EGT2
ENGINEERING TRIPOS PART IIA

Monday 5 May 2014   9.30 to 11

Module 3A6

HEAT AND MASS TRANSFER

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A multi-pass, cross-flow heat exchanger is laid out as in Fig. 1 between air and a fluid. Fluid with a constant specific heat capacity c enters a tube at a mass flow rate $\dot{m}$ and temperature T_{s1} into the first pass and leaves at temperature T_{f1} . The flow continues onto a second pass at an inlet temperature $T_{s2} = T_{f1}$, and so on to the next pass. Air flowing with a constant specific heat capacity c_a enters the heat exchanger in cross flow at a mass flow rate $\dot{m}_a$ and temperature T_{a1} . The overall heat exchange coefficient between the two fluids is constant and equal to U, and the corresponding air temperature during the exchange can be taken as the inlet temperature. The tubes have a perimeter P, and length L. Assume that the ratio $C = \dot{m}c/(\dot{m}_a c_a) < 1$. Tube end effects can be neglected.

(a) Determine the exit temperature of each i-th pass, T_{fi}, as a function of the inlet temperature of each pass, the oncoming air temperature, and the geometric and operating parameters of the heat exchanger. [20%]

(b) Show that the effectiveness of each pass is $\varepsilon = 1 - \exp[-UPL/(\dot{m}c)]$. [20%]

(c) Show that the ratio of the heat rate exchanged between successive passes is $1-(1+C)\varepsilon$. [20%]

(d) Sketch the temperature evolution of both fluids as a function of the number of passes. Assume $T_{a1} > T_{s1}$ and $(1+C)\varepsilon < 1$. [20%]

(e) Show that for a very large number of passes, the total heat exchange reaches a limit of $\dot{Q} = \dot{m}c(T_{a1} - T_{s1})/(1+C)$. Discuss the meaning of the result. [20%]

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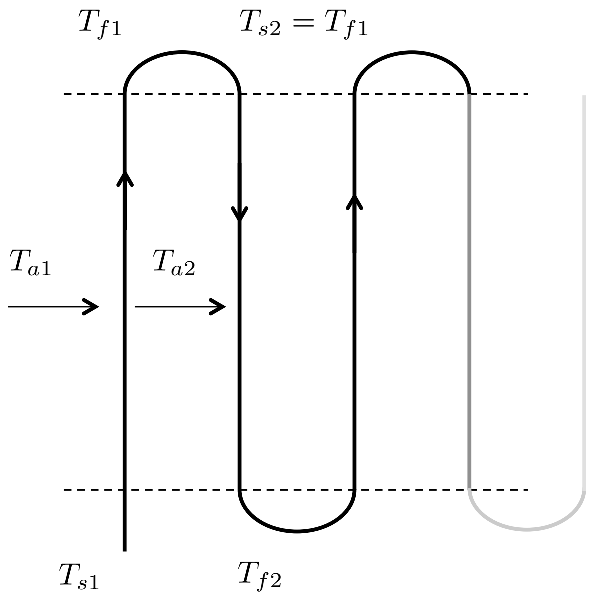

Fig. 1

2 A nuclear fuel rod with thermal conductivity λ_1 generates heat at a uniform volumetric rate $\dot{g}$. The fuel rod is encased in a tube with inner radius a, outer radius b and thermal conductivity λ_2. The surface area of the tube is enhanced with radial fins of thermal conductivity λ_2, thickness t and length L. The fin tips are insulated, and the edges lose heat to their surroundings at temperature T_∞ with a constant convective heat transfer coefficient, h. The cross section of the fuel rod is shown in Fig. 2.

(a) (i) Derive an expression for the temperature T within the fuel rod as a function of radius r and the boundary temperatures. Assume that conduction is steady and in the radial direction only. [15%]

(ii) Ignoring the effect of the fins, derive an expression for the temperature in the encasing tube. [15%]

(b) (i) Show that the temperature at a distance x from the base of the fin is given by

$$\frac{T(x)-T_\infty}{T(b)-T_\infty}=\frac{\cosh\left[m(L-x)\right]}{\cosh(mL)}$$

where $m=\sqrt{\frac{2h}{\lambda_2 t}}$. [35%]

(ii) Using your answers to parts (a) and (b)(i), sketch the complete temperature profile from the centre of the fuel rod to a fin tip. Assume that $\lambda_2=\lambda_1$. Label any important features in the profile. [20%]

(iii) Discuss the factors which might affect the assumption of constant heat transfer coefficient h. [15%]

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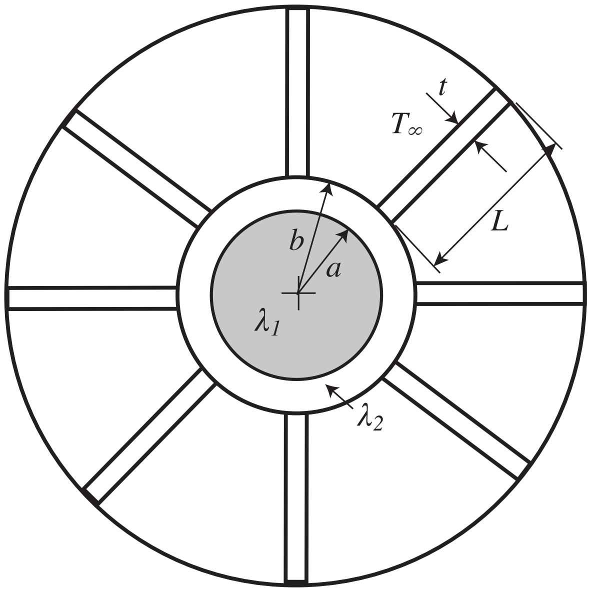

