1.

A single phase concentrated winding of N turns produces an mmf distribution around the airgap of a single-phase induction motor as shown in Fig. 1 when carrying a current of I amps.

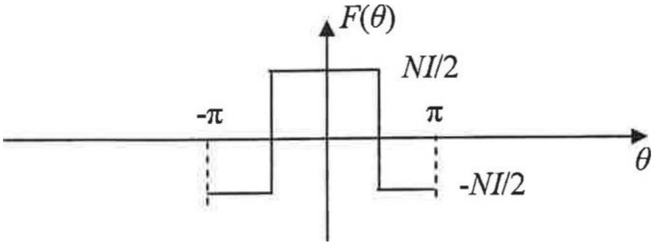


Fig. 1

a) Show that the fundamental component of this mmf distribution is

$$F(\theta) = \frac{2NI}{\pi}cos\theta$$

Fourier expansion of a square wave is

$$f(\theta) = 4/\pi \sum_{n=1.3.5..}^{\infty} \frac{\cos(n\theta)}{n}$$

Assuming only the first term is of interest and that the amplitude is NI/2 then we obtain:

$$F(\theta) = \frac{2NI}{\pi}cos\theta$$

b) Assuming that the winding current is given by $I(t) = \hat{I}cos\omega t$, derive an expression for the mmf as a function of θ and t, and hence show that a single-phase induction motor produces two counter-rotating mmf waves of equal amplitude.

Substituting in for I using $I(t) = \hat{I}cos\omega t$ gives us

$$F(\theta, t) = \frac{2N\hat{I}cos\omega t}{\pi}cos\theta$$

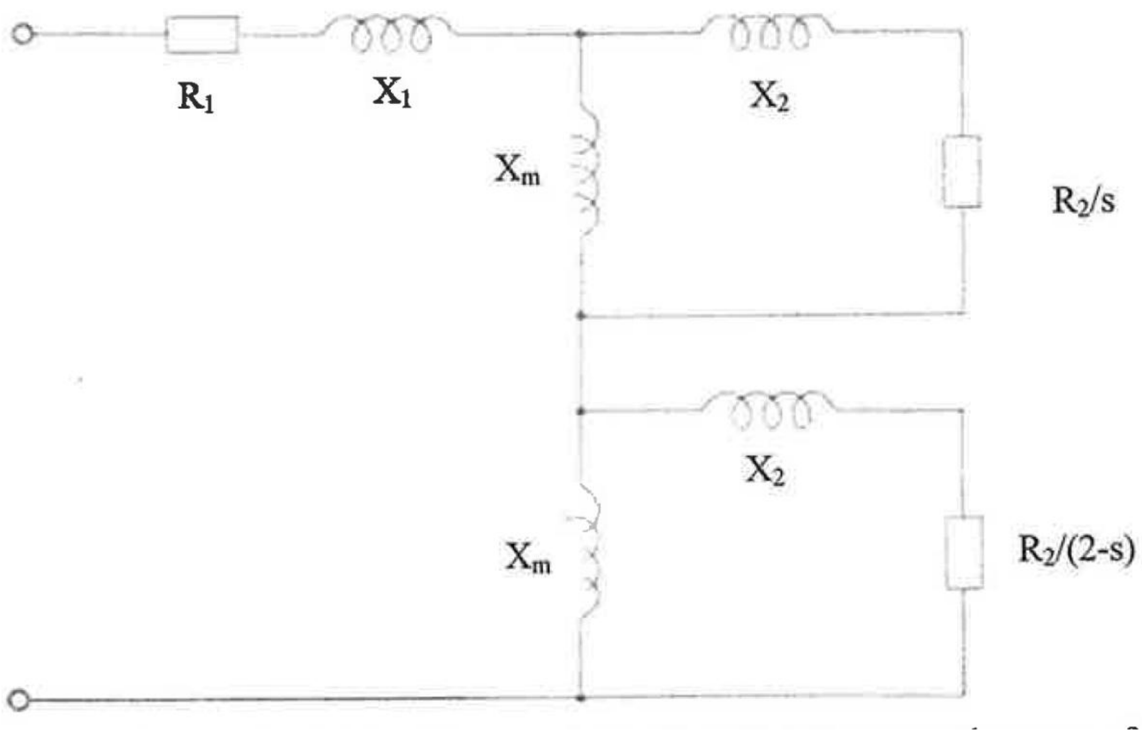
Which is $= \frac{N\hat{I}}{\pi}[\cos(\omega t - \theta) + \cos(\omega t + \theta)]$ which is two rotating waves one in the positive x direction the other the negative x direction

c) By considering a single-phase induction motor as the superposition of two induction motors, one with a forwards-rotating mmf wave and the other with a backwards-rotating mmf wave, draw the equivalent circuit of the single-phase induction motor. and sketch a typical torque-speed curve.

