Version RV/4

EGT2
ENGINEERING TRIPOS PART IIA

Friday 30 April 2021 9.00 to 10.40

Module 3F4

DATA TRANSMISSION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet and at the top of each answer sheet.*

STATIONERY REQUIREMENTS
Write on single-sided paper.

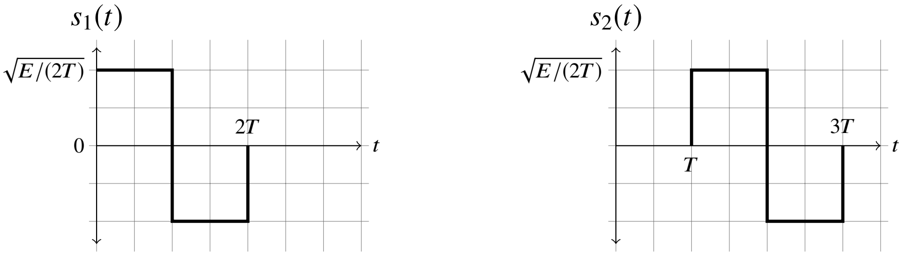
SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
You are allowed access to the electronic version of the Engineering Data Books.

10 minutes reading time is allowed for this paper at the start of the exam.

The time taken for scanning/uploading answers is 15 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers.

1 (a) A transmitter wishes to communicate one bit using the waveforms shown below.



The transmitted signal $x(t)$ is either $s_1(t)$ or $s_2(t)$. The two signals are equally likely. The received signal is $y(t) = x(t) + n(t)$, where $n(t)$ is white Gaussian noise with power spectral density $N_0/2$.

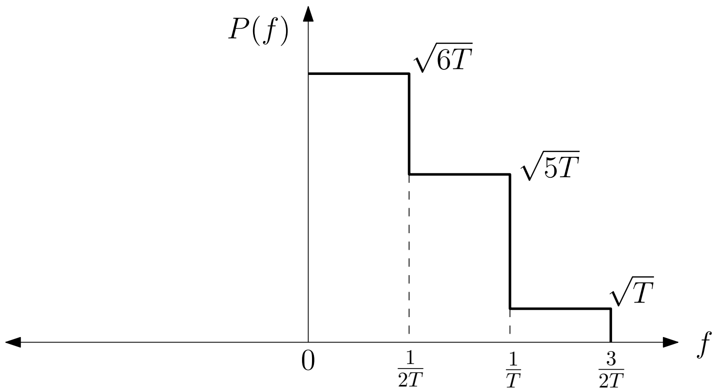
(i) Specify a pair of orthonormal basis functions for $s_1(t)$ and $s_2(t)$, and express the waveforms as linear combinations of the orthonormal basis functions. [30%]

(ii) Specify an optimal receiver to detect the transmitted waveform from $y(t)$. [20%]

(iii) Compute the probability of detection error for the receiver in part (ii). Your answer should be expressed in terms of the ratio $\frac{E}{N_0}$ and the Q-function, where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-u^2/2} du.$ [20%]

(b) Let $x(t) = \sum_{k=-\infty}^{\infty} X_k p(t - kT)$ be a PAM signal, where T is the symbol period. Assume that there is no noise, so that the received signal is $y(t) = x(t)$. The signal is passed through a filter with impulse response $p(-t)$. The output of the filter is $r(t) = \sum_{k=-\infty}^{\infty} X_k g(t - kT)$, where $g(t) = p(t) \star p(-t)$. [30%]

The pulse $p(t)$ is real-valued and even, and its Fourier transform $P(f)$ is shown in the figure below. (Only the positive frequencies are plotted.) Determine the sampled filter output $r(nT)$, for $n = 0, 1, 2, \ldots$.



