Version MPFS/4

EGT2
ENGINEERING TRIPOS PART IIA

Thursday 26 April 2022 2 to 3.40

Module 3C8

MACHINE DESIGN

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: Module 3C8 data sheet (9 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 Figure 1 shows a pair of identical gears, each consisting of a central disc and 10 radial pegs. The pegs are cylinders of radius r. The length of the pegs is such that there is always one pair of pegs in contact. The distance between the centres of the two gears is R. For the purposes of calculating the contact geometry and kinematics, it can be assumed that r is negligibly small compared with R. The axes A_1 and A_2 about which Gears 1 and 2, respectively, rotate are perpendicular to each other, and both lie in the horizontal plane. The angle between the centre-line of the peg on Gear 1 (labelled C-L) and the horizontal is θ. A constant torque T is applied to Gear 1, which rotates at a constant angular velocity $\dot{\theta}$. Rotation of Gear 1 is clockwise, so that $\dot{\theta}$ is negative. Unit vectors $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ define an orthogonal coordinate system as shown on the figure.

(a) Assuming Hertzian contact theory can be applied to the contact between the pegs, derive an expression for the Hertz stress at the contact in terms of θ, r, R, T, and the Young's modulus E and Poisson's ratio ν of the pegs. Describe also how the contact pressure varies across the contact patch. [15%]

(b) Find an expression, as a function of θ, for the angle between the contact normal of the pegs and the vertical direction. [20%]

(c) Find expressions for:

(i) the sliding velocity at the centre of the contact patch; [35%]

(ii) the spin velocity at the contact. [20%]

(d) Comment on the variation with angular position θ of the power lost due to friction. [10%]

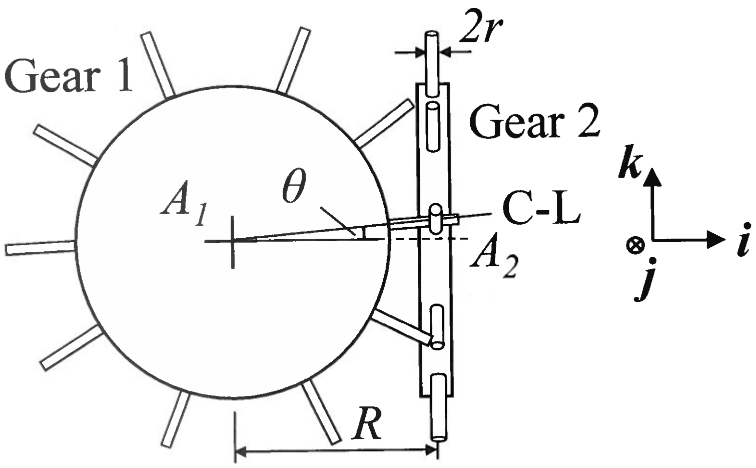


Fig. 1

2 Figure 2 shows a cross-section through part of a compound gearbox used in an automatic transmission. Two input shafts A and B rotate at speeds ω_A and ω_B, respectively, while the output shaft D rotates at ω_D. Element C rotates at angular speed ω_C about the same axis as shafts A, B and D, and carries with it planetary gear elements E which are free to rotate relative to C. The different elements are connected via bevel gears, with tooth numbers a, b, d, e_1 and e_2, as marked on Fig. 2 at the relevant meshing points. Because of the geometry of the gears, note that $a = b$.

(a) Element C is held fixed (i.e. $\omega_C = 0$). Derive expressions for the angular speeds ω_B and ω_D, in terms of ω_A and the tooth numbers a, d, e_1 and e_2. [25%]

(b) Element C is now released. Show that the angular speeds of the three shafts A, B and D can be related by a linear equation of the form

$$\omega_D = \alpha\omega_A + \beta\omega_B$$

finding expressions for α and β in terms of the tooth numbers a, d, e_1 and e_2. [45%]

(c) With element C released and with $\omega_A = 2\omega_B$, find an expression, in terms α and β, for the power input through shaft A as a proportion of the output power through shaft D. [30%]

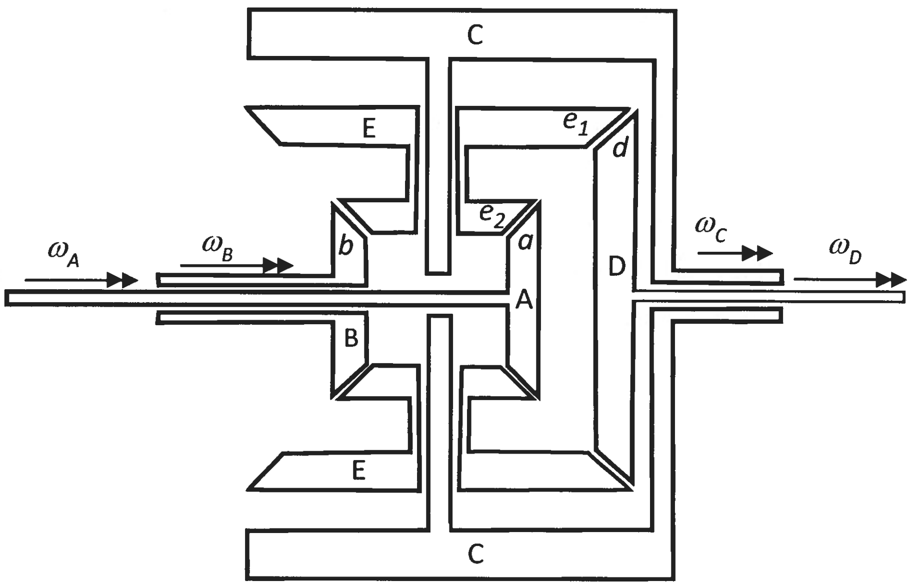


Fig. 2

(TURN OVER

3 A car has mass $m = 1500$ kg, rolling resistance coefficient $C_r = 0.02$, frontal area $A = 2.0$ m^2, aerodynamic drag coefficient $C_d = 0.3$, and wheel radius $R = 0.35$ m. The force F resisting forward motion at constant speed V on horizontal ground is:

$$F = mgC_r + \frac{1}{2}\rho A C_d V^2$$

where g is the gravitational force per unit mass and air density $\rho = 1.2$ kg m^{-3}. The vehicle is driven by an electric motor with the output characteristic shown in Fig. 3, where rpm means revolutions per minute. The permitted operating region is indicated by the shaded region.

(a) The electric motor drives the wheels of the vehicle through a single fixed speed ratio.

(i) Calculate the maximum possible speed of the vehicle on horizontal ground and the corresponding speed ratio (motor speed : wheel speed). [25%]

(ii) For the speed ratio found in (a)(i), use numerical integration of data presented on a plot with axes V and F, or otherwise, to calculate an approximate value for the minimum time taken to accelerate the car from rest to 80% of the maximum speed. [35%]

(b) It is decided to equip the vehicle with a gearbox to include a second speed ratio, to reduce the time calculated in (a)(ii). The vehicle initially accelerates from rest in the new ratio. The speed ratio is then changed to the ratio found in (a)(i) before the vehicle reaches maximum speed.