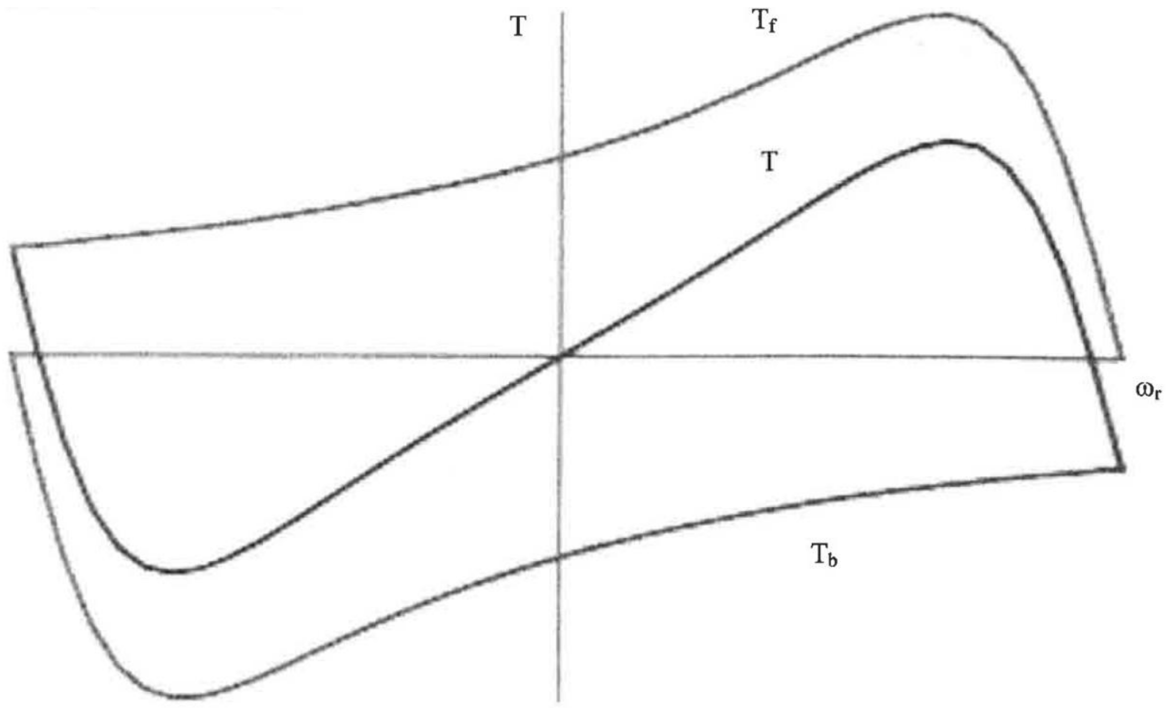
The two waves have different slips the forward slip is $s_f = \frac{\omega_s - \omega_r}{\omega_s} = s$

The backward is $s_b = \frac{-\omega_s - \omega_r}{-\omega_s} = 2 - s$

And the equivalent circuit is



The torque speed curve is



d) Given that the forward and backward torques are

$$T_f = \frac{1}{\omega_s}[I_1]^2 Re(Z_f)$$

$$T_b = \frac{1}{\omega_s}[I_1]^2 Re(Z_b)$$

Find an expression for the value of s at which torque is zero.

. Under what conditions will the motor fail to run altogether even if given a start.

$$T_f = \frac{1}{\omega_s}|I_1|^2 Re(Z_f) \quad (15.8)$$

$$T_b = \frac{1}{\omega_s}|I_1|^2 Re(Z_b) \quad (15.9)$$

$$T = T_f - T_b \quad (15.10)$$

Where Z_f and Z_b are the impedances of the forward and backward branches, respectively.

