Answers

Question 1.

(a) Semi-infinite bar, perfectly insulated sides, constant thermal properties (diffusivity), perfect heat transfer at quenched end (T = room temp for all t>0), 1D heat flow.

(b) Dimensions of dT/dt: $\theta\ T^{-1}$

Dimensions of $C_1\ a/x^n$: $\theta\ (L^2\ T^{-1})\ L^{-n}$

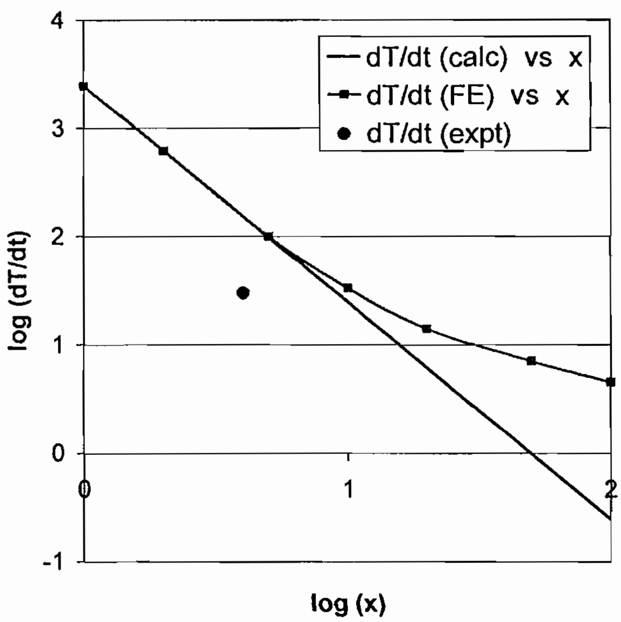
Hence n = 2.

(c) 1D elements only model temperature variations in one direction. With insulated sides and end-quenching, heat flow is purely axial and therefore 1D.

Two linear elements only give accurate temperature histories at the nodes, so only one point between the ends is well-modelled. To give the required accuracy a finer mesh of linear elements is required, either with nodes at all the target distances or with sufficiently short elements that linear interpolation between nodes is precise enough. Alternatively use a small number of higher order elements (e.g. quadratic). (Note: using an axisymmetric mesh, realistic heat transfer, temperature-dependent properties etc are *not* the required solution here, since the aim is to improve an FE model that replicates the assumptions of the analytical solution).

Log-log plot is most sensible, so that analytical solution has constant slope of -2. FE solution plotted below. Same slope for small x, and solutions agree up to approx. x = 5mm. Analytical solution extrapolates to lower cooling rates than FE for larger x. This is due to the finite length of the bar in the FE solution – the semi-infinite solution gives a lower cooling rate due to the continued supply of heat from the model bar beyond 125mm.

(d) Experimental cooling rate lower than predicted – primarily because heat transfer is not perfect in practice.



Examiner's comments: Although quite similar to an examples paper problem, candidates were good only on the most straightforward parts. Poor ability to think about the right graph to sketch in part (c) – only one candidate sketched a log-log plot.

Question 2

(a) (i)

$$\frac{\partial T}{\partial t} = 0 = -\frac{C}{2} t^{-3/2} \exp\left(-\frac{x^2}{4at}\right) + C\frac{x^2}{4a} t^{-5/2} \exp\left(\frac{x^2}{4at}\right) \Rightarrow t_p = \frac{x^2}{2a}$$

Substituting back into equation for T:

$$T_p(x) - T_o = C\frac{\sqrt{2a}}{x}\exp(-1/2)$$

(ii) From equation for time to peak: thermal diffusivity $a = \frac{x^2}{2t_p} = 8.9 \times 10^{-6}$ m^2/s.

Peak temperature rise is inversely proportional to distance x, hence $(\Delta T\, x)_1 = (\Delta T\, x)_2$

Hence: $300 \times 5 = \Delta T \times 2$, so $\Delta T = 750$ and thus $T_p = 770$°C.

(b)

(a) Mention should be made of: defects within weld (cracks in weld and at junction with host metal; inclusions and cavities); HAZ phase transformation effects; geometrical effects with contact angle of weld bead causing stress concentration; thermal residual stresses. Mild steel so hydrogen cracking probably not important (yield stress below critical value).

Solutions: Care over cooling rates to prevent martensite formation; pre-heating plus slow cooling to reduce residual stresses; grind and maybe hammer peen to remove bead.

(b) Liquid metal embrittlement: zinc migrates into steel during welding. Break in the protective layer may also lead to corrosion.

Solution: don't weld galvanised plate. Remove zinc from vicinity of weld, and paint afterwards (cold galvanising).

Examiner's comments: In part (a) several candidates lost marks by failing to substitute back t_p once they had found it, and many took a laborious route to calculate T_p (by first calculating C). In part (b) there was some confusion over the role of surface grinding which does not reduce residual stress.

Question 3

(a) Photodegradation of HDPE leads to slight embrittlement; surface cracks form under alternating stresses of use. Dust wedges cracks open, so accelerating failure. Avoid by using UV-stabilised grade of polymer (e.g. contains pigment/filler to scatter/absorb UV), use other polymer less susceptible to photodegradation (but NB cost), change design to reduce stress in use.

(b) Residual mechanical stresses from rolling of I-beam. Flame cutting introduces extra thermal stresses. Avoid by using mechanical cutting method, or stress relieve beam after rolling (e.g. via slower cooling rate).

(c) Chromium carbide precipitates form on grain boundaries in HAZ, leading to chromium-depleted regions close to boundaries. At surface, these regions are not protected by chromium oxide and so form anodic regions in electrochemical corrosion. (equations should be given). Deep pits form along grain boundaries: weld decay. Solution: Stabilise the steel by changing the alloying content: add niobium or vanadium to form carbides preferentially to chromium; use steel composition with very low carbon content.