(i) Choose a suitable new ratio, explaining your reasoning using qualitative arguments and stating any assumptions made. [20%]

(ii) Calculate an approximate value for the new minimum time taken to accelerate from rest to 80% of the maximum vehicle speed, and state the vehicle speed at which the ratio should be changed. [20%]

(cont.

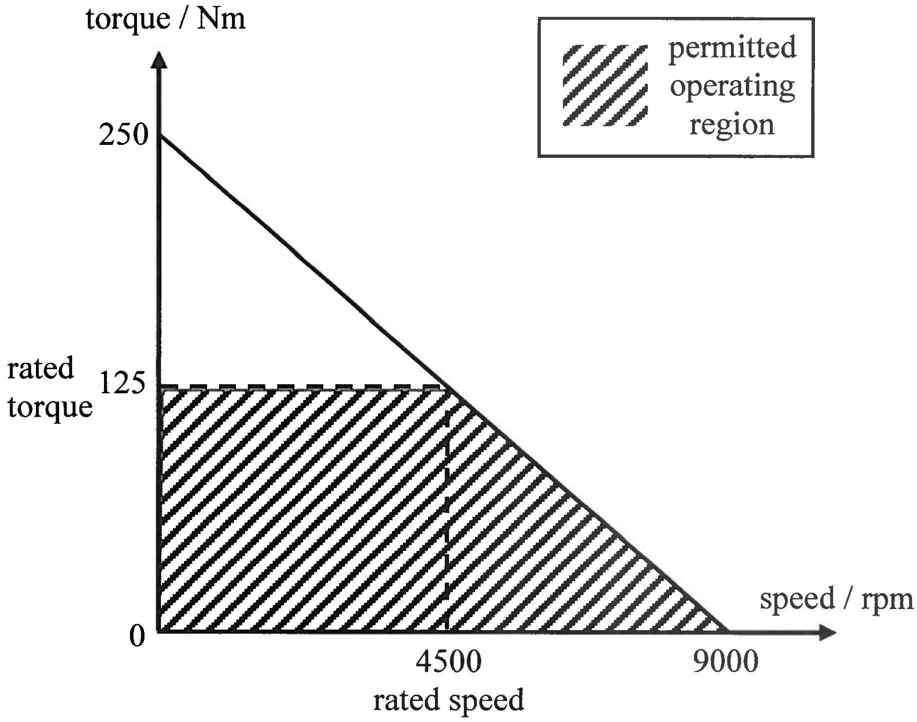


Fig. 3

(TURN OVER

4 (a) A torque T causes surface failure of two accurately-made spur gears, both with 38 involute teeth. The gears have module m, pressure angle $\phi = 20°$, and addendum $a = 1.2m$, which gives a contact ratio of 2. Assuming the same failure mechanism, find the expected safe pinion torque, in terms of T, when one of the spur gears meshes with a rack with standard teeth having a pressure angle $\phi = 20°$ and an addendum $a = m$. [50%]

(b) A pair of identical spur gears, with standard pressure angle $\phi = 20°$, is required to transmit power of 50 kW at a shaft speed of 2,000 revolutions per minute for a duration of 100 hours with 90% reliability. Each gear wheel is supported by a single deep groove ball bearing of outer diameter D. The bearings are well lubricated such that the factor $a_{23} = 1$. The gears are required to have the smallest possible distance a between centres subject to $a \geqslant 1.1D$.

(i) Show that a suitable bearing minimises D, subject to exceeding a minimum value of CD, where C is the basic dynamic load rating of the bearing. [30%]

(ii) Find the minimum allowable value of CD and hence select, from the deep groove ball bearings in the data sheet, the bearing that minimises a subject to $a \geqslant 1.1D$. [20%]

END OF PAPER

ENGINEERING TRIPOS Part IIA

Module 3C8 Data Sheet

ELASTIC CONTACT STRESS FORMULAE

Suffixes 1, 2 refer to the two bodies in contact.

Effective curvature $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

where R_1, R_2 are the radii of curvature of the two bodies (convex positive).

Contact modulus $\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$

where E_1, E_2 and ν_1, ν_2 are Young's moduli and Poisson's ratios

	Line contact width $2b$; load P' per unit length	Circular contact diameter $2a$; load P
Semi contact width or contact radius	$b = 2\left\{\frac{P'R}{\pi E^*}\right\}^{1/2}$	$a = \left\{\frac{3PR}{4E^*}\right\}^{1/3}$
Maximum contact pressure ('Hertz stress')	$p_0 = \left\{\frac{P'E^*}{\pi R}\right\}^{1/2}$	$p_0 = \frac{1}{\pi}\left\{\frac{6PE^{*2}}{R^2}\right\}^{1/3}$
Approach of centres	$\delta = \frac{2P'}{\pi}\left[\frac{1-\nu_1^2}{E_1}\left\{\ln\left(\frac{4R_1}{b}\right)-\frac{1}{2}\right\} + \frac{1-\nu_2^2}{E_2}\left\{\ln\left(\frac{4R_2}{b}\right)-\frac{1}{2}\right\}\right]$	$\delta = \frac{a^2}{R} = \frac{1}{2}\left\{\frac{9}{2}\frac{P^2}{E^{*2}R}\right\}^{1/3}$
Mean contact pressure	$\bar{p} = \frac{P'}{2b} = \frac{\pi}{4}p_0$	$\bar{p} = \frac{P}{\pi a^2} = \frac{2}{3}p_0$
	$\tau_{max} = 0.300\,p_0$ at $x = 0, z = 0.79b$	$\tau_{max} = 0.310\,p_0$ at $r = 0, z = 0.48a$ for $\nu = 0.3$
Maximum tensile stress	zero	$\frac{1}{3}(1-2\nu)p_0$ at $r = a, z = 0$