The speed at which the motor generates no torque, s_0 can be determined by equating the torques produced by the forward and backward fields. Using equations 15.9 and 15.10, this amounts to

$$Re(Z_f) = Re(Z_b)$$

From which it can be shown that

$$s_o = 1, \qquad \pm\sqrt{1-\left[\frac{R'_2}{X'_2+X_m}\right]^2} \qquad (15.11)$$

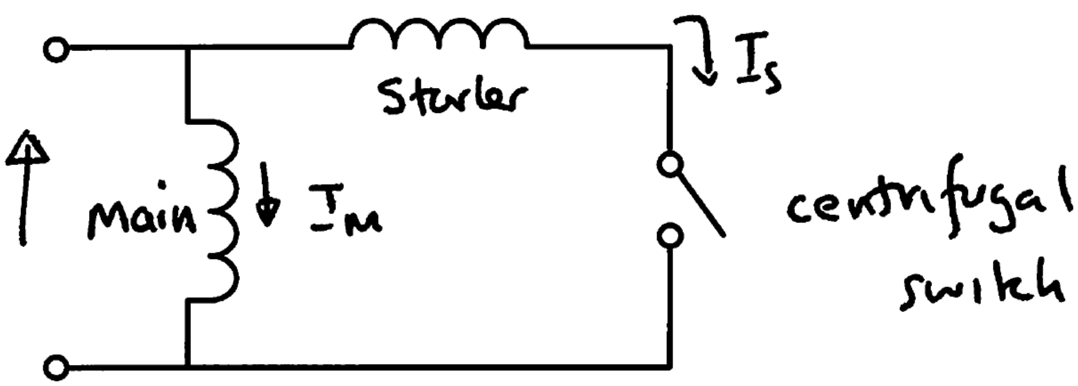
The first solution signifies that there is no torque at starting, while the two other solutions represent the forward and backward speeds at which the motor runs at no load. We can see from equation 15.11 that if $R'_2 > (X'_2 + X_m)$, then there is no real solution for the running speed. The physical consequence of this is that the motor will not run even is given a start.

e) At zero speed the nett Torque is zero. How does the motor start?

1) Explain the principles of using a starter winding to provide a starting torque.

Give at least two methods of providing a starting torque and compare and contrast the merits of each.

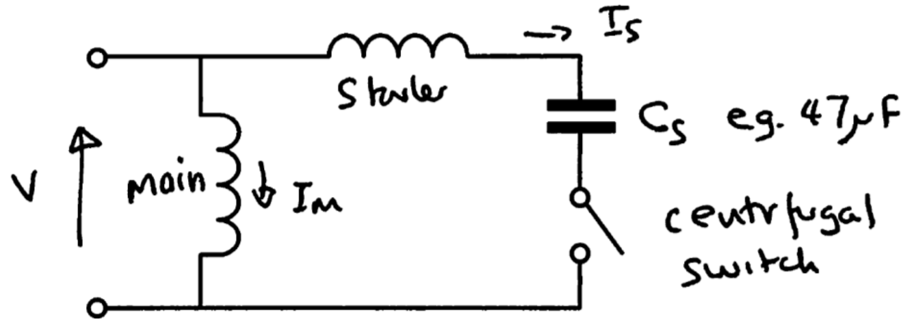
Split phase



Uses starter winding which is switched out

Capacitor start

The phase difference between the current in the main and starter windings can be increased to nearer 90° by inserting a capacitor in series with the starter winding. As for the split-phase motor, the capacitor and starter windings are switched out at some speed so the winding can be rated for intermittent use.



Plus other variations e.g.

The shaded pole motor is a special case of the split-phase motor, in which the auxiliary winding takes the form of a simple copper ring left permanently in the magnetic circuit. The stator is of a salient-pole construction similar to that used in d.c. machines and the thick copper rings embrace one side of the pole tip of each pole in the motor.

