EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 26 April 2023 9.30 to 11.10

Module 3G4

MEDICAL IMAGING & 3D COMPUTER GRAPHICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) In the operation of a Magnetic Resonance Imaging (MRI) scanner, how is the source of a particular magnetic resonance signal localised in three-dimensional space? [30%]

(b) The main field B_z of an MRI scanner is 1.5 T at $z = 0$, with a gradient $\partial B_z/\partial z$ of 40 mT/m applied at the same time as the 90° RF pulse. The flip angle achieved by a rectangular (i.e. finite duration, constant amplitude) RF pulse is given approximately by

$$\alpha = 2\pi\gamma B_1 t_p$$

where α is the flip angle in radians, B_1 is the pulse's amplitude in Teslas, t_p is the pulse's duration in seconds and the gyromagnetic ratio $\gamma = 42.58$ MHz/T for protons.

(i) If $B_1 = 2\,\mu$T, calculate t_p for a 90° flip angle. [5%]

(ii) For a rectangular RF pulse of frequency 63699680 Hz and duration 2.936 ms, calculate and sketch the range of z values that would be imaged. [30%]

(iii) How might the axial imaging resolution be improved? What trade-offs must be made in clinical practice? [35%]

2 (a) Briefly outline two different ways in which a polygon mesh like the one shown in Fig. 1 can be stored in a computer. Give two examples of different features of the mesh, or operations on the mesh, that are affected by how it is stored. Explain in each case what effect the storage scheme has. [20%]

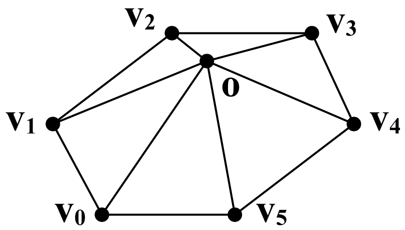

Fig. 1

(b) The expression $\mathbf{n} = \frac{1}{2}\mathbf{a} \times \mathbf{b}$ is known as the *vector area* of a triangle, where $\mathbf{a}$ and $\mathbf{b}$ are two edges of the triangle. Show how $\mathbf{n}$ represents both the triangle's normal and area. [10%]

(c) A user wants to estimate the surface normal at the centre vertex $\mathbf{o}$ of the mesh in Fig. 1. To this end, the user calculates the *vertex normal* by summing $\mathbf{n}$, as defined in part (b), over all of the triangles containing $\mathbf{o}$.

(i) Why might the user want a vertex normal rather than a triangle normal? [10%]

(ii) Find the simplest expression for the vertex normal in terms of the vertices $\mathbf{o}$ and $\mathbf{v}_i$, where in this case $i \in \{0 \ldots 5\}$. [30%]

(iii) Comment on the likely accuracy of the vertex normal (as an estimate of the underlying surface normal) as the position of the centre vertex $\mathbf{o}$ is varied. [15%]

(iv) What other useful property does the vertex normal possess? [15%]

3 (a) Explain how the different types of cubic parametric curves achieve different degrees of continuity at the join points between segments. Does the same degree of continuity apply to the entire curve? [15%]

(b) How is the convex hull property defined for cubic parametric curves, why is this useful, and which types of curves exhibit this property? [15%]

(c) Two segments A and B of a multi-segment curve are shown in Fig. 2. Segment A is defined using points $\mathbf{p}_0 \ldots \mathbf{p}_3$ and segment B is defined using overlapping points $\mathbf{p}_1 \ldots \mathbf{p}_4$. They are both B-splines, using basis matrix M_B, where

$$\mathrm{M}_B = \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$

(i) In terms of the points $\mathbf{p}_0 \ldots \mathbf{p}_3$, where is the left-hand end of segment A if $\mathbf{p}_1$ is moved such that it is coincident with $\mathbf{p}_2$? Sketch the points and both curves for this case. [20%]

(ii) In terms of the points $\mathbf{p}_0 \ldots \mathbf{p}_3$, where is the left-hand end of segment A if both $\mathbf{p}_0$ and $\mathbf{p}_1$ are moved such that they are coincident with $\mathbf{p}_2$? Sketch the points and both curves for this case. [20%]

(iii) Sketch the points and both curves if $\mathbf{p}_0$ remains in its original location as in Fig. 2, but both $\mathbf{p}_1$ and $\mathbf{p}_3$ are moved such that they are coincident with $\mathbf{p}_2$. [20%]

(iv) What geometric and parametric continuity does each part of the curve exhibit in the three cases (i)–(iii)? How might this help in designing curves? [10%]

