EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 28 April 2021   9 to 10.40

Module 3G4

MEDICAL IMAGING & 3D COMPUTER GRAPHICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet and at the top of each answer sheet.*

STATIONERY REQUIREMENTS
Write on single-sided paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
You are allowed access to the electronic version of the Engineering Data Books.

10 minutes reading time is allowed for this paper at the start of the exam.

The time taken for scanning/uploading answers is 15 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers.

1 (a) Iterative reconstruction algorithms have traditionally been preferred over filtered backprojection for PET and SPECT. In recent times, however, they are also being used increasingly for CT. Why do you think this might be? [30%]

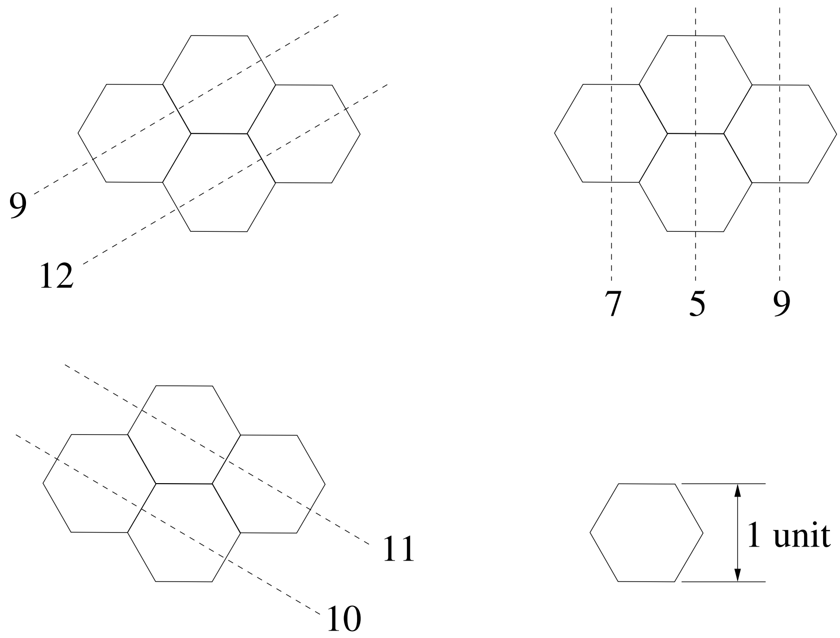

Fig. 1

(b) An X-ray phantom consists of four homogeneous rods. The cross-section of each rod is a regular hexagon of unit width, as shown in Fig. 1. The phantom is exposed to mono-energetic X-rays of incident intensity I_0. The transmitted X-ray intensity measured on the other side of the object is I. The ratio of the transmitted to the incident intensity is recorded, and a value Q is calculated in each case using $Q = -\ln(I/I_0)$. The values of Q measured along lines at three different angles are shown in Fig. 1.

(i) Without using any reconstruction algorithm, write down the values of the X-ray linear attenuation coefficients for each of the rods. [10%]

(ii) Show how the additive algebraic reconstruction technique (AART) can be used to calculate the linear attenuation coefficients of the rods. Start by assuming that each of the rods has an attenuation coefficient of 5, use a relaxation factor of 1, and continue the iteration process until your value for each rod is within 0.2 of the correct answer. [40%]

(c) Explain why a polychromatic X-ray source makes computed tomography reconstruction more difficult. How can the beam be made more monochromatic? [20%]

2 (a) Describe how multi-segment cubic parametric curves are defined, and how they can be used to interpolate or approximate a series of points. What are the advantages and disadvantages of using either a Catmull-Rom or a B-spline basis for such a curve? [20%]

