

EGT2  
ENGINEERING TRIPOS PART IIA

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Tuesday 6 May 2025 14.00 to 15.40

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**Module 3B4**

**ELECTRIC DRIVE SYSTEMS**

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

**STATIONERY REQUIREMENTS**

Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**

CUED approved calculator allowed

Engineering Data Book

**10 minutes reading time is allowed for this paper at the start of the exam.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

**You may not remove any stationery from the Examination Room.**

1 A single phase concentrated winding of  $N$  turns produces an mmf distribution around the airgap of a single-phase induction motor as shown in Fig. 1 when carrying a current of  $I$  amps.

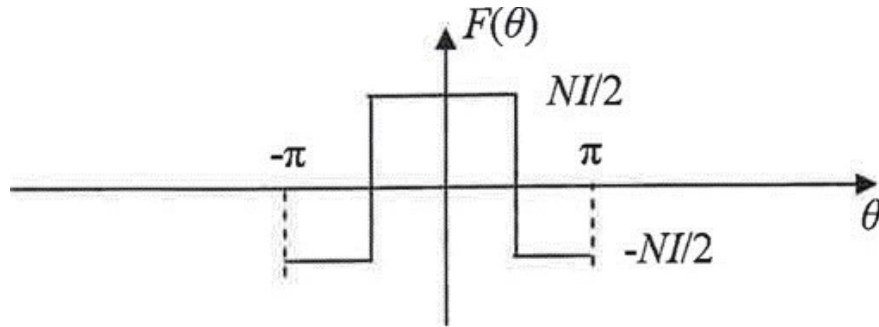


Fig. 1

- (a) Show that the fundamental component of this mmf distribution is [15%]

$$F(\theta) = \frac{2NI}{\pi} \cos \theta$$

- (b) Assuming that the winding current is given by

$$I(t) = \hat{I} \cos \omega t$$

derive an expression for the mmf as a function of  $\theta$  and  $t$ , and hence show that a single-phase induction motor produces two counter-rotating mmf waves of equal amplitude.

[25%]

- (c) By considering a single-phase induction motor as the superposition of two induction motors, one with a forwards-rotating mmf wave and the other with a backwards-rotating mmf wave, draw the equivalent circuit of the single-phase induction motor and sketch a typical torque-speed curve. [25%]

- (d) Given that the forward and backward torques are

$$T_f = \frac{1}{\omega_s} [I_1]^2 \operatorname{Re}(Z_f)$$

$$T_b = \frac{1}{\omega_s} [I_1]^2 \operatorname{Re}(Z_b)$$

(i) Find an expression for the values of slip,  $s$ , at which torque is zero. [15%]

(ii) Under what conditions will the motor fail to run altogether even if given a start.

[10%]

(e) At zero speed the net torque is zero. How does the motor start? Explain the principles of using a starter winding to provide a starting torque. Give at least two methods of providing a starting torque and compare and contrast the merits of each. [10%]

2 A three-phase induction motor is driving a spinning load at the rated output power  $P_{out} = 8.5 \text{ kW}$  at 1458 rpm. The motor has 4 poles and the stator is delta connected. The rated stator voltage is 360 V, 50 Hz. At the rated voltage, the parameters of the motor are: stator resistance  $R_1 = 1.4 \Omega$ , stator leakage reactance  $X_1 = 2.5 \Omega$ , referred rotor resistance  $R_2 = 1.047 \Omega$ , referred rotor leakage reactance  $X_2 = 4.4 \Omega$ , and magnetising reactance  $X_m = 85 \Omega$ .

(a) Ignoring only the iron losses of the motor find:

- (i) the power factor; [20%]
- (ii) the efficiency; [15%]
- (iii) the mechanical loss. [15%]

(b) If the thermal capacity of the motor is  $C$  and the dissipation coefficient is  $k$ ,

- (i) derive an expression for the temperature of the motor after time  $t$  when dissipating a loss power  $P$  in an ambient temperature  $\Theta_0$ ; [10%]
- (ii) if  $C = 4000 \text{ J K}^{-1}$  and  $k = 10 \text{ W K}^{-1}$  determine the temperature of the motor after 100 s when operated as described above in part (a). Assume that the starting motor temperature is the ambient temperature of  $40^\circ\text{C}$ ; [15%]
- (iii) if the motor is operated as described in part b(ii), then stopped and allowed to cool to  $50^\circ\text{C}$  before being run for 1000 s, determine the peak temperature of the motor; [15%]
- (iv) The maximum temperature allowed for the motor is  $100^\circ\text{C}$ . If the motor were to exceed this temperature, what measures should be taken? [10%]