<u>Mildly elliptical contacts</u>

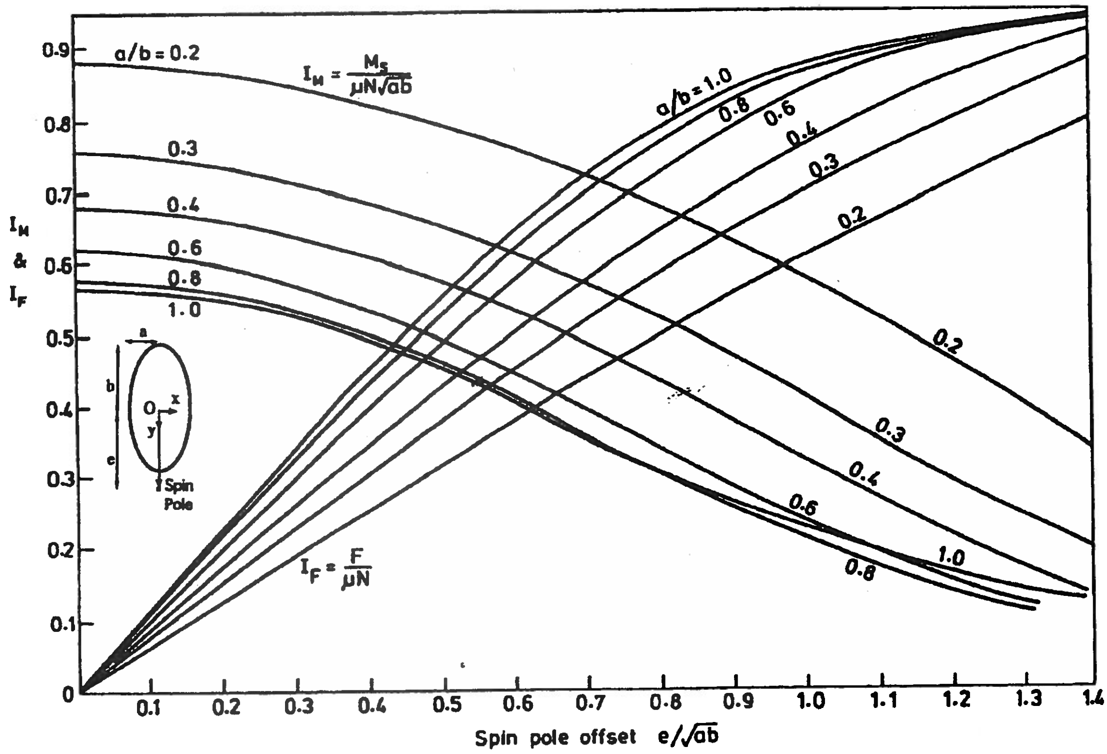
If the gap at zero load is $h = \frac{1}{2}\mathrm{A}x^2 + \frac{1}{2}\mathrm{B}y^2$ and $0.2 < \mathrm{A/B} < 5$ then

ratio of semi-axes $b/a \cong (\mathrm{A/B})^{2/3}$

To calculate the contact **area** or Hertz **stress** use the circular contact equations with $R = (\mathrm{AB})^{-1/2}$ or better $R_e = [\mathrm{AB(A+B)}/2]^{-1/3}$.

For **approach** use circular contact equation with $R = (\mathrm{AB})^{-1/2}$ **(not** R_e)

Hertzian contact frictional losses



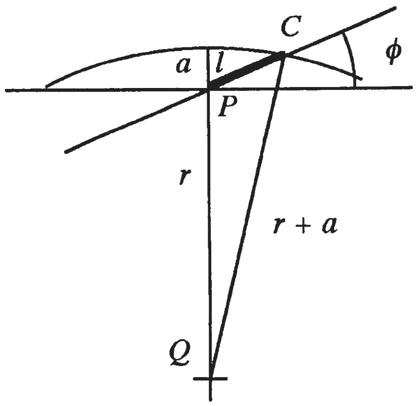
INVOLUTE GEARING

Spur gears

pitch cylinder radii	r	with suffix 1 or 2
base cylinder radii	r_b	with suffix 1 or 2
addendum cylinder radii	r_a	with suffix 1 or 2
number of teeth	N	with suffix 1 or 2
addendum	$a = r_a - r$	
pressure angle	ϕ	

circumferential pitch	$p = 2\pi r/N$
base pitch	$p_b = p\cos\phi$
module	$m = p/\pi = 2r/N$
ratio of contact	r_c
radius of curvature at pitch point	$\rho = r\sin\phi$

Path of contact



$$l = \left\{r^2\sin^2\phi + a(2r+a)\right\}^{1/2} - r\sin\phi$$

For a standard 20° spur wheel with N teeth of module m this becomes

$$\frac{l}{m} = \left(0.02924N^2 + N + 1\right)^{1/2} - 0.1710N$$

Standard tooth forms

Addendum $a = m$, Dedendum $= \frac{7}{6}m$, pressure angle = 20°.

Modules:

0.3 – 1.0 mm in 0.1 mm steps
1.0 – 4.0 mm in 0.25 mm steps
4.0 – 7.0 mm in 0.5 mm steps
7.0 – 16.0 mm in 1.0 mm steps
16.0 – 24.0 mm in 2.0 mm steps
24.0 – 45.0 mm in 3.0 mm steps
45.0 – 75.0 mm in 5.0 mm steps

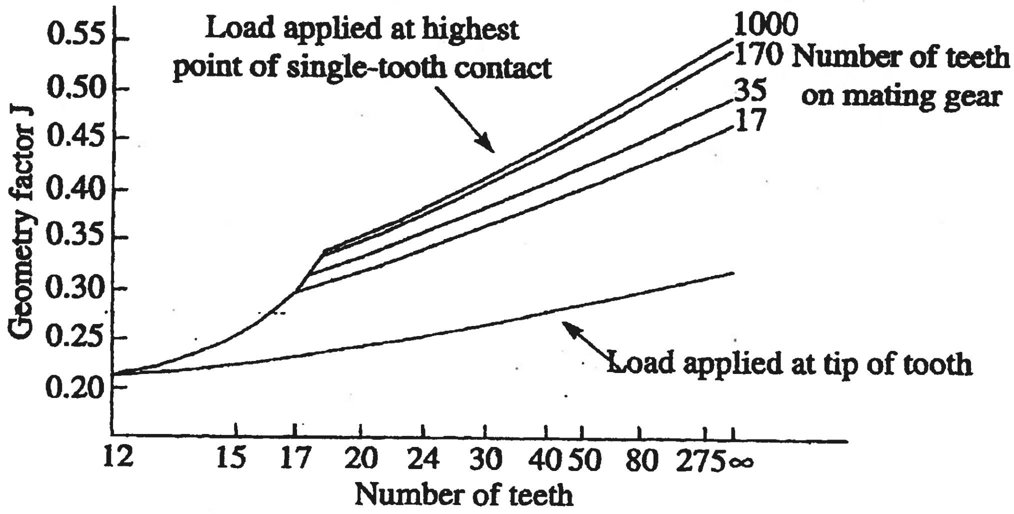
Friction in spur gears

$$\frac{\text{average friction loss}}{\text{power transmitted}} \approx \mu\pi\left\{\frac{1}{N_1}+\frac{1}{N_2}\right\}$$

Tooth failure

Allowable bending stress σ_b according to AGMA guidelines given by $\sigma_b = \frac{P_T'}{Jm}$

where P_T' is force per unit face-width acting tangentially to pitch circle and J given in the figure below for 20° spur gears. Typical values of σ_b shown in table.