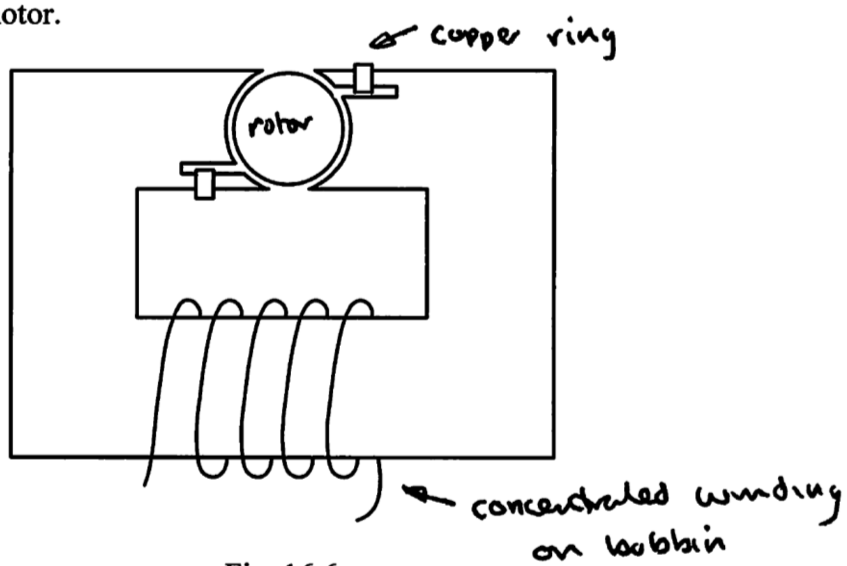
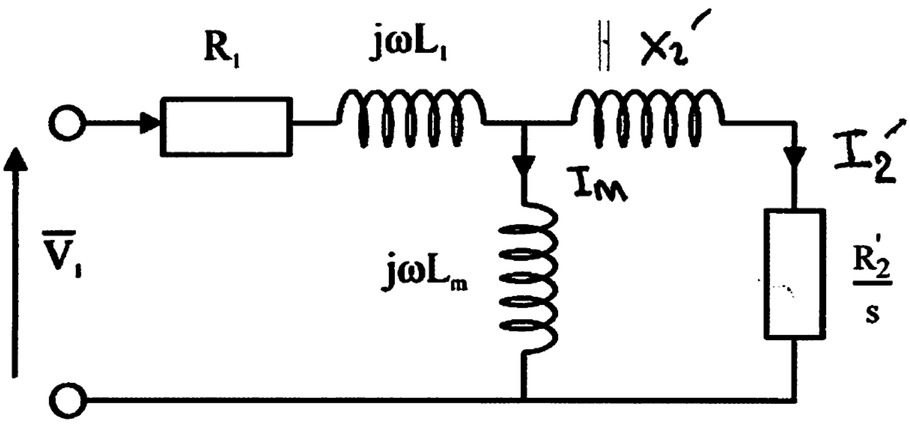


Fig. 16.6

2

A three-phase induction motor is driving a spinning load at the rated output power P_{out} = 8.5 kW at 1458 rpm. The motor has 4 poles and the stator is delta connected. The rated stator voltage is 360 V, 50 Hz. At the rated voltage, the parameters of the motor are: stator resistance R_1 = 1.4 Ω, stator leakage reactance X_1 = 2.5 Ω, referred rotor resistance R_2 = 1.047 Ω, referred rotor leakage reactance X_2 = 4.4 Ω, and the magnetising reactance X_m = 85 Ω.

Per-phase equivalent circuit



- R_1 - Stator resistance
- ωL_1 - Stator leakage reactance
- ωL_m - Magnetizing reactance
- ωL_2 - Rotor leakage reactance
- $\frac{R_2'}{s}$ - Referred rotor resistance (s is slip)

(a) Ignoring only the iron losses of the motor find:

P = 2

F=50*60

S= f/P-1438/F

$$X_m = 85$$

$$Z_{in} = \frac{jX_m(\frac{R_2'}{s} + jX_2')}{\frac{R_2'}{s} + jX_2' + jX_L} + R_1 + jX_1$$

Subbing in we get Zin = 35.5 at an angle of 31.8 degrees.

(i) The power factor;

Hence power factor = cos 32.2 which is 0.85 lagging

(ii) the efficiency;

$$I_1 = \frac{V1}{Zin} = \frac{360}{35.3@32.3} = 10.1\ @ -31.8$$

Pin =3*I*V= 3*360*10.1@-31.8 = 9305.6 W
Power out is given in the question at 8.5k w
Hence efficiency is 8500/9305 = 91.3%

(iii) the mechanical output power loss.

