Version AHG/2

EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 23 April 2014 9.30 to 11

Module 3G4

MEDICAL IMAGING & 3D COMPUTER GRAPHICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Explain the meaning of the terms *projection* and *sinogram* as used in computed tomography reconstruction. [20%]

(b) The function

$$f(x,y) = \begin{cases} 1, & \text{if } (|x|+|y|) \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

is defined in a right-handed coordinate system (x,y). The projection angle, ϕ, is measured anticlockwise from the positive x axis to the line onto which the data is projected. Sketch projections at $\phi = 0$ and $\phi = \pi/4$ for the function f. [20%]

(c) Sketch the outline (silhouette) of the sinogram $p(s,\phi)$ of the function f defined in (b). There is no need to depict the intensities inside the silhouette. [20%]

(d) Define the Radon transform in two dimensions. [20%]

(e) Calculate the value of the Radon transform of the function f defined in (b) when $s = 0$ and $\phi = \pi/8$. [20%]

2 (a) A laser range scanner is a device for digitizing the surface geometry of a 3D object by scanning it with a stripe of laser light and viewing the reflection with a camera. Describe the various scanning problems associated with this technique, how each affects the measurement accuracy, and what can be done to correct them. [30%]

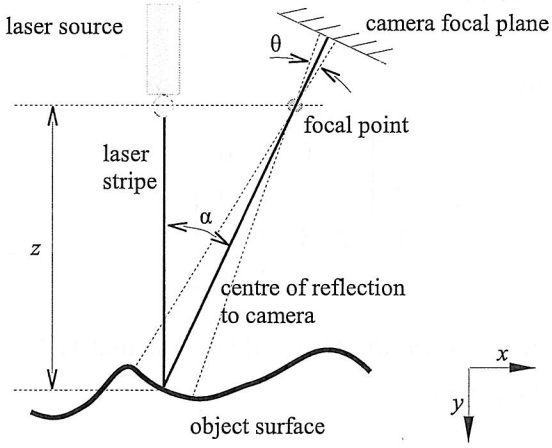

Fig. 1

(b) Figure 1 shows a schematic of a laser scanner with a single camera scanning the surface of an object. Calculate the resolution of the digitized surface, in each of the x and y directions, as a function of the distance from the scanner z, the angle of the reflected light α and the minimum angle which the camera can resolve θ. Assume that the laser stripe is very thin and that θ is small. What are the consequences of this finite resolution when scanning large objects? [30%]

(c) The area of a polygon with vertices $\mathbf{v}_i$, $i \in \{1 \ldots N\}$, can be calculated from $|\mathbf{p}|$, where

$$\mathbf{p} = \frac{1}{2} \sum_{i=1}^{N} \mathbf{v}_i \times \mathbf{v}_{i \oplus 1} \qquad (\oplus \text{ represents addition modulo } N)$$

(i) By appropriate substitution, show that $|\mathbf{p}|$ correctly returns the area of a triangle with vertices **a**, **b** and **c**. [20%]

(ii) Explain what the direction of **p** signifies, and suggest how this might be used in the process of mesh simplification. [20%]

3 (a) Describe the different properties of Bézier, Catmull-Rom and B-spline cubic parametric curves, and suggest in what context each might be useful. [30%]

(b) A spline *curve* in one parameter is defined as $\mathbf{p}(t) = \mathbf{TMG}$. Define the terms in this equation, provide a similar equation which describes a spline *surface* $\mathbf{q}(s,t)$, and define any new terms in this new equation. [20%]

(c) A matrix $\mathbf{L}$ is given by

$$\mathbf{L} = \frac{1}{8}\begin{bmatrix} 8 & 0 & 0 & 0 \\ 4 & 4 & 0 & 0 \\ 2 & 4 & 2 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix}$$

(i) Explain the process of *sub-division* and how the matrix $\mathbf{L}$ can be used to sub-divide a Bézier surface patch. [20%]

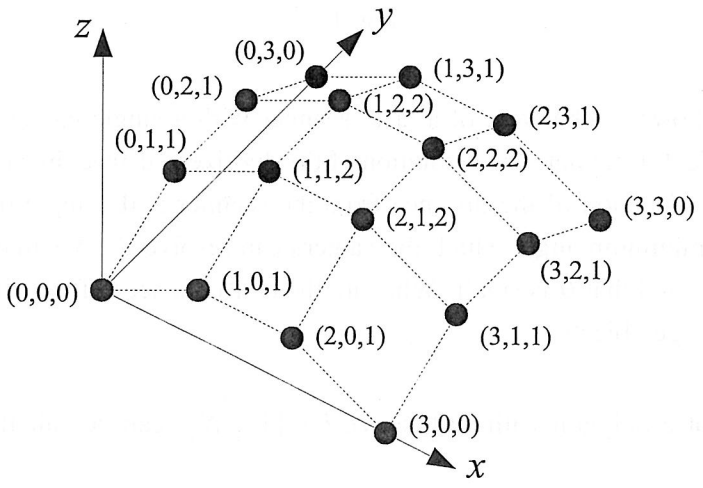

Fig. 2

(ii) A Bézier surface patch is defined using a net of control points with coordinates (x,y,z) as in Fig. 2. By use of the matrix $\mathbf{L}$, or otherwise, calculate the coordinates of the highest point (i.e. with maximum z value) on the surface patch. [30%]

4 (a) The basic Phong reflection model and a common variant are given below.

$$I_\lambda = c_\lambda (I_a k_a + I_p k_d \mathbf{L}.\mathbf{N}) + I_p k_s (\mathbf{R}.\mathbf{V})^n$$
$$I_\lambda = c_\lambda (I_a k_a + I_p k_d \mathbf{L}.\mathbf{N}) + I_p k_s (\mathbf{N}.\mathbf{H})^n$$

Explain the meanings of the various terms, and discuss the relative merits of the two formulations. [40%]

(b) Distinguish between *Gouraud shading* and *Phong shading*. What are the relative strengths and weaknesses of the two techniques? [30%]

(c) Explain how Phong shading might be accelerated on modern graphics hardware. What is the minimum number of scalar values that the rasterization unit must interpolate from vertices to pixels? [30%]

END OF PAPER

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Part IIA 2014

Module 3G4: Medical Imaging & 3D Computer Graphics

Numerical Answers

1. (e) 1.5307
2. (b) $r_x \approx 2z\theta/(1+\cos 2\alpha)$, $r_y \approx 2z\theta/\sin 2\alpha$
3. (c) (ii) $(1.5, 1.5, 1.5)$