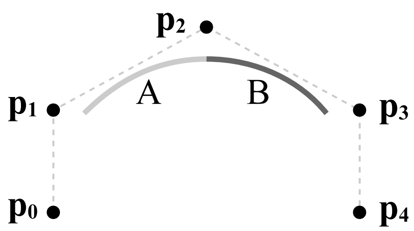

Fig. 2

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4 (a) The augmented Phong reflection model is given by

$$I_\lambda = c_\lambda I_a k_a + \sum_i S_i f_{\text{att}} I_{pi} (c_\lambda k_d \mathbf{L}_i.\mathbf{N} + k_s (\mathbf{R}_i.\mathbf{V})^n)$$

Explain the meaning of the various terms. [20%]

(b) The S_i terms are to be calculated using the shadow z-buffer algorithm. The scene is first "rendered" from the light source, with a "shadow" z-buffer filled in the usual way. The scene is then rendered from the viewpoint with one extra step. 3D screen coordinates (x_s, y_s, z_s), as seen from the viewpoint, are transformed to 3D screen coordinates (x'_s, y'_s, z'_s), as seen from the light source. (x'_s, y'_s) are used to index the shadow z-buffer. The retrieved value z_b is compared with z'_s, with $z'_s > z_b$ indicating that the point is in shadow.

In practice, the shadow z-buffer entries z_b are stored with a limited precision of k bits. What rendering artefacts might arise as a result of this limited precision? How might these artefacts depend on the positions of the shadow rendering's near and far clipping planes? [20%]

(c) Figure 3 shows part of a plane surface viewed from above. There is a single light source and the shadow z-buffer algorithm is used to render shadows, though the surface in Fig. 3 is not in the shadow of any other objects. The black circles in Fig. 3 are the frame buffer's pixel *centres* projected onto the surface, while the rectangles are the shadow z-buffer's pixel *boundaries* projected onto the surface. The shading indicates the contents z_b of the shadow z-buffer, with light representing near and dark representing far.

(i) Sketch a configuration of surface, viewpoint and light position that is consistent with Fig. 3. [20%]

(ii) By considering the shading of pixels A, B and C, identify self-shadowing artefacts that are likely to appear in the rendering. [20%]

(iii) Suggest how these artefacts might be suppressed. [20%]

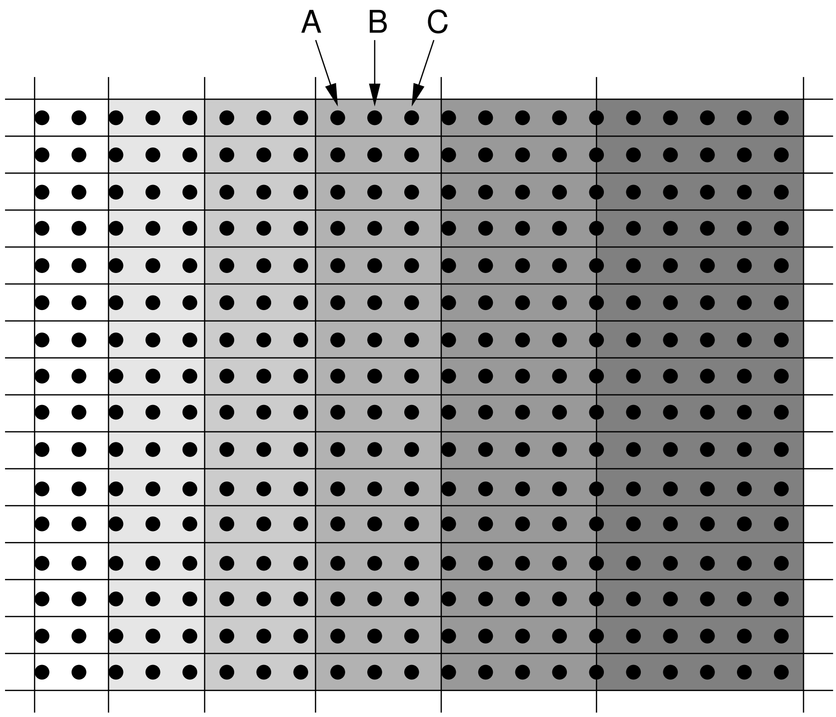

Fig. 3

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Part IIA 2023

Module 3G4: Medical Imaging & 3D Computer Graphics

Numerical Answers

1. (b) (i) 2.936 ms

2. (c) (ii) $\frac{1}{2}\sum_{i=0}^{5}(\mathbf{v}_i \times \mathbf{v}_{i\oplus 1})$ where $\oplus$ represents addition modulo 5

3. (c) (i) $\frac{1}{6}\mathbf{p}_0 + \frac{5}{6}\mathbf{p}_2$ (ii) $\mathbf{p}_2$