(d) Season cracking: stress corrosion cracking. Requires tensile stress and plastic strain (residual stress from cold working), and specific environment (ammonia). Solution: brass is notoriously susceptible to SCC by ammonia; avoid contact with ammonia and stress-relieve if possible.

(e) Graphite coated the insides of the pipe, and gradually wore away in service. Once graphite-free patches developed, electrochemical couples were set up, resulting in large cathode areas (the graphite) and very small anodes which led to rapid localised corrosion of the copper. Avoid use of graphite (e.g. by using other lubricant such as an oil or soap) or clean scrupulously after tube drawing to remove all traces of graphite.

Examiner's comments: Although all the examples had been discussed in the lectures the level of answers was disappointing and showed poor ability to apply the knowledge which should have been acquired. There was confusion over: role of water in degradation of non-reinforced polymer; existence of residual stresses in rolled steel beams; galvanic corrosion. Several candidates failed to suggest methods by which the failures could be avoided and so threw marks away.

Question 4.

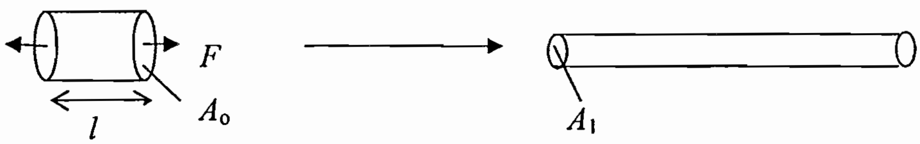
(a)

The work formula method assumes that the shape is changed in the most efficient way possible, and estimates the forces from the work they do. It takes account of initial and final states only without considering the path taken by the material.

Redundant work is work done on the system in excess of that predicted by the work formula method.

$$\text{Efficiency} = \frac{\text{work formula estimate of work or force}}{\text{actual work or force}}$$

Assume shape change by uniaxial tension:



$F = A\,\sigma_y = A\,2k$ (since for Tresca $\sigma_y = 2k$)

Increase length by dl: work done $= F\,dl = 2k\,A\,dl$

Total work done in changing length $= \int F dl = \int 2kA dl$

Work done per unit volume $= \int 2kAdl / Al = 2k \int_{lo}^{l1} \frac{dl}{l} = 2k \ln \frac{l_1}{l_o} = 2k \ln \frac{A_o}{A_1} = 2k \ln R$

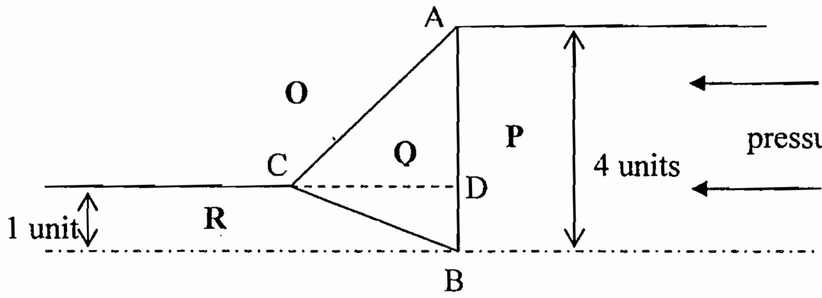
(since by conservation of volume $A_o l_o = A_1 l_1$)

In direct extrusion, pressure p is applied to area A_o. Volume deformed = $A_o\, l_o$

$\therefore$ Work done in extruding (input) length $l_o = p\, A_o\, l_o = 2k \ln R\, A_o\, l_o$

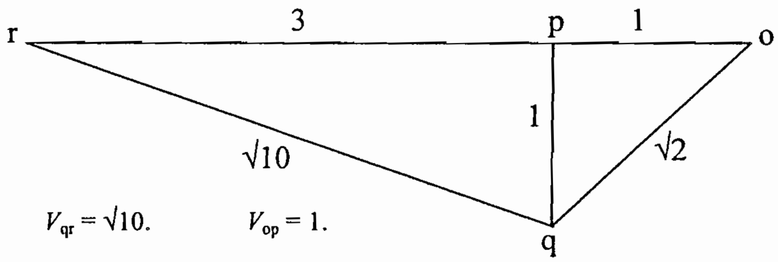
$$\therefore p = 2k \ln R$$

(b)



from diagram AD = CD = 3. BC = √10. AB = 4

Hodograph:



$V_{pq} = 1$. $V_{qr} = \sqrt{10}$. $V_{op} = 1$.

(i)

Assume unit depth into diagram and consider only top half of die. Pressure p acts on ram. Work done by $p = p.\ V_{op}.\ 4$

$$4\,p = k\,[\,PQ.\,V_{pq} + QR.\,V_{qr}\,] = k\,[\,4 \times 1 + \sqrt{10} \times \sqrt{10}\,] = 14\,k$$

$$\therefore\ p\,/\,2k = 14\,/\,8 = 1.75$$

For $k = 100$ MPa, then $p = 350$ MPa.

(ii)

Work formula (from part (a)): $p\,/\,2k = \ln R = \ln 4 = 1.386$

$$\therefore\ p = 277\ \text{MPa}$$

Efficiency = 277 / 350 = 0.79 i.e. 79%.

(iii)

The best upper bound is the one giving the lowest force. The position of B in the diagram should be varied (i.e. moved along the axis) to find the minimum. There may be other deformation patterns which give lower values as well. The current model assumes no friction – this is probably unrealistic and friction would need to be taken into account.

Examiner's comments: It was disappointing how many candidates were unable to calculate lengths in this hodograph from simple geometry (i.e. Pythagoras) and resorted to measurement from a sketch drawing.