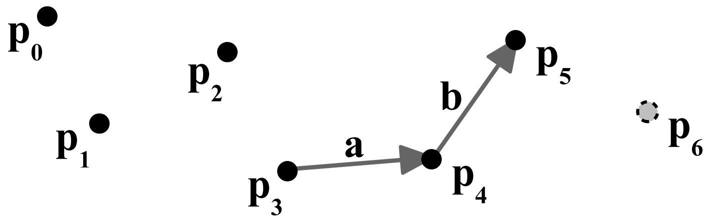

Fig. 2

(b) Figure 2 shows a set of six user-defined points $\{\mathbf{p}_0, \ldots, \mathbf{p}_5\}$ which will form the control points for a multi-segment cubic parametric curve. A seventh point $\mathbf{p}_6$ is to be added automatically to this set, creating an additional segment which changes how the curve ends. Vectors **a** and **b** are between the points $\{\mathbf{p}_3, \mathbf{p}_4\}$ and $\{\mathbf{p}_4, \mathbf{p}_5\}$ respectively. The basis matrices for the Catmull-Rom spline $\mathbf{M}_{CR}$ and the B-spline $\mathbf{M}_B$ are as follows:

$$\mathrm{M}_{CR} = \frac{1}{2}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 2 & -5 & 4 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \quad \mathrm{M}_B = \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$

(i) For the Catmull-Rom basis, determine the location of $\mathbf{p}_6$ so that the first parametric derivative is zero at the end of the curve. What is the direction of the curve at the end point, in terms of **a** and **b**, in this case? [25%]

(ii) For the B-spline basis, determine the location of $\mathbf{p}_6$ so that the curve ends at $\mathbf{p}_5$. If the curve is a motion path, what is the acceleration, in terms of **a** and **b**, at this point? [20%]

(iii) Again for the B-spline basis, determine the location of $\mathbf{p}_6$ so that the first parametric derivative is zero at the end of the curve. What is the location of the end point, in terms of $\mathbf{p}_5$, **a** and **b**, in this case? [20%]

(iv) Discuss the possible uses of each of the three schemes in (i)–(iii). [15%]

3 (a) What are the differences between nearest-neighbour interpolation and bi-linear interpolation when applied to rectilinear two-dimensional data? Briefly explain how bi-linear interpolation can be applied to unstructured two-dimensional data. [20%]

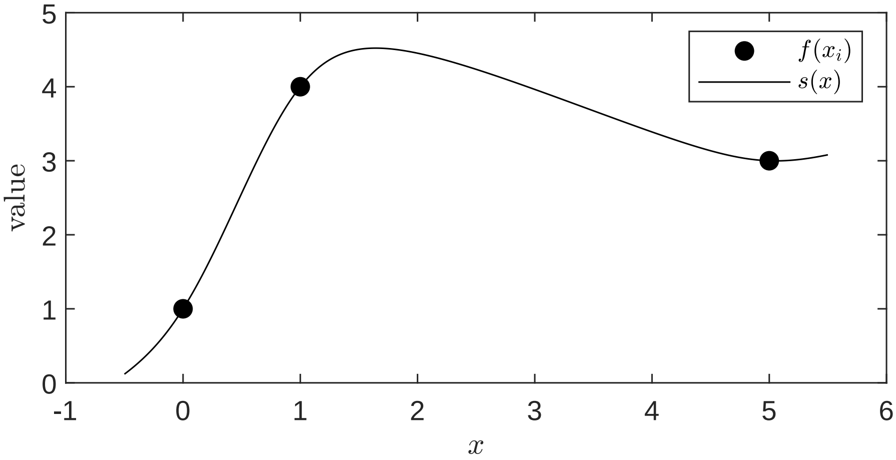

Fig. 3

(b) Three data points, shown as dots in Fig. 3, have locations $x_i = \{0,\ 1,\ 5\}$ and values $f(x_i) = \{1,\ 4,\ 3\}$. They are interpolated by a radial basis function (RBF) $s(x)$, shown as a solid line in Fig. 3. The resulting matrix equation is:

$$\begin{bmatrix} 0.6 & 1.166 & 5.036 & 1 & 0 \\ 1.166 & 0.6 & 4.045 & 1 & 1 \\ 5.036 & 4.045 & 0.6 & 1 & 5 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 5 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 3 \\ 0 \\ 0 \end{bmatrix} \qquad (1)$$