Fig. 2

3 A cylindrical perfusion tube of inner radius R is used to sublimate a substance present on the tube walls into an air stream, as shown in Fig. 3. The flow of air can be considered as steady, fully developed, and laminar, so that the axial velocity is $u = 2u_b\left(1-(r/R)^2\right)$, where r is the radial coordinate, and u_b is the bulk velocity. The mass fraction of the sublimating substance is sufficiently small so that the total mass flow rate through the tube and the mixture density ρ are not affected. The gradient of species mass fraction in the axial direction, z, is assumed to be constant. The diffusion coefficient for the sublimating species in air is given as $\mathscr{D}$.

(a) Using a species balance across an element dz, for the substance mass fraction Y, show that the radial mass diffusion flux of the substance at the tube surface, j_s, is constant and equal to $\frac{1}{2}\rho u_b R\frac{dY}{dz}$. [10%]

(b) Starting from the differential species conservation equations in cylindrical coordinates,

$$u\frac{\partial Y}{\partial z} + v\frac{\partial Y}{\partial r} = \mathscr{D}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial Y}{\partial r}\right) + \frac{\partial^2 Y}{\partial z^2}\right]$$

show that :

$$Y - Y_c = \frac{u_b r^2}{2\mathscr{D}}\frac{dY}{dz}\left(1-\frac{r^2}{4R^2}\right)$$

where Y_c is the mass fraction at the centreline. [30%]

(c) Show that the bulk mass fraction of the substance Y_b is given as: [20%]

$$Y_b = Y_c + \frac{7}{48}\frac{u_b R^2}{\mathscr{D}}\frac{dY}{dz}$$

(d) Calculate the radially diffusive flux at the surface $r = R$, and compare it to your answer in (a). Discuss why the flux does not depend on $\mathscr{D}$. [20%]

(e) Show that the non-dimensional mass transfer coefficient is given by the Sherwood number below:

$$\text{Sh} = h_s(2R)/\mathscr{D} = 48/11$$

Comment on your answer compared to the equivalent Nusselt number for a laminar tube. [20%]

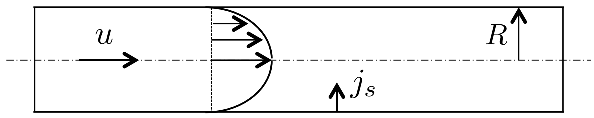

Fig. 3

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4 (a) Show that the net rate of radiative heat loss $\dot{Q}_i$ from a grey diffuse surface i of area A_i and total emissivity ε_i i, s

$$\dot{Q}_i = \frac{\varepsilon_i A_i \left(E_{bi} - J_i\right)}{(1-\varepsilon_i)}$$

where E_{bi} is the black body radiation and J_i is the radiosity of the surface. [15%]

(b) If a surface i is radiating to N others, show that the explicit expression for its radiosity J_i is

$$J_i = \frac{1}{1 - F_{ii}\left(1-\varepsilon_i\right)} \left[(1-\varepsilon_i) \sum_{j=1, j\neq i}^{N} F_{ij} J_j + \varepsilon_i E_{bi} \right]$$

where F_{ij} is the view factor between the i^{th} and j^{th} surface. [25%]

(c) Figure 4 shows two parallel plates of equal area in a large enclosure at temperature T_3 (surface 3). Surface 1 is held at T_1 and has a total emissivity ε_1 of 0.8. Surface 2 is held at T_2 and has a total emissivity ε_2 of 0.5. Both surfaces are grey and diffuse.

(i) The view factor between the parallel surfaces is $F_{12} = F_{21} = 0.2$. Evaluate the remaining view factors in the system. [10%]

(ii) Using the equation derived in part (b) find the 2×2 matrix A and vector C, such that

$$AJ = C$$

where J is the vector of the unknown radiosities. Express your answer in terms of the temperatures given, as well as the Stefan-Boltzmann constant, σ. [30%]

(iii) Find the radiant heat lost by Surface 1 per unit surface area. Express your answer as a function of the same parameters as in Part (b)(ii). [10%]

(d) Surface 1 is now subject to convective heat loss to a bulk flow with temperature T_∞, and its net heat transfer flux is q_1. Describe how you would modify the equations in Part (b)(ii) to solve for T_1 and J_1. [10%]

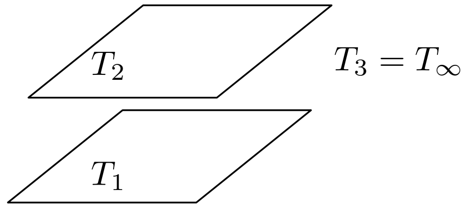

Fig. 4

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