2 (a) Let $x(t) = \sum_{k=-\infty}^{\infty} X_k p(t-k)$ be a transmitted PAM signal, where $p(t)$ is the rectangular pulse

$$p(t) = \begin{cases} 1, & 0 \leq t < 1, \\ 0, & \text{otherwise.} \end{cases}$$

The PAM signal is transmitted through a multipath channel with impulse response

$$h(t) = \delta(t) \; - \; 0.4\delta(t-0.5) \; + \; 0.1\delta(t-1.5),$$

so that the received signal is $y(t) = x(t) \star h(t) + n(t)$, where $n(t)$ is white Gaussian noise. At the receiver, $y(t)$ is passed through a filter with impulse response $p(-t)$. The filter output $r(t)$ is sampled at times $m = 0, 1, 2, \ldots$.

(i) Show that the filter output $r(t)$ is given by [10%]

$$r(t) = \sum_{k=-\infty}^{\infty} X_k \Big[g(t-k) \; - \; 0.4g(t-k-0.5) \; + \; 0.1g(t-k-1.5) \Big] \; + \; \tilde{n}(t),$$

where $g(t) = p(t) \star p(-t)$, and $\tilde{n}(t) = n(t) \star p(-t)$.

(ii) Obtain a compact expression for the sampled filter output $r(m)$, for $m = 0, 1, 2, \ldots$. Your expression should be a linear combination of the symbols $\{X_k\}$, plus $\tilde{n}(m)$. [25%]

(iii) Design a three-tap FIR zero-forcing equaliser to recover the transmitted symbols from the sequence $\{r(m)\}_{-\infty < m < \infty}$. [25%]

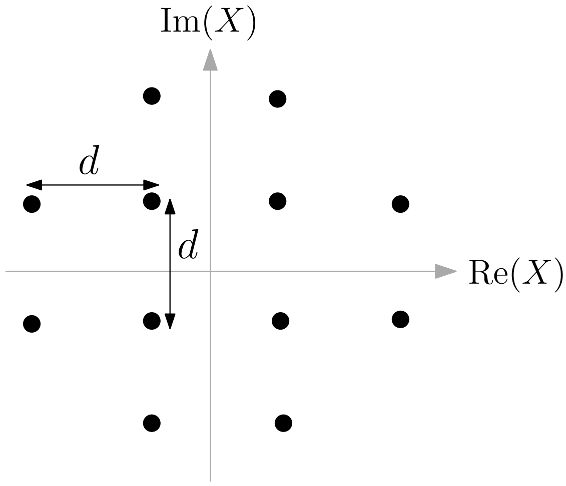
(iv) Compute the noise enhancement factor for the equaliser in part (iii), and name another 3-tap FIR equaliser with a better tradeoff between residual interference and noise enhancement. [10%]

(b) Consider a network with N nodes, where N is large. We run the Bellman-Ford algorithm to find a minimum-cost path from each node in the network to every other node. The algorithm terminates when such has a path has been found between each pair of nodes. Assume that the network has no isolated nodes, i.e., there exists a finite cost path between each pair of nodes.

(i) Specify a network configuration (i.e., a set of links) for which the Bellman-Ford algorithm is guaranteed to converge in two iterations, *regardless* of the costs on the links. [15%]

(ii) Specify a network configuration for which the Bellman-Ford algorithm is guaranteed to take $(N-1)$ iterations, regardless of the costs on the links. Justify your answer briefly. [15%]

3 (a) Consider the 12-point QAM constellation shown in the figure below, with neighbouring symbols in the vertical and horizontal directions spaced d apart.



The constellation is used for signalling over the discrete-time AWGN channel $Y = X + N$, where the noise N is a complex random variable whose real and imaginary parts are each i.i.d. Gaussian $\sim \mathcal{N}(0, \frac{N_0}{2})$. Assume that each of the constellation symbols is equally likely to be transmitted.