(i) Carefully explain where the non-zero numbers in the matrix and vector come from, and also the meanings of the variables λ_i and c_i. [10%]

(ii) What equations are expressed by each row of Eq. (1), and how do they contribute to the solution of the RBF? [10%]

(iii) $s(x)$ uses a multiquadric basis $\phi(r) = \sqrt{(r^2 + \alpha^2)}$. Write down the full expression for $s(x)$ in terms of x_i, x, α, λ_i, c_0 and c_1. What value of α has been used in Eq. (1)? [25%]

(iv) Sketch, over the range $0 \le x \le 5$, the result of fitting (to the same three points) a similar RBF but with $\alpha = 0$. [15%]

(v) Suggest what is the purpose of the parameter α. How would you arrive at a sensible value of α for a given set of points? [20%]

4 Figure 4 shows a point on an infinite planar floor being viewed from a height h above the floor. The single light source is at infinity behind the viewer, at an angle ϕ above the floor. All vectors in the diagram are unit length and coplanar.

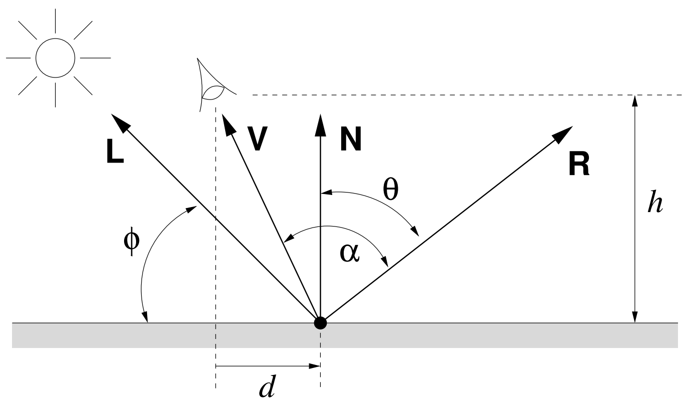

Fig. 4

(a) Using the same notation as in Fig. 4, write down an expression for I_s, the specular term in the Phong reflection model. [10%]

(b) The following line of code is used to partially calculate I_s in a GLSL implementation of Phong shading:

```
spec = pow(max(dot(V, R), 0.0), n);
```

What is the purpose of the variable `n`? Would you expect this line to appear in the vertex shader or the fragment/pixel shader? Justify your answer. [20%]

(c) Referring again to Fig. 4, derive an expression for α in terms of ϕ, h and the horizontal distance d between the viewer and the point on the floor. You may assume that $d > 0$. Sketch a graph showing the variation of α with d. [30%]

(d) Hence, identify an artefact that is likely to be apparent in Phong-shaded renderings of the floor, when `n` is small. [20%]

(e) Discuss the advantages of reformulating the specular term in terms of **N.H**, where **H** is Blinn's 'halfway' vector. [20%]

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Part IIA 2021

Module 3G4: Medical Imaging & 3D Computer Graphics

Numerical Answers

1. (b) (i) 2, 9, 3 and 7 (clockwise from top)

2. (b) (i) $\mathbf{p}_4$, $5\mathbf{b} - \mathbf{a}$
   (ii) $\mathbf{p}_5 + \mathbf{b}$, 0
   (iii) $\mathbf{p}_4$, $\mathbf{p}_5 - \mathbf{b}/3$

3. (b) (iii) $s(x) = c_0 + c_1 x + \sum_{i=1}^{3} \lambda_i \sqrt{(|x_i - x|^2 + \alpha^2)}$, $\alpha = 0.6$

4. (a) $I_s = \begin{cases} I_p k_s \cos^n \alpha = I_p k_s \left(\mathbf{R}.\mathbf{V}\right)^n & \text{for } \alpha, \theta < 90^\circ \\ 0 & \text{otherwise} \end{cases}$

   (c) $\alpha = \pi - \phi - \tan^{-1}(h/d)$