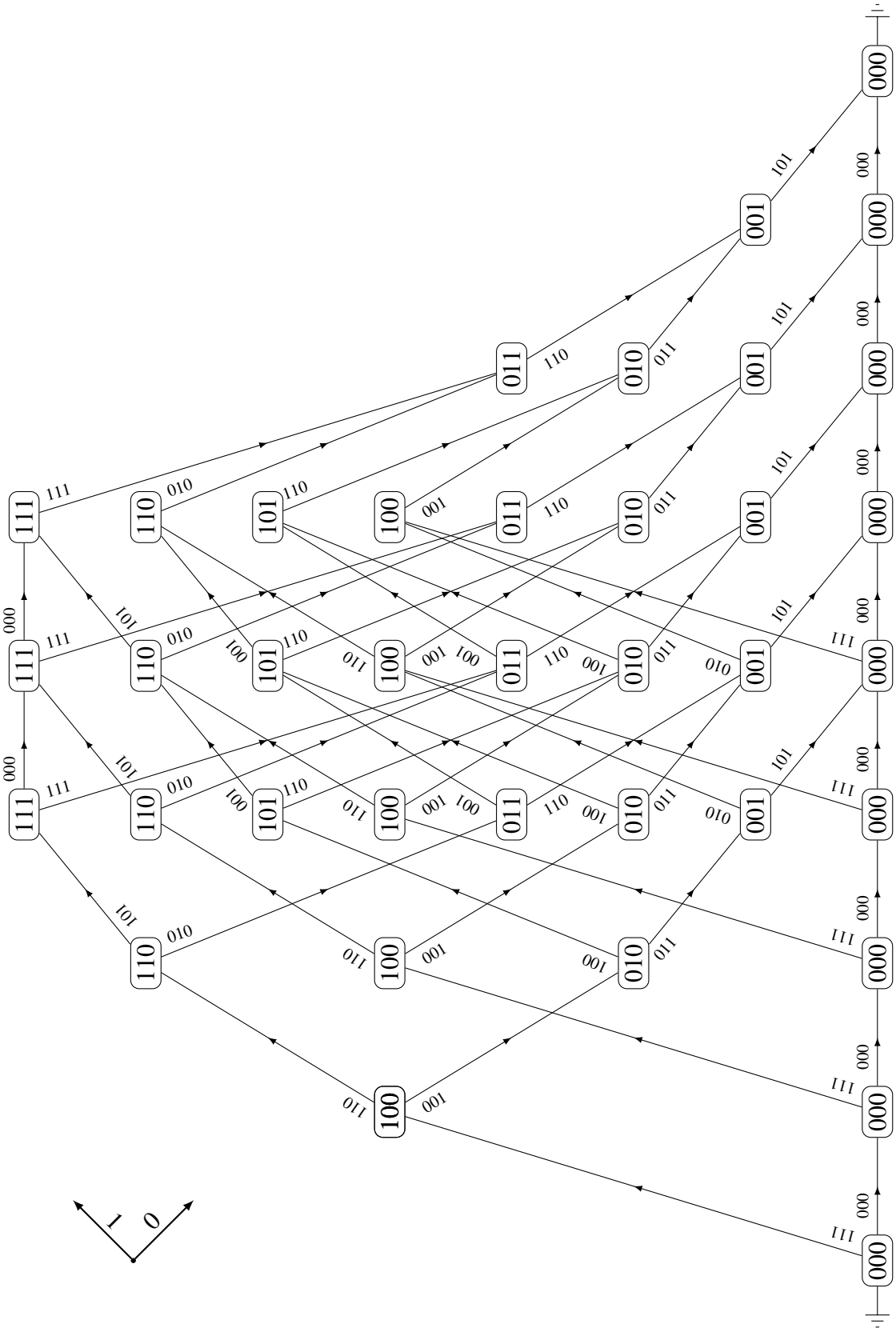


Fig. 2

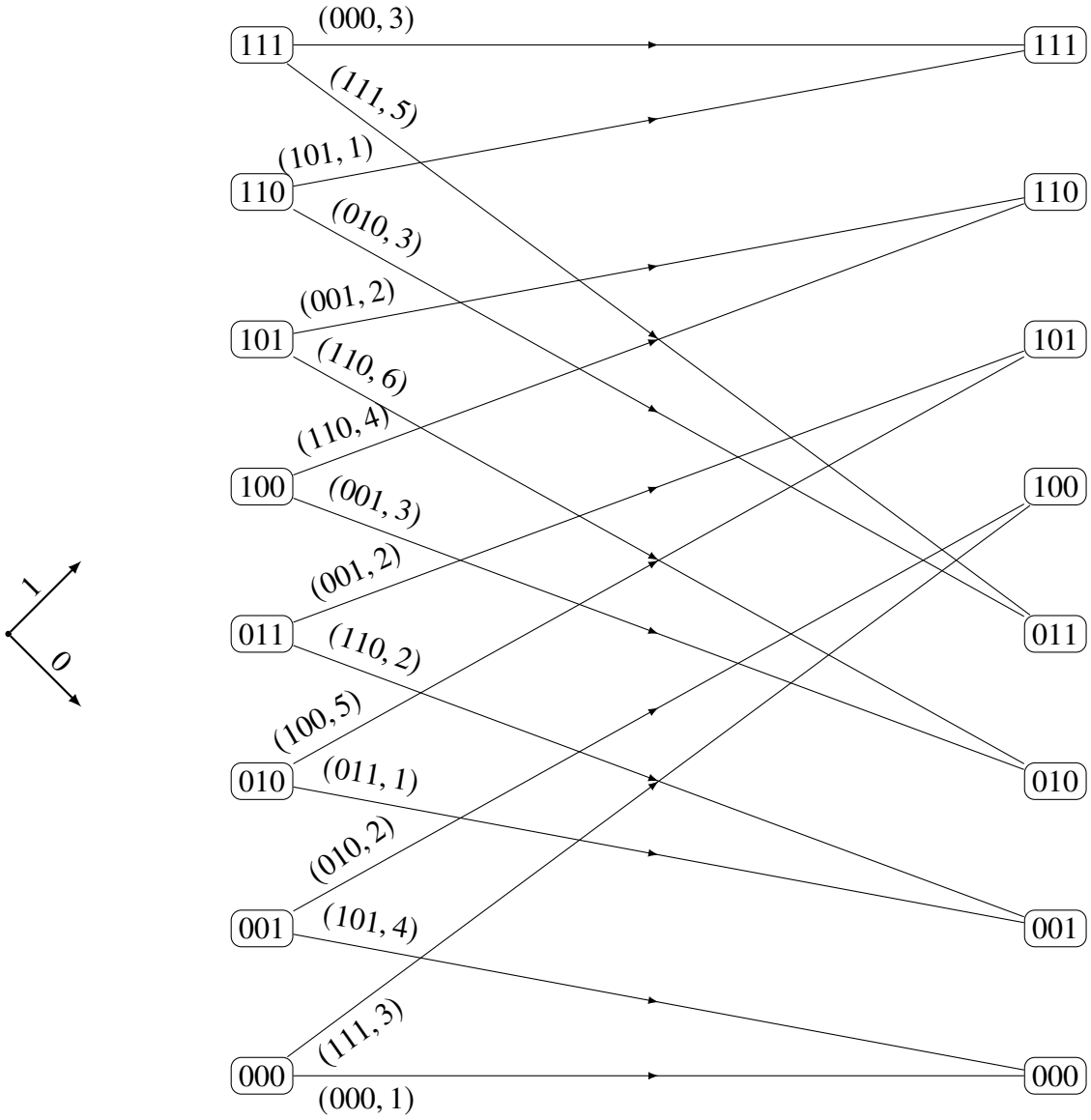
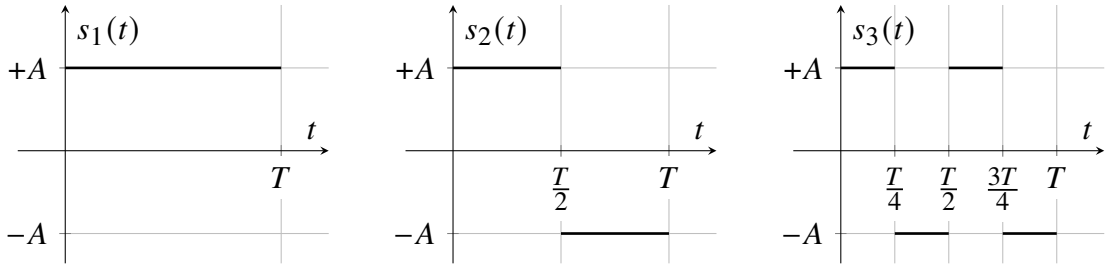

111
110
101
100
011
010
001
000
(000, 3)
(111, 5)
(101, 1)
(010, 3)
(001, 2)
(110, 6)
(110, 4)
(001, 3)
(001, 2)
(110, 2)
(100, 5)
(011, 1)
(010, 2)
(101, 4)
(111, 3)
(000, 1)
111
110
101
100
011
010
001
000
1
0