Electrical loss in R1 is 431.8W
Electrical loss in R2 = 248.5W
➔Total electrical loss is 631.6 W
(Input power)9305.6 – (motor power) 8500 – (electrical loss) 631.6 w…difference is 174w equals mechanical loss

(b) If the thermal capacity of the motor is *C* and the dissipation coefficient is *k*,

(i) derive an expression for the temperature of the motor after time *t* when dissipating a loss power *P* in an ambient temperature Θ_0;

In time δt, $P\delta t = C\delta\theta + k\theta\delta t$

where C = heat stored per °C rise (J/K)
$1/k$ = 'thermal resistance' to surroundings
k = dissipation coefficient (W/K)

Case 1 ***Motor switched on from cold; P constant with time thereafter***

At $t = 0, \theta = 0$ so that

$$\theta = \frac{P}{k}\left(1 - e^{-t/\tau}\right)$$

where $\tau = C/k$ - **'thermal time constant'**

At $t = \infty$, $\theta = \theta_\infty = P/k$

or $\theta = \theta_\infty \left(1 - e^{-t/\tau}\right)$

(ii) if C = 4000 J K^{-1} and k = 10 W K^{-1} determine the temperature of the motor after 100 s when operated as described above in part (a). Assume that the starting motor temperature is the ambient temperature of 40 °C;

where $\tau = C/K$ **- 'thermal time constant'**

total P = 806 W from previous part

$$\theta_m = \frac{P_{in}}{k}$$

$$\theta_1 = \theta_m(1 - e^{-T_1/\tau})$$

$$temp = \theta_1 + ambient$$

Hence temperature after 100s is 57.8 degrees

(iii) if the motor is operated as described in part (ii), then stopped and allowed to cool to 50 °C before running for a further 1000s, determine the peak temperature of the motor;

same as part 1 except starting temperature is now 50 degrees $\theta_1 = 50 - ambient = 10$

$$\theta_m = \frac{P_{in}}{k}$$

$$\theta_2 = \theta_1 + (\theta_m - \theta_1)(1 - e^{-T_1/\tau})$$

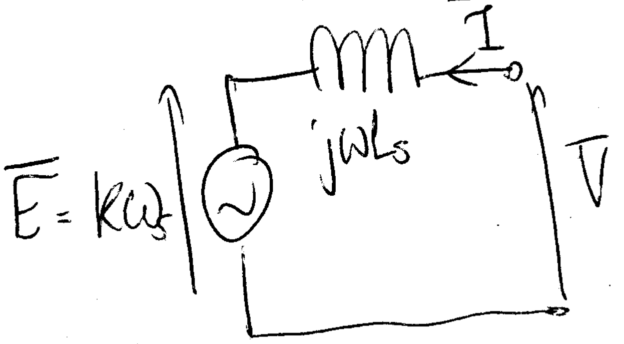
$$temp = \theta_2 + ambient$$

Hence temperature after 1000s is 115 degrees

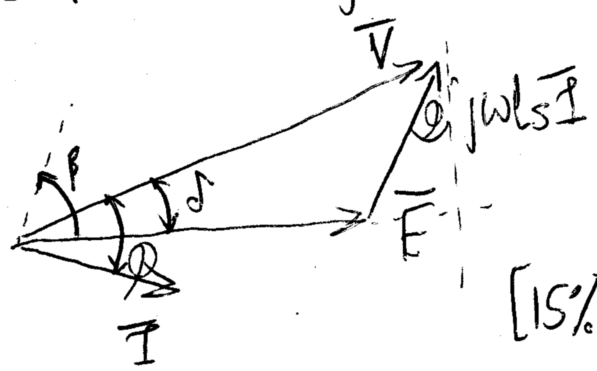
(iv) the maximum temperature allowed for the motor is 100 °C. If the motor exceeds this temperature, what measures should be taken?

Reduction of power losses but better would be to increase the dissipation coefficient

3 (a) The sinusoidal BLDCM has a balanced 3ϕ stator winding which produces a rotating magnetic field in the air gap when connected to an inverter which produces balanced 3ϕ voltages. The rotor has permanent magnets mounted on it which are radially magnetized, and produce an air gap field of the same pole number as the stator. Thus, the BLDCM is similar to the synchronous machine except that the rotor field is fixed. Other than that its principles of operation are identical.



$\bar{V} = \bar{E} + j\omega L_s \bar{I}$



[15%]

(b) $P = 3VI\cos\phi$ $\qquad \omega L_s I\cos\phi = E\sin\delta \qquad E = k\omega_s$

$= \dfrac{3VE\sin\delta}{\omega L_s} \qquad \omega L_s I\sin\beta = V\sin\delta$

$= \dfrac{3E\,\omega L_s I\sin\beta}{\omega L_s} = 3EI\sin\beta = 3k\omega_s I\sin\beta$

$P = T\omega_s \Rightarrow T = \dfrac{3k\omega_s I\sin\beta}{\omega_s} = 3kI\sin\beta$

k: emf constant

I: phase current

β: torque angle.