- 3 (a) Explain the principles of operation of the three-phase sinusoidal brushless DC motor (BLDCM). Draw the per-phase equivalent circuit, assuming that the winding resistance is negligible, and draw a phasor diagram showing operation at a lagging power factor. Mark on the angle that gives rise to the power factor, the load angle  $\delta$  and the torque angle  $\beta$ . [15%]
- (b) Using your phasor diagram, or otherwise, show that the torque of a sinusoidal BLDCM may be expressed as  $T = 3kI \sin \beta$ , defining all the terms in the equation. Hence explain why operating the sinusoidal BLDCM with a torque angle of 90 degrees in the context of a drive system gives optimum efficiency. [10%]
- (c) Explain why variable voltage, variable frequency (VVVF) control is needed in variable speed drives that utilise the sinusoidal BLDCM. Hence outline the operation of a drive system based around the sinusoidal BLDCM, making reference to the main system components. [15%]
- (d) A 4-pole, three-phase, star-connected sinusoidal BLDCM has an emf constant of  $1.5 \text{ V s rad}^{-1}$ , its rated phase current is 30 A, its phase inductance is 20 mH and its phase resistance may be neglected. It forms part of a drive system which includes a three-phase inverter with maximum frequency and line-line voltage of 100 Hz and 415 V, respectively.
- (i) Find the rated torque, speed and power of the drive, and the load angle of the BLDCM when operating at rated torque and speed. [15%]
- (ii) At speeds greater than rated the drive operates the BLDCM with a fixed load angle corresponding to that found in part d(i) above. Using a phasor diagram, explain how this results in field-weakening, and enables the drive to operate at greater speeds at the expense of torque. Derive an expression for the resulting output power, and show that with this control method, it is constant. Find the maximum torque available at the maximum speed of the drive system. [25%]
- (iii) Hence construct a torque-speed characteristic for the drive that covers the entire range of inverter frequencies, marking on the key points. [10%]
- (e) Explain why iron losses become more significant at high rotational speeds, and outline the principles of their calculation. [10%]

4 (a) The two stack, two phase hybrid stepper motor, with 50 teeth on each of the two rotor wheels has come to be an industry-standard design. Explain the principles of operation of this type of hybrid stepper motor, and show that its step angle in full-stepping mode is 1.8 degrees. Also explain why this type of motor is very popular in cheap consumer devices that require precise position control. Give two methods for increasing the precision of the position control. [20%]

(b) Sketch typical graphs on the same axes of static torque vs rotor position for the hybrid stepper motor of part (a) at rated current, 50% rated current and zero current. Explain what is meant by *detent torque* and how this can be a useful feature. [15%]

(c) Derive the equation of motion for small rotor oscillations about the step position, and show that the natural frequency  $f_0$  of such oscillations is given by

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{N_t \hat{T}}{J}}$$

defining all the terms in this expression. [15%]

(d) A standard hybrid stepper motor is to be used in full stepping mode with one phase excited at a time. Its peak static torque is 0.5 Nm at its rated phase current of 1.5 A, and its detent torque is 0.1 Nm. Assume that its peak static torque varies linearly with phase current between these values. The manufacturers data sheet of the motor states that at rated current, and with the motor running without any mechanical load coupled to it, the speed of 60 rpm should be avoided.

(i) Explain why certain speeds should be avoided, and determine the moment of inertia of the stepper motor. [15%]

(ii) The stepper motor is to be used to drive a mechanical load of moment of inertia  $4 \times 10^{-5} \text{ kg m}^2$ . Find the speeds that should be avoided assuming operation at rated current and 50% of rated current. [15%]

(iii) Suggest how these results could be used to arrive at a means for accelerating the load up to its final speed of 40 rpm. [10%]

(e) Sketch a typical torque-speed characteristic for a hybrid stepper motor. Referring to your sketch, explain why care has to be taken to specify maximum acceleration and deceleration rates in open-loop stepper motor drive applications. [10%]

**END OF PAPER**