Typical allowable tooth stresses (AGMA)

Material	Condition	Bending fatigue strength σ_b (MPa)	Surface fatigue strength σ_s (MPa)
Steel	Through hardened and tempered	170-390	590-1200
	Carburised and case hardened	380-480	1250-1550
Cast iron	As cast	69-90	450-590
Nodular iron	Quenched, annealed and tempered	150-300	500-800
Malleable iron	Pearlitic	70-145	500-650

EPICYCLIC SPEED RULE

$$\omega_s = (1+R)\omega_c - R\omega_a \quad \text{where} \quad R = \frac{A}{S}$$

ROLLING ELEMENT BEARINGS

Fatigue life

$L = a_1 a_{23}(C/P)^p$ p = 3 for ball and 10/3 for roller bearings

Fatigue probability %	10	5	4	3	2	1
Life adjust factor a_1	1	0.62	0.53	0.44	0.33	0.21

Bearing choice

The information on the following pages concerning loads, viscosities and standard bearing sizes and ratings is extracted from the SKF General Bearing Catalogue and is copied with permission. It is SKF copyright and is not to be further reproduced.

MPFS, DJC, JAW
September 2021

Required viscosities and the effect of viscosity ratio on a_{23}

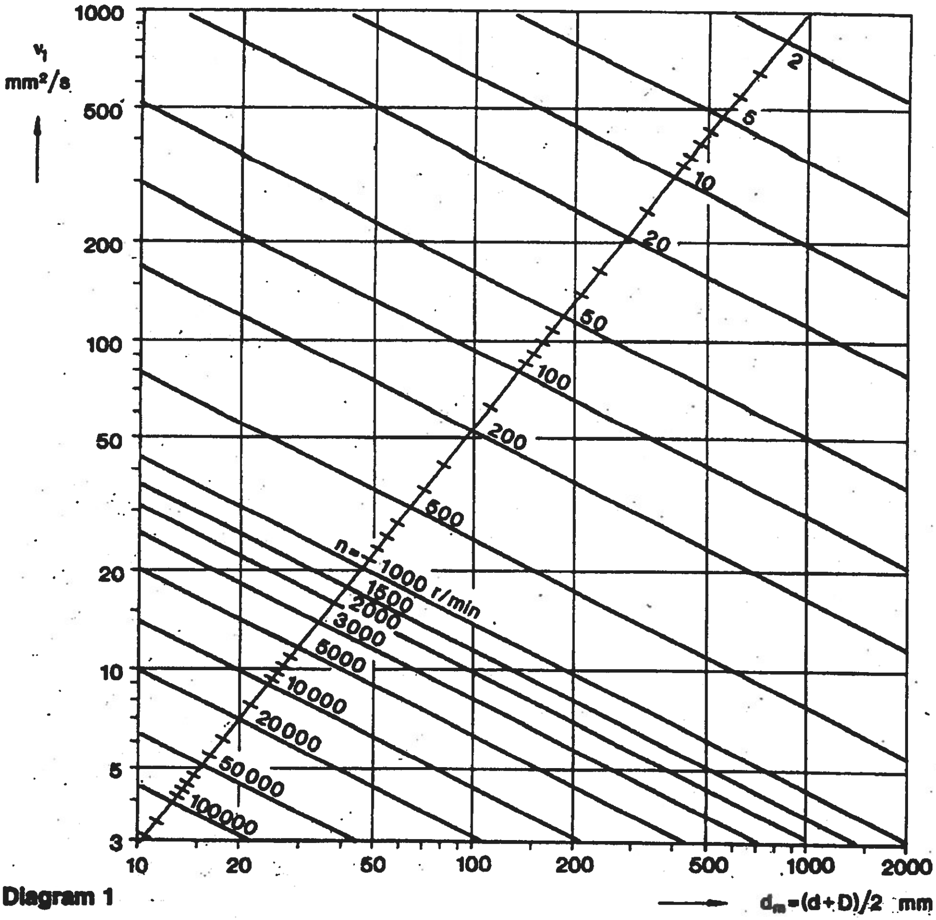


Diagram 1

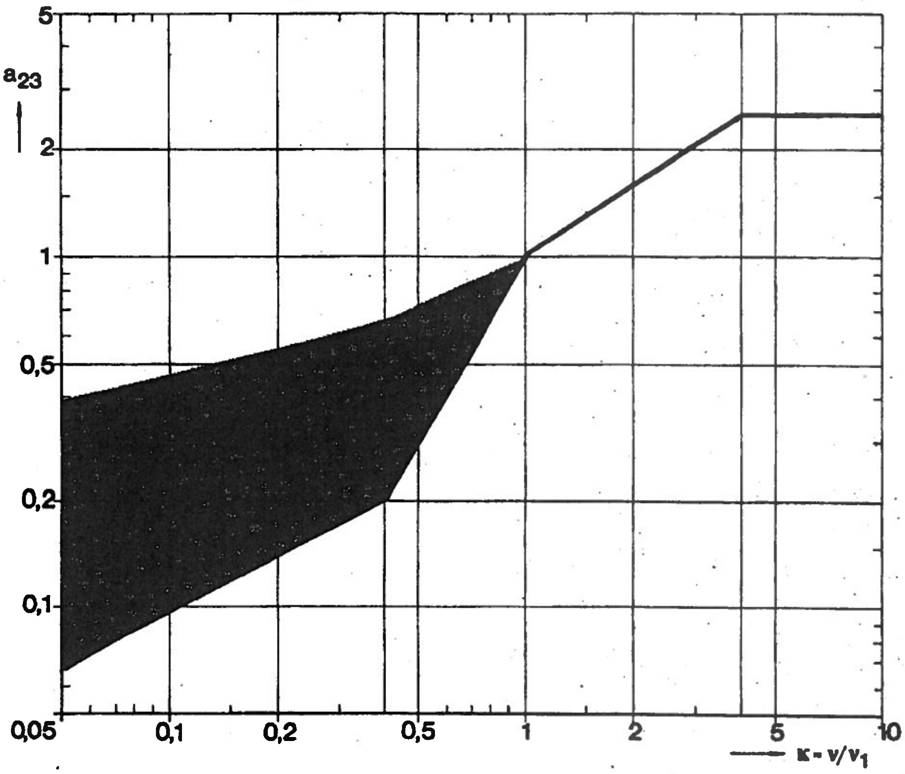
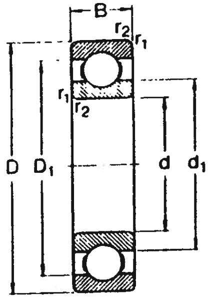
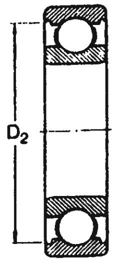


