Version AGiF/3

EGT2
ENGINEERING TRIPOS PART IIA

Monday 22 April 2024   14.00 to 15.40

Module 3F4

DATA TRANSMISSION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Write on single-sided paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Engineering Data Book.

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) What is the rate of the code in Fig. 1(a) opposite? [5%]

(b) Describe the encoder in Fig. 1(a) in octal form. [5%]

(c) Four inputs $\underline{U} = U_0, \ldots, U_3$ are fed into the encoder in Fig. 1(a) and its output is transmitted through a binary symmetric channel with outputs $\underline{y} = y_0, \ldots, y_7$. A Viterbi decoder returns the estimation $\hat{\underline{U}} = 1, 1, 0, 1$, and $P(\underline{U} = 1, 1, 0, 1 \mid \underline{y}) = 1/8$. Based on the properties of Viterbi decoders, state which of the following statements are incompatible with the scenario described:

(i) $P(\underline{U} = 0, 1, 0, 0 \mid \underline{y}) = 1/4$, (ii) $P(\underline{U} = 0, 0, 0, 1 \mid \underline{y}) = 1/3$, and (iii) $P(U_0 = 0 \mid \underline{y}) = 3/4$. Justify your answers. [15%]

(d) Is the encoder in Fig. 1(a) catastrophic? Justify your answer. [10%]

(e) Express the detour enumerator transfer function $T(J, D, N)$ for the encoder in Fig. 1(a) and hence determine the free distance d_{free} of the encoder [25%]

(f) Now consider the recursive systematic encoder in Fig. 1(b). Is this encoder catastrophic? Justify your answer. [10%]

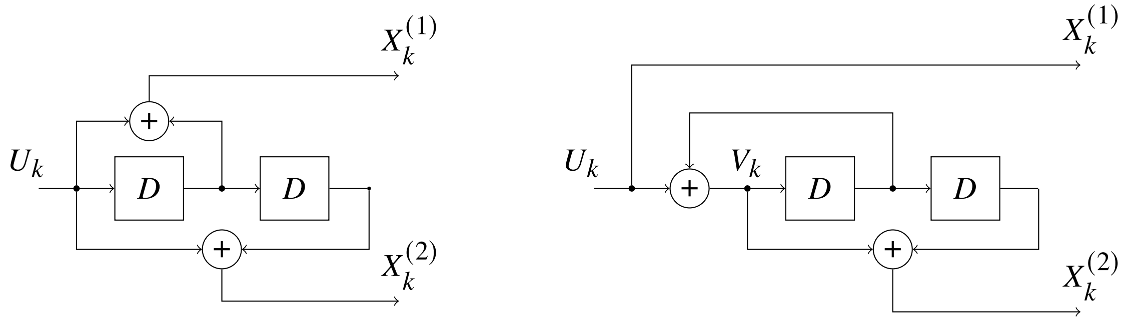
(g) Using the D transform, express the output sequences $X^{(1)}(D)$ and $X^{(2)}(D)$ as a function of $U(D)$ for the encoder in Fig. 1(b). You may want to use the auxiliary sequence V_k as labeled in the figure to help determine the transfer function from $U(D)$ to $X^{(2)}(D)$. [15%]

(h) Which of the following are valid output sequences for the encoder in Fig. 1(a) and which are valid for the encoder in Fig. 1(b)? Both encoders are used with six input symbols followed by two zero-valued termination symbols. Justify your answers.

(i) $X^{(1)} = 1, 0, 1, 1, 0, 1, 0, 0$, and $X^{(2)} = 1, 1, 1, 0, 1, 1, 1, 0$. [5%]

(ii) $X^{(1)} = 1, 1, 0, 0, 1, 1, 1, 1$, and $X^{(2)} = 1, 0, 1, 0, 1, 0, 0, 0$. [5%]

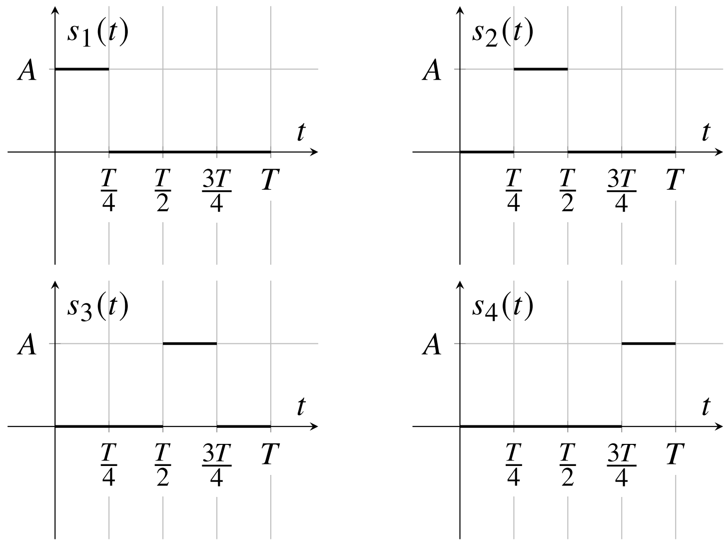
(iii) $X^{(1)} = 1, 0, 1, 1, 0, 1, 0, 0$, and $X^{(2)} = 1, 0, 1, 0, 0, 1, 1, 0$. [5%]



