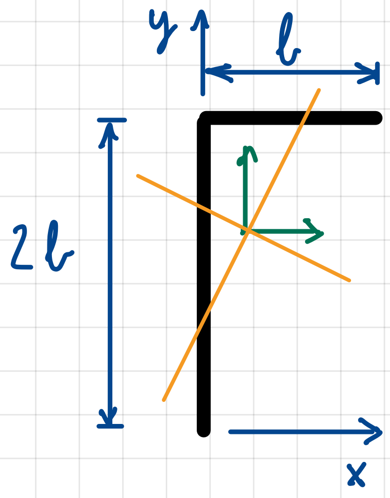
Q1) a)



$$x_s = \frac{1}{3l}\left(l\,\frac{l}{2}\right) = \frac{1}{6}l$$

$$y_s = \frac{1}{3l}\left(2l^2 + 2l^2\right) = \frac{4}{3}l$$

$$I_{xx} = 8\frac{l^3t}{12} + 2lt\,l^2 + lt\,4l^2 = \frac{20}{3}l^3t$$

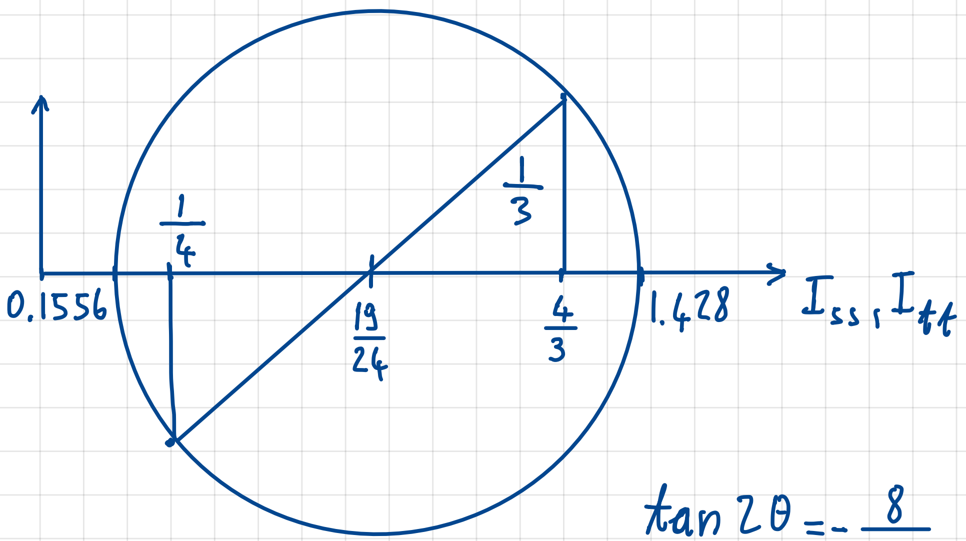
$$I_{yy} = \frac{l^3t}{12} + lt\,\frac{l^2}{4} = \frac{1}{3}l^3t$$

$$I_{xy} = lt\cdot 2l\cdot\frac{l}{2} = l^3t$$

$$I_{xx} = I_{ss} + 3lt\,\frac{16}{9}l^2 \Rightarrow I_{ss} = \frac{4}{3}l^3t$$

$$I_{yy} = I_{tt} + 3lt\,\frac{1}{36}l^2 \Rightarrow I_{tt} = \frac{1}{4}l^3t$$

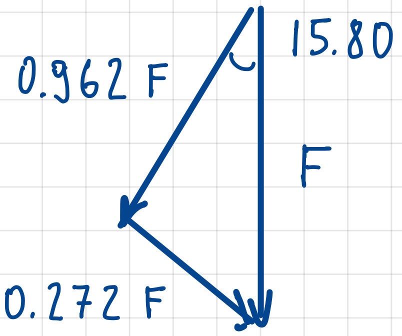
$$I_{xy} = I_{st} + 3lt\,\frac{1}{6}l\,\frac{4}{3}l \Rightarrow I_{st} = \frac{1}{3}l^3t$$



$$\tan 2\theta = -\frac{8}{3} \Rightarrow \theta = -15.80°$$

l) i) Setion rotates around shear centre A.