(i) Calculate the average energy per symbol E_s, and the average energy per bit E_b of the constellation, in terms of d. [15%]

(ii) Sketch the decision regions for the optimal detector. [10%]

(iii) Compute an upper bound on the probability of detection error. Your upper bound should be in terms of the ratio $\frac{E_b}{N_0}$ and the Q-function, where [25%]

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2} du.$$

(b) In a binary-input optical communication channel, the number of photons Y observed at the receiver is Poisson distributed with mean λ_0 if the input $X = 0$, and Poisson distributed with mean λ_1 if $X = 1$. That is, the probability mass function of Y when $X = 0$ is $P_0(Y = y) = \frac{\lambda_0^y}{y!} e^{-\lambda_0}$, for $y = 0, 1, 2, \ldots$. Similarly, the probability mass function of Y when $X = 1$ is $P_1(Y = y) = \frac{\lambda_1^y}{y!} e^{-\lambda_1}$, for $y = 0, 1, 2, \ldots$. Assume that $\lambda_1 > \lambda_0$.
We wish to decode X from the observed number of photons $Y = y$.

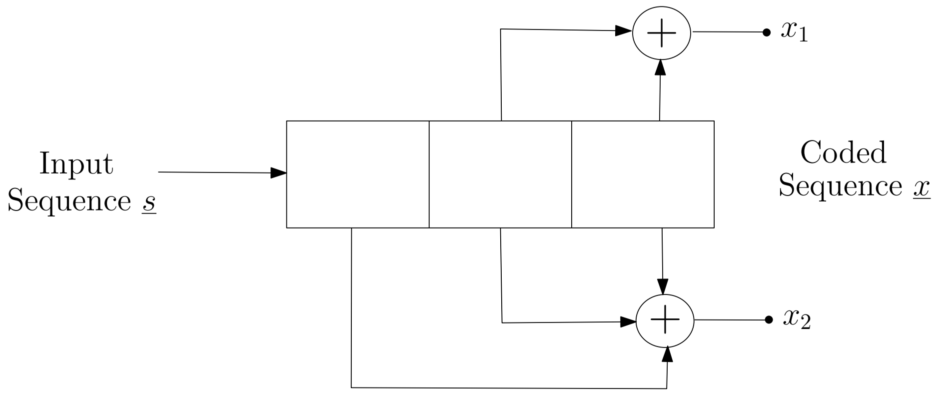
(i) If the distribution on the input X is $P(X = 1) = p$, $P(X = 0) = 1 - p$, derive the optimal decision rule that minimizes the probability of decoding error. Your answer must be expressed in terms of a threshold T such that the decoded symbol is 0 if $Y \leq T$, and is 1 otherwise. [20%]

(ii) For $p = \frac{1}{2}$, $\lambda_0 = 2$, and $\lambda_1 = 10$, specify the values of y for which the decision rule in (i) decodes 0. [10%]

(iii) Derive an expression for the probability of error of the detection rule derived in part (i). Your expression should be in terms of p, λ_0, λ_1, and the threshold T. [20%]

4 (a) A digital broadcast system uses coded OFDM with 16-QAM as the underlying modulation scheme. The signal bandwidth is 10 MHz, the spacing between sub-carriers is 4 kHz, and the duration of the guard interval is 10 μs. A rate $1/2$ code is used for error correction. 10% of the sub-carriers are reserved for pilot tones for carrier phase and amplitude recovery (i.e., no user information is transmitted on these sub-carriers).
Calculate the user data rate of the system, and the length of the cyclic prefix. [20%]

(b) Consider the convolutional code defined by the encoder shown in the figure below.



(i) What is the rate of the code? [5%]

(ii) Draw the state diagram of the code, labelling each branch with the output bits corresponding to that branch. [20%]

(iii) Suppose that a coded sequence from the convolutional encoder is transmitted over a binary symmetric channel, and the sequence at the output of the channel is

$$\underline{y} = (00\,10\,10\,01).$$

Determine the decoded sequence $\hat{\underline{x}}$ and the corresponding input sequence $\hat{\underline{s}}$. [45%]

(iv) Briefly explain why the decoded sequence is not unique. [10%]

END OF PAPER