Fig. 3

2 Consider the signal set shown below.



(a) Determine an orthonormal basis using the Gram-Schmidt method and find the dimension of the signal space. [20%]

(b) What is the transmission rate in bits/second assuming equiprobable symbols? [10%]

(c) Find the vector representation of the signal space, calculate the average energy per symbol, E_s, and the minimum Euclidean distance $d_{\min}$ of the constellation. [15%]

(d) Derive the optimal detector assuming transmission over an AWGN channel with noise power spectral density $N_0/2$. [15%]

(e) Which of the three signals is more vulnerable to Gaussian noise errors? Justify your answer. [10%]

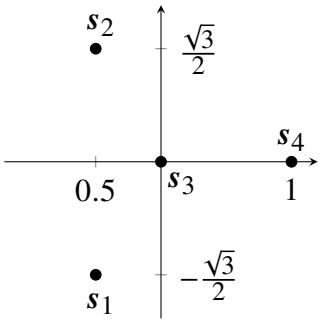
(f) Show that the average error probability p_e can be upper bounded by

$$p_e \leq 2Q\left(\frac{d_{\min}}{\sqrt{2N_0}}\right)$$

[15%]

(g) In what signal-to-noise ratio (SNR) regime is this bound accurate? Justify your answer. [15%]

3 (a) Consider the signal constellation below.



(i) What is the dimension of the signal space? [10%]

(ii) Propose an orthonormal basis for this vector representation and a suitable set of signals. [20%]

(iii) Sketch the decision regions for the optimal detector assuming equiprobable symbols. [20%]

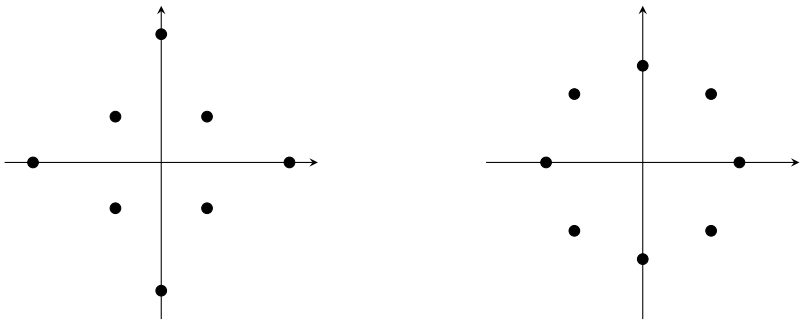
(iv) Sketch the decision regions when symbols s_1, s_2, s_3, s_4 are transmitted with probabilities $\frac{1}{3}, \frac{1}{3}, 0, \frac{1}{3}$, respectively. [10%]

(b) Consider the two constellations below and assume they have unit energy.

(i) Which constellation has a lower error probability at high signal-to-noise ratio (SNR)? [10%]

(ii) Explain the concept of cyclic prefix in OFDM as well as its role. [10%]

(iii) The two constellations are to be used for transmission over an inter-symbol interference channel with OFDM. Which of the two induces a time-domain signal with lower peak-to-average power ratio? Justify your answer. [20%]



4 (a) Consider pulse amplitude modulation (PAM) with symbol period T and pulse $p(t)$. The Fourier transform of the pulse is given below

$$P(f) = \begin{cases} \sqrt{A} & |f| \leq \frac{1-\alpha}{2T} \\ \sqrt{\frac{AT}{\alpha}(|f| - \frac{1+\alpha}{2T})} & \frac{1-\alpha}{2T} < |f| \leq \frac{1+\alpha}{2T} \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

for $0 \leq \alpha \leq 1$.

(i) Sketch $|P(f)|^2$. [5%]

(ii) Determine the value of A such that the pulse has unit energy. [20%]

(iii) Using the value of A found in part (a)(ii) discuss whether the pulse given in Eq. (1) induces inter-symbol interference for $\alpha = 0$ and $\alpha = 1$. [10%]

(iv) Using the value of A found in part (a)(ii) determine the overall pulse in the time domain $g(t)$ for $\alpha = 0$ and $\alpha = 1$. [15%]

(b) Describe how TCP Reno responds after

(i) an in sequence acknowledgement (`ack`) is received,

(ii) duplicate acknowledgements (`dupacks`) are received. [10%]

(c) Assume that each packet is delayed/lost with probability q, that packet loss events are independent and that no timeouts occur.