(a) (b)

Fig. 1

2 Consider the signal set shown below.



(a) Find the dimension of the signal space and determine an orthonormal basis. [10%]

(b) Find the transmission rate in bits/second assuming equiprobable symbols. [10%]

(c) Write the vector representation of the signals $s_1(t), \ldots, s_4(t)$ as a function of the average energy per bit E_b. [10%]

(d) Calculate the minimum Euclidean distance d_min of the constellation. [10%]

(e) Consider transmission over an AWGN channel with noise power spectral density $\frac{N_0}{2}$ with non-equiprobable symbols. Derive and draw the optimal receiver. Explain the main operations of the optimal receiver. [15%]

(f) State the property of the noise that allows to ignore the noise contribution outside the signal space for detection purposes. [20%]

(g) Show that the probability of error with equiprobable symbols can be bounded as [15%]

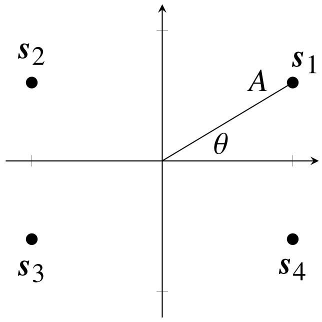
$$p_e \leq 3\mathbb{P}[N_2 - N_1 \geq \sqrt{2E_\mathrm{b}}]$$

(h) Show that the error probability can be bounded as

$$p_e \leq 3Q\left(\sqrt{\frac{2E_\mathrm{b}}{N_0}}\right)$$

Justify your answer. [10%]

3 Consider a data transmission system employing the modulation drawn in the figure below, where the angle θ is assumed to be such that $0 \leq \theta < \frac{\pi}{4}$.



(a) What is the dimension of the signal space? [10%]

(b) Write down the vector representation of the symbols and find the average energy per bit E_b of the constellation. [20%]

(c) Specify the optimal decision region. [15%]

(d) After the transmission of a single symbol over an AWGN with power spectral density $\frac{N_0}{2}$, the received sample at the input of the detector is

$$y = h \cdot x + n$$

where h is a positive real coefficient.

(i) Find the signal-to-noise ratio at the receiver. [10%]

(ii) Calculate the symbol error probability as a function of h, θ and $\frac{E_b}{N_0}$. [20%]

(iii) Suppose we have two different receivers with received signals

$$y_1 = h_1 \cdot x + n_1 \qquad y_2 = h_2 \cdot x + n_2$$

where the real coefficients h_1, h_2 satisfy that $h_2 < h_1$. Receiver 1 intends to detect the whole constellation, while receiver 2 detects only the real part. What is the value of θ such that the error probability at high SNR of each receiver is the same? Justify your answer. Find θ when $h_1 = \frac{1}{10}$ and $h_2 = \frac{1}{40}$. [25%]

4 (a) Show that the error probability of a K-dimensional signal set composed of points $\boldsymbol{s}_1, \ldots, \boldsymbol{s}_M$ can be upper bounded as

$$p_e \leq \frac{1}{M} \sum_{m=1}^{M} \sum_{m' \neq m} P(\boldsymbol{s}_m \to \boldsymbol{s}_{m'})$$

where $P(\boldsymbol{s}_m \to \boldsymbol{s}_{m'})$ is the probability of the detector deciding in favour of $\boldsymbol{s}_{m'}$ when $\boldsymbol{s}_m$ was sent. [15%]

(b) Show that the term $P(\boldsymbol{s}_m \to \boldsymbol{s}_{m'})$ in (a) is [25%]

$$P(\boldsymbol{s}_m \to \boldsymbol{s}_{m'}) = Q\left(\sqrt{\frac{\|\boldsymbol{s}_m - \boldsymbol{s}_{m'}\|^2}{2N_0}}\right)$$

(c) Explain the behaviour of the error probability at high signal-to-noise (SNR) ratio. [10%]

(d) Distance vector and link-state protocols either share their knowledge of the network with their neighbours or their knowledge of their neighbours with the whole network. Which is which? Give examples of protocols of each kind, stating which algorithm they use to find shortest paths. [20%]

(e) Describe the benefits and disadvantages of each approach, using as an example the network shown in Fig. 2 with the link from node C to D failing at $t = T$, which C then communicates to its neighbour or the whole network. [30%]

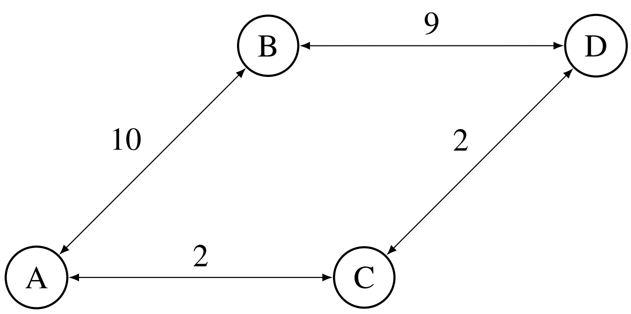


Fig. 2

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