Dat book: $w = \dfrac{F\ell^3}{3EI}$

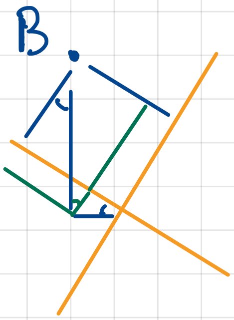


$$w_t = \frac{0.962}{1.43\,\ell^3 t}\,\frac{F\ell^3}{3E} = 0.225\,\frac{F\ell^3}{E\ell^3 t}$$

$$w_s = \frac{0.272}{0.156\,\ell^3 t}\,\frac{F\ell^3}{3E} = 0.583\,\frac{F\ell^3}{E\ell^3 t}$$

$$\Rightarrow \quad w_{tot} = 0.625\,\frac{F\ell^3}{E\ell^3 t}$$

ii) Coords of B in centroidal system: $\frac{2}{3}\ell,\ -\frac{1}{6}\ell$



$$s_B = \frac{2}{3}\ell \sin(15.8) + \frac{1}{6}\ell\cos(15.8)$$

$$= 0.342\,\ell$$

$$t_B = \frac{2}{3}\ell\cos(15.8) - \frac{1}{6}\ell\sin(15.8)$$

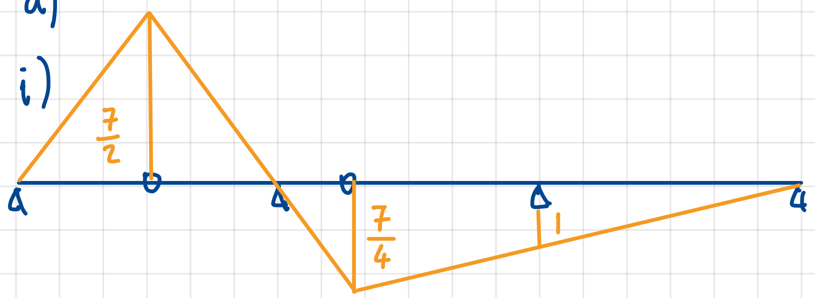
$$= 0.596\,\ell$$

$$\Rightarrow \quad \sigma = \frac{0.962F}{1.43\,\ell^3 t}\cdot 0.596\,\ell + \frac{0.272F}{0.156\,\ell^3 t}\cdot 0.342\,\ell$$

$$= 1.022\,\frac{F}{\ell^2 t}$$

Q2) a)

i)

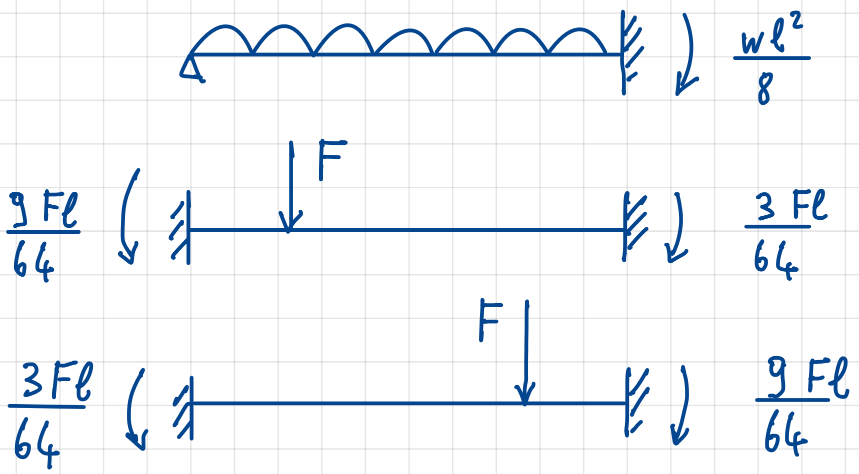


ii) Support reaction: $\frac{1}{2}\,\frac{7}{4}\,\frac{wl}{4} + \frac{1}{2}\left(\frac{7}{4}+1\right)\frac{3wl}{4} - \frac{7}{2}F$

$$= \frac{5}{4}w - \frac{7}{2}F$$

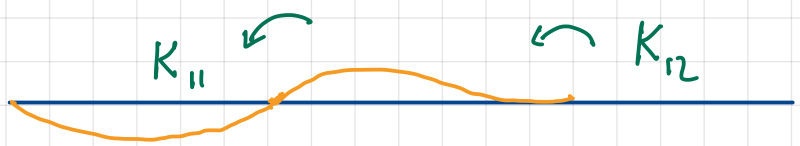
b) i

From Databook:



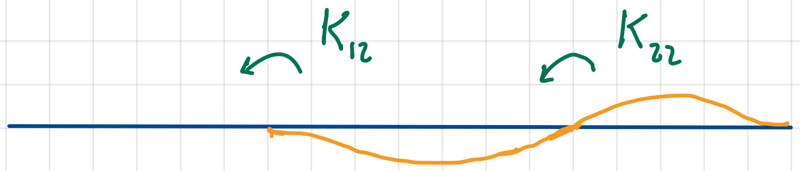
Degree of kinematic indeterminacy: 2

From element stiffness matrices



$$K_{11} = 4\frac{EI}{l} + 3\frac{EI}{l} = 7\frac{EI}{l}$$

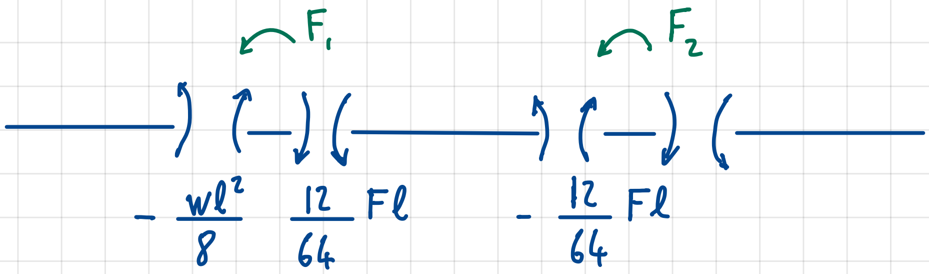
$$K_{12} = 2\frac{EI}{l}$$



$$K_{22} = 4\frac{EI}{\ell} + 4\frac{EI}{\ell} = 8\frac{EI}{\ell}$$

$$K_{21} = 2\frac{EI}{\ell}$$

Restraining moments



$$F_1 = -\frac{w\ell^2}{8} + \frac{12}{64}F\ell = \frac{w\ell^2}{16}$$

$$F_2 = -\frac{12}{64}F\ell = -\frac{3w\ell^2}{16}$$

Stiffness matrix

$$K = \frac{EI}{\ell}\begin{pmatrix} 7 & 2 \\ 2 & 8 \end{pmatrix} \Rightarrow K^{-1} = \frac{\ell}{EI}\begin{pmatrix} 0.154 & -0.035 \\ -0.035 & 0.135 \end{pmatrix}$$

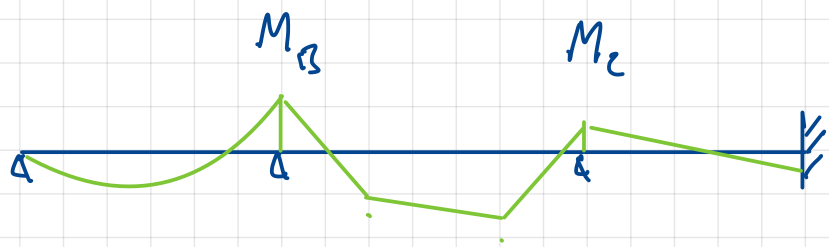
$$K^{-1}\begin{pmatrix} r_1 \\ r_2 \end{pmatrix} = -\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} \Rightarrow r_1 = -0.0168\frac{w\ell^3}{EI}$$

$$r_2 = 0.0276\frac{w\ell^3}{EI}$$

ii) $M_B^{l} = -\frac{wl^2}{8} - 0.0168\,\frac{wl^3}{EI}\cdot 3\frac{EI}{l} = -0.18\,wl^2$

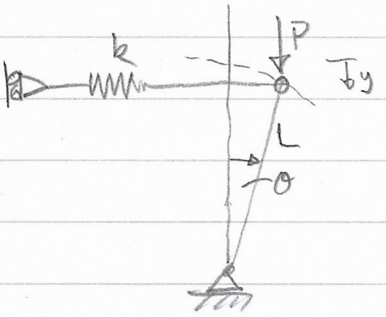
$M_C^{r} = 0.0276\,\frac{wl^3}{EI}\cdot 4\,\frac{EI}{l} = 0.110\,wl^2$

iii)