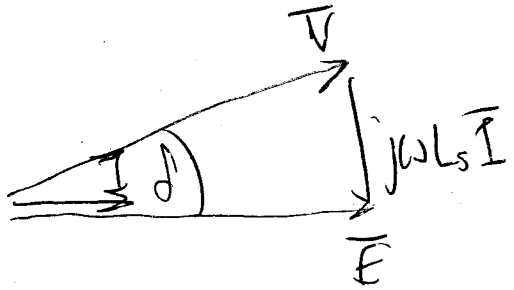
[10%]

For a given amount of output power, from $T = 3kI\sin\beta$ I is minimized when $\sin\beta = 1$, so $\beta = 90°$. Minimising I minimises I^2R losses, hence maximises efficiency. [10%]

(c) Because the BLDCM is a type of synchronous machine, it is constrained to rotate at $\omega_s = 2\pi f/p$. Thus in a variable speed drive it is essential to be able to vary f. But as ω_s increases, because the rotor field is fixed, $E = k\omega_s$ must increase in proportion to ω_s, hence the terminal voltage V has to be increased too. Hence VVVF control is needed.

By measuring the drive speed (Hall effect sensors) and rotor position, the drive can determine the error between the reference and actual rotor speeds. The error signal feeds into a PID controller which provides a reference for the inverter voltage magnitude. Knowledge of the phase currents (current sensors) enable torque to be estimated if a seperate torque loop is used. Alternatively, the micro controller can calculate the required phase of the inverter voltages. The inverter connects to the BLDCM which completes the drive system. [15%]

d) (i) $T_{rated} = 3kI_{rated} = 3 \times 1.5 \times 30 = \underline{135 \text{ Nm}}$



At ω_{rated}, $V = V_{max} = 415/\sqrt{3}$ and $I = I_{rated}$

$E = k\omega_s = \frac{k\omega}{p} = 0.75\omega \qquad \omega L_s I_{rated} = \omega \times 20 \times 10^{-3} \times 30 = 0.6\omega$

$V_{max}^2 = (0.75\omega)^2 + (0.6\omega)^2 \Rightarrow \left(\frac{415}{\sqrt{3}}\right)^2 = \omega^2(0.75^2 + 0.6^2)$

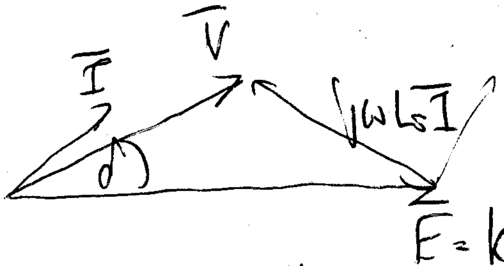
$\omega = 249 \text{ rad s}^{-1}$ at rated speed, so $\omega_{rated} = \frac{249}{p} = \underline{125 \text{ rad s}^{-1}}$ $\underline{(1191 \text{ rpm})}$

$P = T_{rated}\,\omega_{rated} = 135 \times 125 = \underline{16.8 \text{ kW}}$

$\tan\delta = \frac{\omega L_s I_{rated}}{k\omega/p} = \frac{0.6}{0.75}$ so $\delta = \underline{38.7°}$ (for all speeds up to rated)

[15%]

(ii) Now $E \gg V_{max}$ so



$\overline{I}$ no longer in phase with $\overline{E}$

so it produces a component of field that is cancelling the rotor field. Hence field weakening and reduced torque.