Diagram 3

Deep groove ball bearings
single row
d 35–55 mm



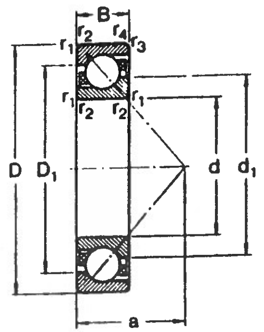
With full outer ring shoulders



With recessed outer ring shoulders

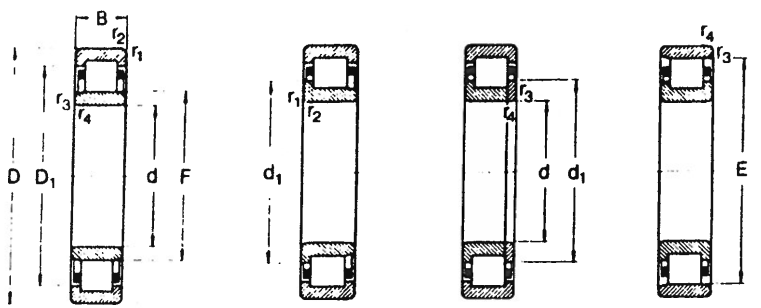
Principal dimensions d	D	B	Basic load ratings dynamic C	static C_0	Fatigue load limit P_u	Speed ratings Lubrication grease	oil	Mass	Designation
mm			N		N	r/min		kg	–
35	47	7	4 750	3 200	166	13 000	16 000	0,030	61807
	55	10	9 560	6 200	290	11 000	14 000	0,080	61907
	62	9	12 400	8 150	375	10 000	13 000	0,11	16007
	62	14	15 900	10 200	440	10 000	13 000	0,16	6007
	72	17	25 500	15 300	655	9 000	11 000	0,29	6207
	80	21	33 200	19 000	815	8 500	10 000	0,46	6307
	100	25	55 300	31 000	1 290	7 000	8 500	0,95	6407
40	52	7	4 940	3 450	186	11 000	14 000	0,034	61808
	62	12	13 800	9 300	425	10 000	13 000	0,12	61908
	68	9	13 300	9 150	440	9 500	12 000	0,13	16008
	68	15	16 800	11 600	490	9 500	12 000	0,19	6008
	80	18	30 700	19 000	800	8 500	10 000	0,37	6208
	90	23	41 000	24 000	1 020	7 500	9 000	0,63	6308
	110	27	63 700	36 500	1 530	6 700	8 000	1,25	6408
45	58	7	6 050	4 300	228	9 500	12 000	0,040	61809
	68	12	10 100	6 700	285	9 000	11 000	0,14	61909
	75	10	15 600	10 800	520	9 000	11 000	0,17	16009
	75	16	20 800	14 600	640	9 000	11 000	0,25	6009
	85	19	33 200	21 600	915	7 500	9 000	0,41	6209
	100	25	52 700	31 500	1 340	6 700	8 000	0,83	6309
	120	29	76 100	45 000	1 900	6 000	7 000	1,55	6409
50	65	7	6 240	4 750	250	9 000	11 000	0,052	61810
	72	12	14 600	10 400	500	8 500	10 000	0,14	61910
	80	10	16 300	11 400	560	8 500	10 000	0,18	16010
	80	16	21 600	16 000	710	8 500	10 000	0,26	6010
	90	20	35 100	23 200	980	7 000	8 500	0,46	6210
	110	27	61 800	38 000	1 600	6 300	7 500	1,05	6310
	130	31	87 100	52 000	2 200	5 300	6 300	1,90	6410
55	72	9	8 320	6 200	325	8 500	10 000	0,083	61811
	80	13	15 900	11 400	560	8 000	9 500	0,19	61911
	90	11	19 500	14 000	695	7 500	9 000	0,26	16011
	90	18	28 100	21 200	900	7 500	9 000	0,39	6011
	100	21	43 600	29 000	1 250	6 300	7 500	0,61	6211
	120	29	71 500	45 000	1 900	5 600	6 700	1,35	6311
	140	33	99 500	62 000	2 600	5 000	6 000	2,30	6411

Angular contact ball bearings
single row
d 10–65 mm



Principal dimensions d	D	B	Basic load ratings dynamic C	static C_0	Fatigue load limit P_u	Speed ratings Lubrication grease	oil	Mass	Designation
mm			N		N	r/min		kg	–
10	30	9	7 020	3 350	140	19 000	28 000	0,030	7200 BE
12	32	10	7 610	3 800	160	18 000	26 000	0,036	7201 BE
	37	12	10 600	5 000	208	17 000	24 000	0,060	7301 BE
15	35	11	8 840	4 800	204	17 000	24 000	0,045	7202 BE
	42	13	13 000	6 700	280	15 000	20 000	0,080	7302 BE
17	40	12	11 100	6 100	260	15 000	20 000	0,065	7203 BE
	47	14	15 900	8 300	355	13 000	18 000	0,11	7303 BE
20	47	14	14 000	8 300	355	12 000	17 000	0,11	7204 BE
	52	15	19 000	10 400	440	11 000	16 000	0,14	7304 BE
25	52	15	15 600	10 200	430	10 000	15 000	0,13	7205 BE
	62	17	26 000	15 600	655	9 000	13 000	0,23	7305 BE
30	62	16	23 800	15 600	655	8 500	12 000	0,20	7206 BE
	72	19	34 500	21 200	900	8 000	11 000	0,34	7306 BE
35	72	17	30 700	20 800	880	8 000	11 000	0,28	7207 BE
	80	21	39 000	24 500	1 040	7 500	10 000	0,45	7307 BE
40	80	18	36 400	26 000	1 100	7 000	9 500	0,37	7208 BE
	90	23	49 400	33 500	1 400	6 700	9 000	0,63	7308 BE
45	85	19	37 700	28 000	1 200	6 700	9 000	0,42	7209 BE
	100	25	60 500	41 500	1 730	6 000	8 000	0,85	7309 BE
50	90	20	39 000	30 500	1 290	6 000	8 000	0,47	7210 BE
	110	27	74 100	51 000	2 200	5 300	7 000	1,10	7310 BE
55	100	21	48 800	38 000	1 630	5 600	7 500	0,62	7211 BE
	120	29	85 200	60 000	2 550	4 800	6 300	1,40	7311 BE
60	110	22	57 200	45 500	1 930	5 000	6 700	0,80	7212 BE
	130	31	95 600	69 500	3 000	4 500	6 000	1,75	7312 BE
65	120	23	66 300	54 000	2 280	4 500	6 000	1,00	7213 BE
	140	33	108 000	80 000	3 350	4 300	5 600	2,15	7313 BE