3D4 Q3 2022.

a)
i)



spring extension $= L(\sin\theta - \sin\theta_0)$

Internal strain energy $= \frac{1}{2}kL^2(\sin\theta - \sin\theta_0)^2$

$WD = -Py$ with $y = L(1-\cos\theta)$

$\therefore$ Total Potential Energy $\Pi(\theta) = \frac{1}{2}kL^2(\sin\theta - \sin\theta_0)^2 - PL(1-\cos\theta)$

ii) Equilibrium $\frac{\partial \Pi}{\partial \theta} = 0 \Rightarrow$

$$kL^2\cos\theta(\sin\theta - \sin\theta_0) - PL\sin\theta = 0 \quad \underline{\text{EQUILIB. EQN}}$$

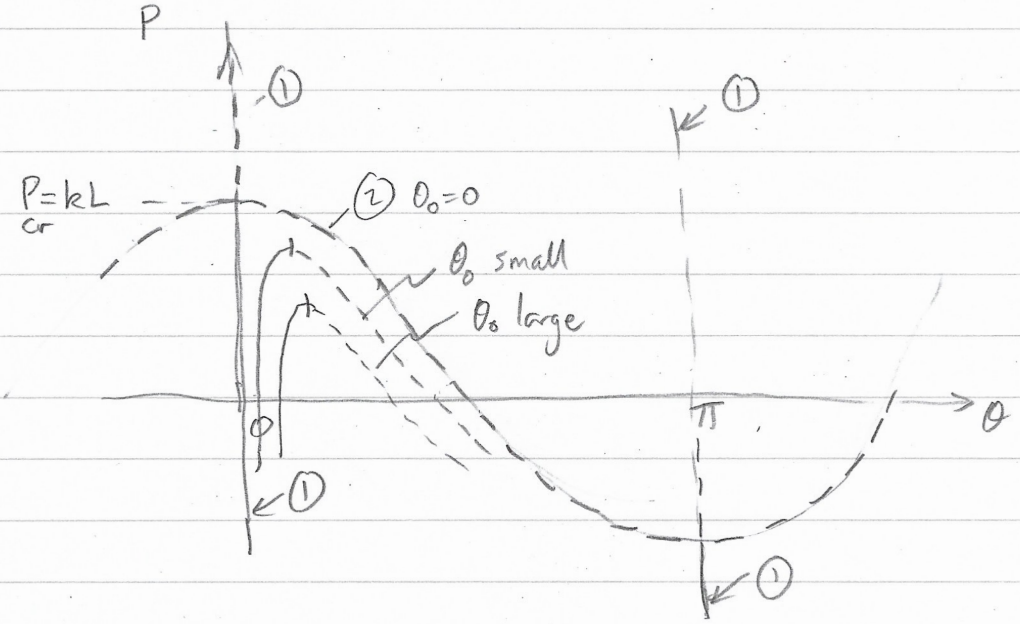
$$\therefore \quad P = kL\cos\theta\left(1 - \frac{\sin\theta_0}{\sin\theta}\right)$$

iii) For $\theta_0 = 0$ $\quad kL^2\cos\theta\sin\theta - PL\sin\theta = 0$

$$(kL\cos\theta - P)L\sin\theta = 0$$

$\rightarrow$ paths ① $\sin\theta = 0 \rightarrow \theta = 0, \pm\pi$, etc.

② $kL\cos\theta - P = 0 \Rightarrow P/kL = \cos\theta$



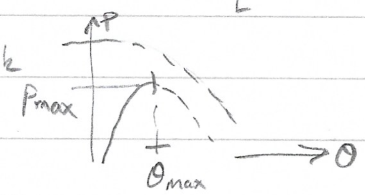
3D4, Q3 2022 cont'd.

a) iv) Equilib path $\rightarrow \dfrac{P}{P_{cr}} = p = \cos\theta\left(1 - \dfrac{\sin\theta_0}{\sin\theta}\right)$ $\quad [P_{cr} = kL]$

$$p = \cos\theta - \sin\theta_0 \cot\theta \quad \leftarrow \cos\theta/\sin\theta$$

$$\frac{dp}{d\theta} = -\sin\theta - \sin\theta_0\left[\frac{-\sin^2\theta - \cos^2\theta}{\sin^2\theta}\right] = -\sin\theta + \frac{\sin\theta_0}{\sin^2\theta}$$