(i) Explain why the expected value of the rate of change of the window size $W(t)$ is proportional to

$$(1-q)(1/W(t)) - qW(t)/2$$

and find the expected equilibrium window size as a function of q. [20%]

(ii) What is the maximum packet loss rate compatible with a throughput of 500MB/s over a link with an RTT of 100ms. Is this reasonable? (Consider the mean time between loss events). [5%]

(d) A modification to TCP Reno is proposed where the window size is increased by a fixed factor, $1/20$, per in sequence `ack` received. Repeat the calculations of part (c) for this situation. [15%]

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Version AGiF/3 Candidate Number:

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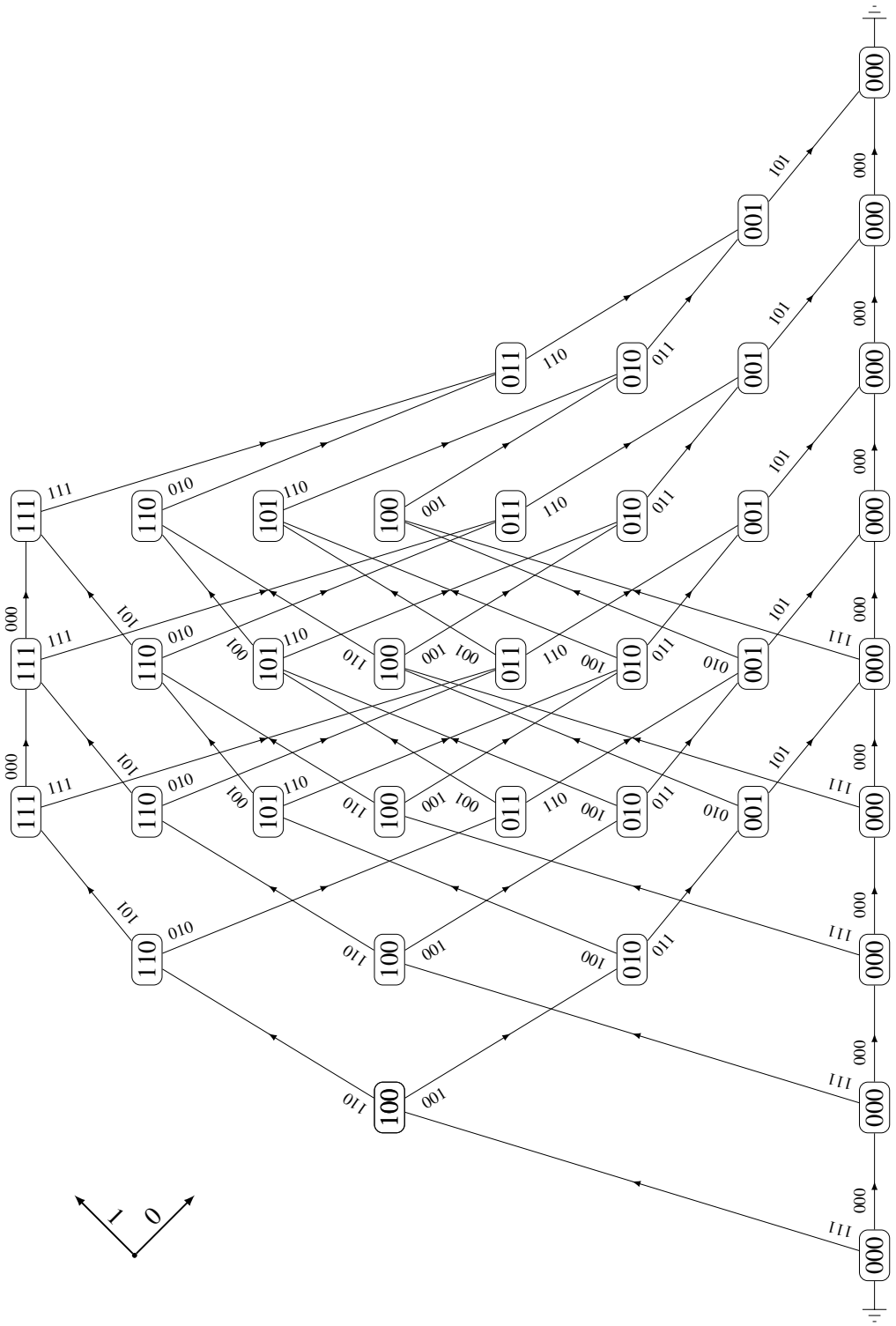
Friday 28 April 2023, Module 3F4, Question 1.



Extra copy of Fig. 2: labeled trellis diagram for Question 1.