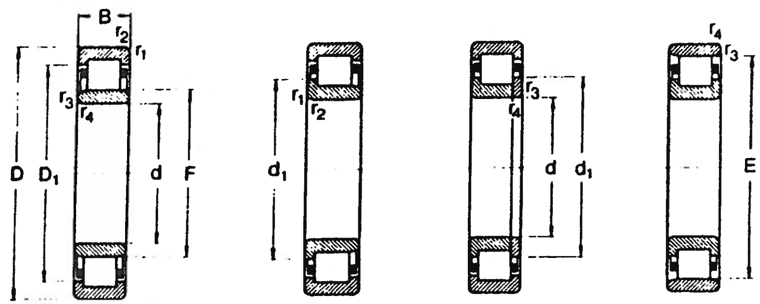
Cylindrical roller bearings
single row
d **40–45** mm



Type NU Type NJ Type NUP Type N

Principal dimensions d	D	B	Basic load ratings dynamic C	static C_0	Fatigue load limit P_u	Speed ratings Lubrication grease	oil	Mass	Designation
mm			N		N	r/min		kg	–
40 (cont.)	90	23	80 900	78 000	10 200	6 700	8 000	0,65	**NU 308 EC**
	90	23	80 900	78 000	10 200	6 700	8 000	0,67	**NJ 308 EC**
	90	23	80 900	78 000	10 200	6 700	8 000	0,68	**NUP 308 EC**
	90	23	80 900	78 000	10 200	6 700	8 000	0,64	**N 308 EC**
	90	33	112 000	120 000	15 300	6 300	7 500	0,94	**NU 2308 EC**
	90	33	112 000	120 000	15 300	6 300	7 500	0,96	**NJ 2308 EC**
	90	33	112 000	120 000	15 300	6 300	7 500	0,98	**NUP 2308 EC**
	110	27	96 800	90 000	11 600	6 000	7 000	1,30	**NU 408**
	110	27	96 800	90 000	11 600	6 000	7 000	1,30	**NJ 408**
	110	27	96 800	90 000	11 600	6 000	7 000	1,35	**NUP 408**
45	75	16	44 600	52 000	6 300	9 000	11 000	0,26	**NU 1009 EC**
	85	19	60 500	64 000	8 150	6 700	8 000	0,43	**NU 209 EC**
	85	19	60 500	64 000	8 150	6 700	8 000	0,44	**NJ 209 EC**
	85	19	60 500	64 000	8 150	6 700	8 000	0,45	**NUP 209 EC**
	85	19	60 500	64 000	8 150	6 700	8 000	0,43	**N 209 EC**
	85	23	73 700	81 500	10 600	6 700	8 000	0,52	**NU 2209 EC**
	85	23	73 700	81 500	10 600	6 700	8 000	0,54	**NJ 2209 EC**
	85	23	73 700	81 500	10 600	6 700	8 000	0,55	**NUP 2209 EC**
	85	23	73 700	81 500	10 600	6 700	8 000	0,52	**N 2209 EC**
	100	25	99 000	100 000	12 900	6 300	7 500	0,90	**NU 309 EC**
	100	25	99 000	100 000	12 900	6 300	7 500	0,92	**NJ 309 EC**
	100	25	99 000	100 000	12 900	6 300	7 500	0,95	**NUP 309 EC**
	100	25	99 000	100 000	12 900	6 300	7 500	0,88	**N 309 EC**
	100	36	138 000	153 000	20 000	5 600	6 700	1,30	**NU 2309 EC**
	100	36	138 000	153 000	20 000	5 600	6 700	1,30	**NJ 2309 EC**
	100	36	138 000	153 000	20 000	5 600	6 700	1,35	**NUP 2309 EC**
	120	29	106 000	102 000	13 400	5 600	6 700	1,65	**NU 409**
	120	29	106 000	102 000	13 400	5 600	6 700	1,65	**NJ 409**
	120	29	106 000	102 000	13 400	5 600	6 700	1,70	**NUP 409**

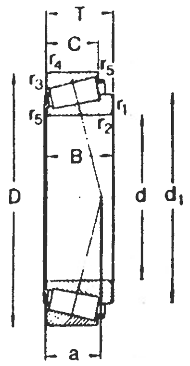
Cylindrical roller bearings
single row
d **50–55** mm



Type NU Type NJ Type NUP Type N

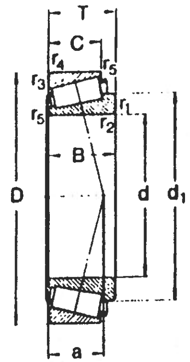
Principal dimensions d	D	B	Basic load ratings dynamic C	static C_0	Fatigue load limit P_u	Speed ratings Lubrication grease	oil	Mass	Designation
mm			N		N	r/min		kg	–
50	80	16	30 800	34 500	4 000	8 500	10 000	0,31	**NU 1010**
	90	20	64 400	69 500	8 800	6 300	7 500	0,48	**NU 210 EC**
	90	20	64 400	69 500	8 800	6 300	7 500	0,49	**NJ 210 EC**
	90	20	64 400	69 500	8 800	6 300	7 500	0,51	**NUP 210 EC**
	90	20	64 400	69 500	8 800	6 300	7 500	0,48	**N 210 EC**
	90	23	78 100	88 000	11 400	6 300	7 500	0,56	**NU 2210 EC**
	90	23	78 100	88 000	11 400	6 300	7 500	0,58	**NJ 2210 EC**
	90	23	78 100	88 000	11 400	6 300	7 500	0,59	**NUP 2210 EC**
	110	27	110 000	112 000	15 000	5 000	6 000	1,15	**NU 310 EC**
	110	27	110 000	112 000	15 000	5 000	6 000	1,15	**NJ 310 EC**
	110	27	110 000	112 000	15 000	5 000	6 000	1,20	**NUP 310 EC**
	110	27	110 000	112 000	15 000	5 000	6 000	1,15	**N 310 EC**
	110	40	161 000	186 000	24 500	5 000	6 000	1,70	**NU 2310 EC**
	110	40	161 000	186 000	24 500	5 000	6 000	1,75	**NJ 2310 EC**
	110	40	161 000	186 000	24 500	5 000	6 000	1,80	**NUP 2310 EC**
	130	31	130 000	127 000	16 600	5 000	6 000	2,00	**NU 410**
	130	31	130 000	127 000	16 600	5 000	6 000	2,05	**NJ 410**
55	90	18	57 200	69 500	8 300	7 000	8 500	0,40	**NU 1011 EC**
	100	21	84 200	95 000	12 200	6 000	7 000	0,66	**NU 211 EC**
	100	21	84 200	95 000	12 200	6 000	7 000	0,67	**NJ 211 EC**
	100	21	84 200	95 000	12 200	6 000	7 000	0,69	**NUP 211 EC**
	100	21	84 200	95 000	12 200	6 000	7 000	0,66	**N 211 EC**
	100	25	99 000	118 000	15 300	6 000	7 000	0,79	**NU 2211 EC**
	100	25	99 000	118 000	15 300	6 000	7 000	0,81	**NJ 2211 EC**
	100	25	99 000	118 000	15 300	6 000	7 000	0,82	**NUP 2211 EC**
	100	25	99 000	118 000	15 300	6 000	7 000	0,79	**N 2211 EC**
	120	29	138 000	143 000	18 600	4 800	5 600	1,45	**NU 311 EC**
	120	29	138 000	143 000	18 600	4 800	5 600	1,50	**NJ 311 EC**
	120	29	138 000	143 000	18 600	4 800	5 600	1,55	**NUP 311 EC**
	120	29	138 000	143 000	18 600	4 800	5 600	1,45	**N 311 EC**