This $= 0$ at peak



p_{max} when $\dfrac{dp}{d\theta} = 0 \rightarrow$ $\boxed{\sin\theta_0 = \sin^3\theta_{max}}$

$$\boxed{\sin\theta_{max} = (\sin\theta_0)^{1/3}}$$

$$\therefore p_{max} = \cos\theta_{max}\left(1 - \frac{\sin\theta_0}{\sin\theta_{max}}\right)$$

$$= (1 - \sin^2\theta_{max})^{1/2}\left(1 - \frac{\sin\theta_0}{\sin\theta_{max}}\right)$$

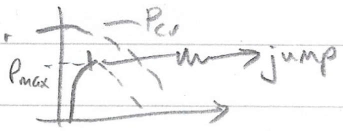
$$= (1 - \sin^2\theta_{max})^{1/2}\left(1 - \frac{\sin^3\theta_{max}}{\sin\theta_{max}}\right)$$

$$= (1 - \sin^2\theta_{max})^{1/2}(1 - \sin^2\theta_{max})$$

$$\boxed{p_{max} = (1 - \sin^2\theta_{max})^{3/2} = \underline{\underline{\cos^3\theta_{max}}}}$$

$$\left(\text{For } \theta_{max} \text{ small}, \quad p_{max} \simeq 1 - \tfrac{3}{2}\theta_{max}^2\right)$$

v) Implications: small deflection eigenvalue analysis would only give $P_{cr} = kL$ and this is an unconservative overestimate of the load at which instability occurs.



Get dangerous jump from limit point at loads below P_{cr}. P_{max} can be substantially less than P_{cr}. ($\theta_{max} \sim c\sqrt{1 - p_{max}}$, so falls off rapidly)

3D4. 2022. Q3(b).

b) Column $EI = a + bx$

Rayleigh Quotient:

$$P_R = \frac{U(\psi)}{y(\psi)}$$

U = strain energy
y = end shortening
ψ = assumed mode shape.

$$P_R = \frac{\int EI(\psi'')^2\,dx}{\int (\psi')^2\,dx}.$$

$$\left.\begin{aligned} \psi &= x(L-x) = Lx - x^2 \\ \psi' &= L - 2x \\ \psi'' &= -2 \end{aligned}\right\} \quad P_R = \frac{\int_0^L (a+bx)(-2)^2\,dx}{\int_0^L (L-2x)^2\,dx}$$

$$= \frac{4\int_0^L a + bx\,dx}{\int_0^L L^2 - 4Lx + 4x^2\,dx} = \frac{4\left[aL + bL^2/2\right]}{\left[\underbrace{L^3 - 2L^3 + \tfrac{4}{3}L^3}_{L^3/3}\right]}$$

$$P_R = \frac{12\left[a + bL/2\right]}{L^2}$$ Estimate of critical load.

Euler load $b = 0$ $a = EI$ $\rightarrow$ $P_{cr} = \frac{\pi^2 EI}{L^2} \simeq 10\frac{EI}{L^2}$

based on eigenvector $\psi = \sin\frac{\pi x}{L}$

whereas approximation based on quadratic $\psi = x(L-x)$ which is not an eigenvector, so the estimate $12EI/L^2$ is an overestimate of the true value $\pi^2 EI/L^2$.

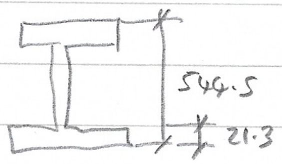
3D4. 2022 Q4

a) LTB . 533 × 210 × 122 UB, 6m $\quad M_{cr}$.