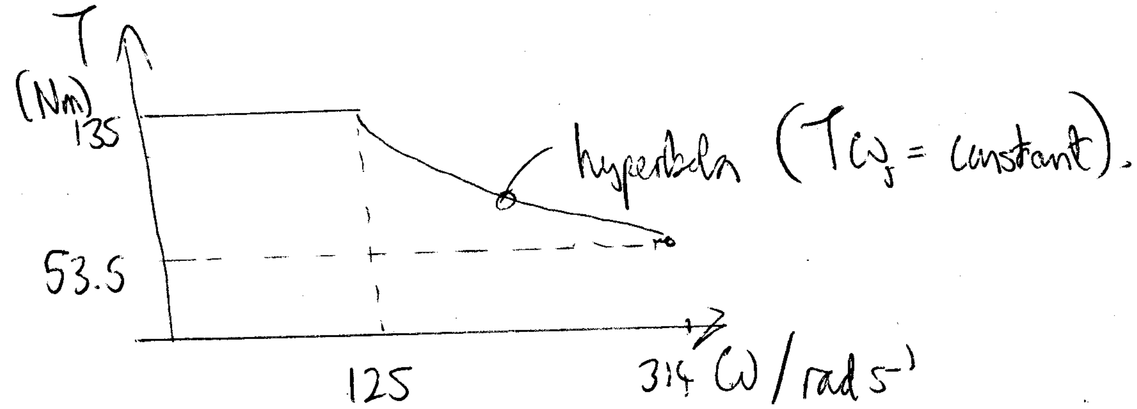
$P = \frac{3V_{max}\,k\omega_s \sin\delta}{\omega L_s} = \frac{3V_{max}\,k \sin\delta}{pL_s}$ = constant if δ is fixed

Thus constant P will be 16.8 kW. Maximum drive speed when $f = 100$ Hz

so $\omega_{s_{max}} = \frac{2\pi \times 100}{2} = 314$ rad s^{-1} (3000 rpm)

$$P = T\omega_{s_{max}} \quad \text{so} \quad T = 16800/314 = 53.5 \text{ Nm}. \qquad [25\%]$$

iii)



[10%]

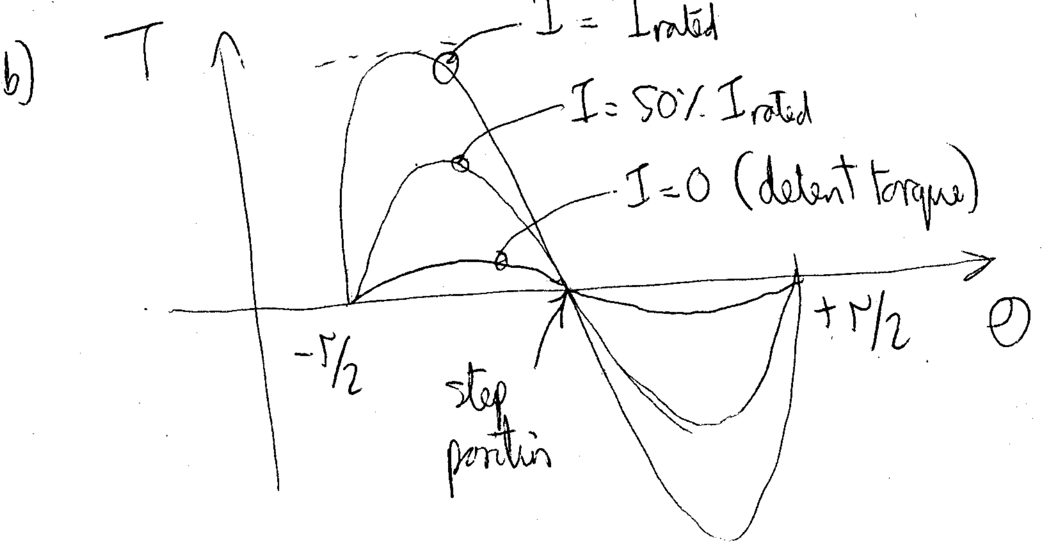
e) Iron losses scale ~ with f^2, and so ω_s^2 hence more significant at high speed. Estimate $\hat{B}$ in teeth and core enables loss density to be found from manufacturers' data. Multiply this by total mass of teeth, core enables iron loss to be estimated. [10%]

4 a) The two rotor wheels are offset by exactly half a rotor tooth pitch, so when e.g. phase A is excited, the N-pole rotor wheel will have its teeth aligned with the S-pole of phase A and vice versa. At this point the rotor wheels are both misaligned by 1/4 rotor tooth pitch from the phase B poles. Thus, on switching between phase A and phase B, the rotor will move to align itself with the phase B poles i.e. it moves by 1/4 rotor tooth pitch. With 50 rotor teeth there will be 200 steps of 1/4 rotor tooth pitch in one revolution, so $\Delta\theta = 360/200 = 1.8°$.

Stepper motors have well-defined positions so that they can be operated without sensors/feedback control, making them easy and cheap to use. To increase the precision can either use a speed reduction gearbox or half-stepping or micro stepping.