Taper roller bearings
single row
d 35–50 mm



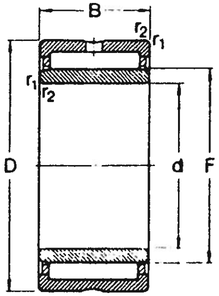
Principal dimensions d	D	T	Basic load ratings dynamic C	static C_0	Fatigue load limit P_u	Speed ratings Lubrication grease	oil	Mass	Designation	Dimension Series to ISO 355
mm			N		N	r/min		kg	–	–
35	80	22,75	72 100	73 500	8 500	5 000	6 700	0,52	30307	2FB
(cont)	80	22,75	61 600	67 000	7 800	4 500	6 000	0,52	31307	7FB
	80	32,75	95 200	106 000	12 200	4 800	6 300	0,73	32307	2FE
	80	32,75	93 500	114 000	13 200	4 500	6 000	0,80	32307 B	5FE
40	68	19	52 800	71 000	7 800	5 300	7 000	0,27	32008 X	3CD
	75	26	79 200	104 000	11 600	5 000	6 700	0,51	33108	2CE
	80	19,75	61 600	68 000	7 650	4 800	6 300	0,42	30208	3DB
	80	24,75	74 800	86 500	9 800	4 800	6 300	0,53	32208	3DC
	80	32	105 000	132 000	15 300	4 300	5 600	0,77	33208	2DE
	85	33	121 000	150 000	17 300	4 500	6 000	0,90	T2EE 040	2EE
	90	25,25	85 800	95 000	11 000	4 500	6 000	0,72	30308	2FB
	90	25,25	73 700	81 500	9 650	4 000	5 300	0,72	31308	7FB
	90	35,25	117 000	140 000	16 300	4 000	5 300	1,00	32308	2FD
	90	35,25	108 000	140 000	16 300	4 000	5 300	1,10	32308 B	5FD
45	75	20	58 300	80 000	8 800	4 800	6 300	0,34	32009 X	3CC
	80	26	84 200	114 000	12 900	4 500	6 000	0,56	33109	3CE
	85	20,75	66 000	76 500	8 650	4 500	6 000	0,48	30209	3DB
	85	24,75	80 900	98 000	11 200	4 500	6 000	0,58	32209	3DC
	85	24,75	73 700	93 000	11 000	4 300	5 600	0,60	32209 B	5DC
	85	32	108 000	143 000	16 300	4 000	5 300	0,82	33209	3DE
	95	29	89 700	112 000	12 900	3 600	4 800	0,92	T7FC 045	7FC
	95	36	147 000	186 000	21 200	4 000	5 300	1,20	T2ED 045	2ED
	100	27,25	108 000	120 000	14 600	4 000	5 300	0,97	30309	2FB
	100	27,25	91 300	102 000	12 500	3 400	4 500	0,95	31309	7FB
	100	38,25	140 000	170 000	20 400	3 600	4 800	1,35	32309	2FD
	100	38,25	134 000	176 000	20 000	3 600	4 800	1,45	32309 B	5FD
50	80	20	60 500	88 000	9 650	4 500	6 000	0,37	32010 X	3CC
	80	24	69 300	102 000	11 400	4 500	6 000	0,45	33010	2CE
	82	21,5	72 100	100 000	11 000	4 500	6 000	0,43	K-JLM 104948/K-JLM 104910	–
	85	26	85 800	122 000	13 700	4 300	5 600	0,59	33110	3CE
	90	21,75	76 500	91 500	10 400	4 300	5 600	0,54	30210	3DB
	90	24,75	82 500	100 000	11 600	4 300	5 600	0,61	32210	3DC
	90	24,75	82 500	104 000	12 500	4 000	5 300	0,65	32210 B	5DC
	90	28	106 000	140 000	16 300	4 000	5 300	0,75	K-JM 205149/K-JM 205110	–
	90	28	106 000	140 000	16 300	4 000	5 300	0,75	K-JM 205149/K-JM 205110 A	–
	90	32	114 000	160 000	18 300	3 800	5 000	0,90	33210	3DE
	100	36	154 000	200 000	22 800	3 800	5 000	1,30	T2ED 050	2ED
	105	32	108 000	137 000	16 000	3 200	4 300	1,20	T7FC 050	7FC