EQ + OPP.

$$M_{cr} = \frac{\pi}{L}\sqrt{EI_{min}GJ}\left(1 + \frac{\pi^2}{L^2}\frac{E\Gamma}{GJ}\right)^{1/2}$$

$$\Gamma = \frac{I_{min}D^2}{4} = \left[3388 \times 10^{-8}\,m^4\right]\frac{\left[0.5445 - 0.0213\right]^2}{4}\,m^2$$



$$= 231.8 \times 10^{-8}\ m^6.$$

$J = 178 \times 10^{-8}\,m^4$ $\qquad E/G = \frac{\cancel{1}}{2(1+\nu)} = \frac{\cancel{1}}{2.6}$

$$\therefore \text{Factor} = \left(1 + \frac{\pi^2}{36}\frac{\cancel{1}}{2.6}\frac{231.8}{178}\right)^{1/2} = \left(\overset{1.928}{\cancel{1.137}}\right)^{1/2} = \overset{1.389}{\cancel{1.067}}$$

$$M_{basic} = \frac{\pi}{6\,m}\left(210 \times 10^9\,N/m^2\right)\sqrt{\frac{(3388 \times 10^{-8})(178 \times 10^{-8})}{2.6}}\quad m^4$$

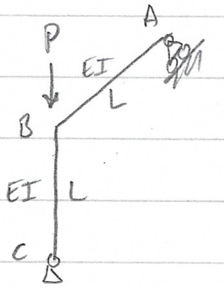
$$= \frac{\pi}{6}\left(210 \times 10^9\right)\left(481.6 \times 10^{-8}\right)$$

$$= \underline{\underline{529.6\ kNm}}$$

$$\therefore M_{crit,\,LTB\ (EQ+OPP)} = 529.6 \times \overset{1.389}{\cancel{1.067}} = \underline{\underline{\overset{735.6}{\cancel{564.8}}\ kNm}}$$

3D4 2022 Q4 cont'd.

b).



i) By inspection:

$$\begin{bmatrix} M_B \\ M_C \end{bmatrix} = K\underline{\theta} = \frac{EI}{L}\begin{bmatrix} s+3 & sc \\ sc & s \end{bmatrix}\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix}$$

(Which is $\begin{bmatrix} s & sc \\ sc & s \end{bmatrix}$ from BC, and $\begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$ from AB (far-end pinned).)

ii) So instability when det = 0

$$\Rightarrow \quad (s+3)s - s^2c^2 = 0$$

$$= \quad s(s+3-sc^2) = 0$$

$$s(s(1-c^2)+3) = 0$$

so either $s=0$ or $s(1-c^2) = -3$

From table, $s=0$ at $P/P_E \approx 2.0^+$

Try other solution

P/P_E	$s(1-c^2)$		
0	$4(1-(1/2)^2)$	=	+3
1.2	$2.0901(1-(1.2487)^2)$	=	−1.1689
1.4	$1.6782(1-(1.6557)^2)$	=	−2.9228 ← here.
1.6	$1.2240(1-(2.4348)^2)$	=	−6.03

So $P/P_E \approx 1.4$, with $s = 1.6782$, $c = 1.6557$.

3D4. 2022, Q4 cont'd.

b) cont'd.

iii) $(s+3-\lambda)\,\theta_B + sc\,\theta_C = 0$ from K matrix

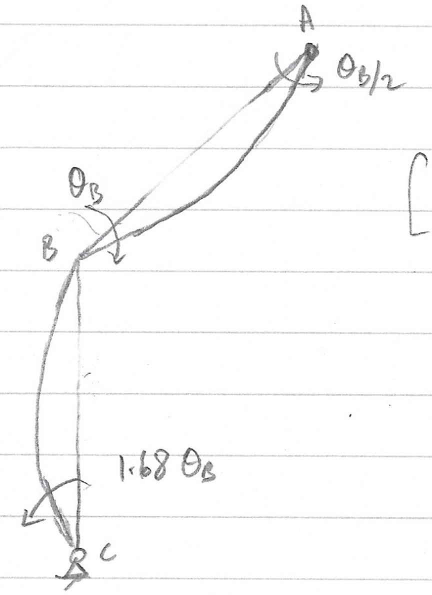
but $\lambda = 0$ at instability, so

$$(s+3)\,\theta_B + sc\,\theta_C = 0$$

so $$\theta_C = -\frac{(s+3)}{sc}\,\theta_B$$

$$= -\frac{(4.6782)}{(1.6782)(1.6557)}\,\theta_B$$

$$= \underline{\underline{-1.68\,\theta_B}}$$



[or the other way ]