[20%]

b)



r is rotor tooth pitch $(360/50 = 7.2°)$

Detent torque is the reluctance torque that occurs when the motor is unexcited. It can be a useful feature in 'remembering' the position when the motor loses power.

[15%]

c) Restoring torque $T_m = -\hat{T}\sin(N_t\theta)$ where N_t is the number of rotor teeth per wheel (50), $\hat{T}_m$ is peak restoring torque.

Linearising about $\theta = 0$ gives $T_m \approx -\hat{T} N_t \theta$

Ignoring damping, equation of motion is $T_m = J\dfrac{d^2\theta}{dt^2}$ where J is the moment of inertia

so $$\frac{d^2\theta}{dt^2} = -\frac{\hat{T} N_t}{J}\theta = -\omega_0^2\theta$$

. Equation of Simple harmonic motion with $\omega_0 = \sqrt{\dfrac{\hat{T} N_t}{J}}$

so $$f_0 = \frac{1}{2\pi}\sqrt{\frac{N_t \hat{T}}{J}}$$ [15%]

d) i) If the stepping frequency f_s matches f_0 then this will excite the natural resonance of the motor, causing the rotor to overshoot and miss steps. Thus the motor becomes uncontrollable.

The step frequency f_s is related to the rotor speed. As there are 200 steps/revolution, in one second f_s steps occur and so the rotor moves $\dfrac{f_s}{200}$ revolutions per second $= \dfrac{60 f_s}{200}$ rpm.

We are told that 60 rpm should be avoided, so the corresponding f_s is given by $\dfrac{60 f_s}{200} = 60$ i.e. $f_s = 200$ Hz should be avoided.

This is because $f_0 = 200\,\text{Hz} = \frac{1}{2\pi}\sqrt{\frac{50 \times 0.5}{J}}$ giving $\underline{J = 1.58 \times 10^{-5}\ \text{kgm}^2}$

[15%]

ii) With load, total $J_t = 1.58 \times 10^{-5} + 4 \times 10^{-5} = 5.58 \times 10^{-5}\ \text{kgm}^2$.

At $I = I_{rated}$ $f_0 = \frac{1}{2\pi}\sqrt{\frac{50 \times 0.5}{5.58 \times 10^{-5}}} = 107\ \text{Hz}$ with corresponding

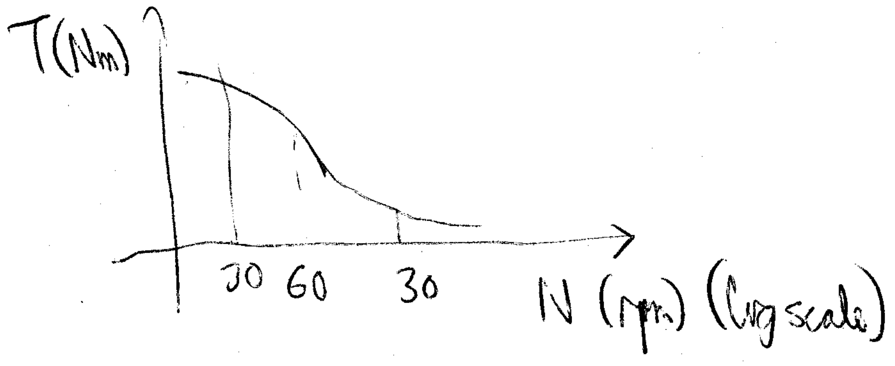
speed $\frac{60 \times 107}{200} = \underline{32\ \text{rpm}}$, which should be avoided.

At $I = 50\%\, I_{rated}$ $\hat{T} = \frac{0.1 + 0.5}{2} = 300\ \text{mNm}$

[15%]

$f_0 = \frac{1}{2\pi}\sqrt{\frac{50 \times 0.3}{5.58 \times 10^{-5}}} = 82.5$, corresponding speed is $\underline{24.8\ \text{rpm}}$.

iii) This could be exploited by initial acceleration taking place at $I = I_{rated}$ until close to 32 rpm, then reduce to 50% I_{rated} (so speed to avoid has been exceeded) for remainder of the acceleration [10%]



As drive speeds up less torque is available. Thus heavy acceleration only possible at low speeds. In open-loop control it is important to specify maximum accelerations at different speeds otherwise missed steps can occur.

[10%]