Taper roller bearings
single row
d 50–65 mm

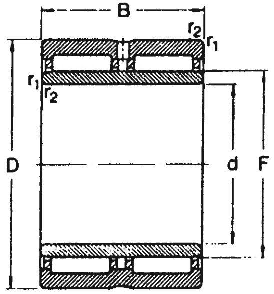


Principal dimensions d	D	T	Basic load ratings dynamic C	static C_0	Fatigue load limit P_u	Speed ratings Lubrication grease	oil	Mass	Designation	Dimension Series to ISO 355
mm			N		N	r/min		kg	–	–
50	110	29,25	125 000	140 000	17 000	3 600	4 800	1,25	30310	2FB
(cont.)	110	29,25	106 000	120 000	14 300	3 200	4 300	1,20	31310	7FB
	110	42,25	172 000	212 000	24 500	3 200	4 300	1,80	32310	2FD
	110	42,25	161 000	216 000	25 000	3 200	4 300	1,85	32310 B	5FD
55	90	23	78 100	112 000	12 500	4 000	5 300	0,56	K-JLM 506849/K-JLM 506810	–
	90	23	80 900	116 000	13 200	4 000	5 300	0,55	32011 X	3CC
	90	27	89 700	137 000	15 300	4 000	5 300	0,67	33011	2CE
	95	30	110 000	156 000	18 000	3 800	5 000	0,86	33111	3CE
	100	22,75	89 700	106 000	12 200	3 800	5 000	0,70	30211	3DB
	100	26,75	106 000	129 000	15 000	3 800	5 000	0,83	32211	3DC
	100	26,75	101 000	127 000	15 300	3 600	4 800	0,87	32211 B	–
	100	35	138 000	190 000	22 000	3 400	4 500	1,20	33211	3DE
	110	39	179 000	232 000	26 500	3 400	4 500	1,70	T2ED 055	2ED
	115	34	125 000	163 000	19 600	3 000	4 000	1,60	T7FC 055	7FC
	120	31,5	142 000	163 000	19 600	3 200	4 300	1,55	30311	2FB
	120	31,5	121 000	137 000	17 000	2 800	3 800	1,55	31311	7FB
	120	45,5	198 000	250 000	29 000	3 000	4 000	2,30	32311	2FD
	120	45,5	190 000	260 000	30 000	2 800	3 800	2,50	32311 B	5FD
60	95	23	82 500	122 000	13 700	3 800	5 000	0,59	32012 X	4CC
	95	24	84 200	132 000	15 000	3 600	4 800	0,62	K-JLM 508748/K-JLM 508710	–
	95	27	91 300	143 000	16 000	3 800	5 000	0,71	33012	2CE
	100	30	117 000	170 000	19 600	3 600	4 800	0,92	33112	3CE
	110	23,75	99 000	114 000	13 400	3 400	4 500	0,88	30212	3EB
	110	29,75	125 000	160 000	19 000	3 400	4 500	1,15	32212	3EC
	110	38	168 000	236 000	27 000	3 000	4 000	1,60	33212	3EE
	115	39	168 000	250 000	27 500	3 000	4 000	1,85	T5ED 060	5ED
	115	40	194 000	260 000	30 000	3 200	4 300	1,85	T2EE 060	2EE
	125	37	154 000	204 000	24 500	2 600	3 600	2,05	T7FC 060	7FC
	130	33,5	168 000	196 000	23 600	3 000	4 000	1,95	30312	2FB
	130	33,5	145 000	166 000	20 400	2 600	3 600	1,90	31312	7FB
	130	48,5	229 000	290 000	34 000	2 600	3 600	2,85	32312	2FD
	130	48,5	220 000	305 000	35 500	2 600	3 600	2,80	32312 B	5FD
65	100	23	84 200	127 000	14 300	3 400	4 500	0,63	32013 X	4CC
	100	27	96 800	156 000	17 600	3 400	4 500	0,78	33013	2CE
	110	28	123 000	183 000	21 200	3 200	4 300	1,05	K-JM 511946/K-JM 511910	–
	110	34	142 000	208 000	24 500	3 200	4 300	1,30	33113	3DE
	120	24,75	114 000	134 000	16 300	3 000	4 000	1,15	30213	3EB
	120	32,75	151 000	193 000	23 200	3 000	4 000	1,50	32213	3EC
	120	39	161 000	240 000	27 500	3 000	4 000	1,95	T5ED 065	5ED

Needle roller bearings with flanges with inner ring
d 40–65 mm



Series NKI(S), NA 49



Series NA 69

Principal dimensions			Basic load ratings		Fatigue load limit	Speed ratings Lubrication		Mass	Designation
			dynamic	static		grease	oil		
d	D	B	C	C_0	P_u				
mm			N		N	r/min		kg	–
40	55	20	27 500	57 000	7 200	6 300	9 000	0,14	**NKI 40/20**
	55	30	40 200	93 000	12 000	6 300	9 000	0,22	**NKI 40/30**
	62	22	42 900	71 000	9 150	5 600	8 000	0,23	**NA 4908**
	62	40	67 100	125 000	16 000	5 600	8 000	0,43	**NA 6908**
	65	22	42 900	72 000	9 150	5 600	8 000	0,28	**NKIS 40**
42	57	20	29 200	61 000	7 650	6 000	8 500	0,15	**NKI 42/20**
	57	30	41 800	98 000	12 900	6 000	8 500	0,22	**NKI 42/30**
45	62	25	38 000	78 000	10 000	5 600	8 000	0,23	**NKI 45/25**
	62	35	49 500	110 000	14 300	5 600	8 000	0,32	**NKI 45/35**
	68	22	45 700	78 000	10 000	5 300	7 500	0,27	**NA 4909**
	68	40	70 400	137 000	17 300	5 300	7 500	0,50	**NA 6909**
	72	22	44 600	78 000	10 000	5 000	7 000	0,34	**NKIS 45**
50	68	25	40 200	88 000	11 200	5 300	7 500	0,27	**NKI 50/25**
	68	35	52 300	122 000	16 000	5 300	7 500	0,38	**NKI 50/35**
	72	22	47 300	85 000	11 000	5 000	7 000	0,27	**NA 4910**
	72	40	73 700	150 000	19 000	5 000	7 000	0,52	**NA 6910**
	80	28	62 700	104 000	13 700	4 500	6 300	0,52	**NKIS 50**
55	72	25	41 800	96 500	12 200	4 800	6 700	0,27	**NKI 55/25**
	72	35	55 000	134 000	17 600	4 800	6 700	0,38	**NKI 55/35**
	80	25	57 200	106 000	13 700	4 500	6 300	0,40	**NA 4911**
	80	45	89 700	190 000	24 000	4 500	6 300	0,78	**NA 6911**
	85	28	66 000	114 000	15 000	4 300	6 000	0,56	**NKIS 55**
60	82	25	44 000	95 000	12 000	4 300	6 000	0,40	**NKI 60/25**
	82	35	60 500	146 000	19 000	4 300	6 000	0,55	**NKI 60/35**
	85	25	60 500	114 000	14 600	4 300	6 000	0,43	**NA 4912**
	85	45	93 500	204 000	26 000	4 300	6 000	0,81	**NA 6912**
	90	28	68 200	120 000	15 600	4 000	5 600	0,56	**NKIS 60**
65	90	25	61 600	120 000	15 300	4 000	5 600	0,46	**NA 4913**
	90	25	52 800	106 000	13 700	4 000	5 600	0,47	**NKI 65/25**
	90	35	73 700	163 000	21 600	4 000	5 600	0,66	**NKI 65/35**
	90	45	95 200	212 000	27 000	4 000	5 600	0,83	**NA 6913**
	95	28	70 400	132 000	17 000	3 800	5 300	0,64	**NKIS 65**

Engineering Tripos Part IIA: Module 3C8
Machine Design

Numerical answers – 2022

1. (b) $\phi = cos^{-1}\left(\frac{1}{\sqrt{1+2tan^2\theta}}\right)$, (c)(i) $\Omega R\phi = (-tan\theta \quad -tan\theta \quad 0)^T$,
(c)(ii) $\frac{-2\Omega tan\theta}{\sqrt{1+2tan^2\theta}}$ along the common normal direction

2. (a) -1; $\frac{ae_1}{be_2}$

2. (b) $\alpha = \frac{\left(\frac{ae_1}{be_2}+1\right)}{2}, \beta = \left(\frac{ae_1}{be_2} - 1\right)/2$

2 (c) $\frac{\alpha}{\alpha+\beta/2}$

3. (a)(i) 50 m/s, 3.30 (a) (ii) 90 s approx., b(ii) 40 s approx., choosing G=6.6

4. (a) 1.26 T (b) 6008