Wednesday, 28 ${ }^{\text {th }}$ April $2010 \quad 9$ to 12

## PAPER 3

Module 3P4: OPERATIONS MANAGEMENT

Module 3P5: INDUSTRIAL ENGINEERING

Answer all questions from Sections A and B.

Answers to sections A and B must appear in two separate booklets.

All questions carry the same number of marks.

The approximate percentage of marks allocated to each part of a question is indicated in the right margin.

| STATIONERY REQUIREMENTS | SPECIAL REQUIREMENTS |
| :--- | :--- |
| 20 page answer booklet x 2 | Engineering Data Book |
| Rough work pad | 3P5 Data Sheet |
|  | CUED approved calculator allowed |

> You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## SECTION A

1 (a) Describe, giving examples, three approaches for matching capacity and demand in the intermediate term.
(b) There are three companies who supply the following quantities of coal, and three consumers who require coal as follows:

| Suppliers |  | Consumers |  |
| :---: | :---: | :---: | :---: |
| Supplier 1 | 14 tons | Consumer A | 6 tons |
| Supplier 2 | 12 tons | Consumer B | 10 tons |
| Supplier 3 | 5 tons | Consumer C | 15 tons |

The cost matrix is as shown below.

|  | Consumer A | Consumer B | Consumer C |
| :--- | :---: | :---: | :---: |
| Supplier 1 | 6 | 8 | 4 |
| Supplier 2 | 4 | 9 | 3 |
| Supplier 3 | 1 | 2 | 6 |

Find the schedule of a transportation policy which minimises the cost.
(c) Why might methods to solve distribution problems sometimes fail to identify the optimal solution? Explain how these failures can be avoided.

2 (a) The Economic Order Quantity (EOQ) model has been criticised due to its 'unrealistic' assumptions. Outline ways in which the various assumptions of EOQ are not matched by reality. To what extent is the validity of the EOQ model affected by these issues?
(b) You are the operations manager at Complete Packaging Services (CPS), a packaging manufacturer based in Cambridge. CPS produces folding cartons (e.g. for frozen food and cereal) for a wide range of customers within the food industry, including the UK's leading supermarket chains. Part of your job as operations manager is to purchase paperboard for use in the production facility. CPS uses 1,500 rolls of paperboard per year in its printing processes. The order cost is $£ 75$ per order; and the holding cost is $1 \%$ per month of the purchase cost of $£ 500$ per roll. The facility operates for 50 weeks per year, and lead time from the supplier is 1.5 weeks.
(i) How many rolls of paperboard should you order at one time? What is the reorder point?
(ii) What would be the change in total annual cost if CPS had storage space for only 50 rolls of paperboard, and thus was forced to use an order quantity of 50 ?
(c) Discuss, with examples, the key differences between fixed-order quantity and fixed-time period ordering systems.
(d) Discuss how the application of the basic EOQ model might be altered in order to reflect quantity discounts?

## SECTION B

3 (a) Define manufacturing strategy. Why is it essential to understand the manufacturing strategy of a company before applying industrial engineering techniques to the redesign of a production system?
(b) Describe the principles of motion economy that relate to the use of the human body.
(c) Describe the ' $5 S$ ' framework, and explain its use, in the context of Lean Production.
(d) Why are standard times necessary in manufacturing operations? An engineer times an operation and observes that it takes 1 minute. He estimates the workers rating at 110 on the BSI scale. He assesses the fixed allowances at $10 \%$, and the variable allowances also at $10 \%$. There are no contingencies for extra work. Calculate the standard time for the operation.
(e) Discuss the benefits of Predetermined Motion Time Systems compared to other methods of work measurement.
(f) An activity sampling study is required to determine the utilisation of a crane to an accuracy of $+/-5 \%$, with $90 \%$ confidence. A pilot study shows the utilisation to be around $70 \%$. How many observations should be planned for the study?
(g) Interpret the following control chart, explaining whether or not the process is in control. Give any recommendations for action.

(j) Explain Juran's Cost of Quality Model. Contrast with Taguchi's Loss to Society notion.

4 (a) Describe the stages of Systematic Layout Planning (R. Muther), clearly stating the objectives of each stage, and identifying the tools used.
(b) Discuss how ergonomic considerations of the following factors influence the design of factories:
(i) the visual environment;
(ii) the auditory environment and noise;
(iii) the climate.
(c) A worker operates a machine in an environment with a background noise of 83 dBA . The machine produces an additional noise, also of 83dBA. What is the maximum length of time the worker could operate the machine to ensure the daily personal noise exposure level does not exceed 85dBA?

Describe three different approaches that could be taken to allow the worker to operate the machine for a full 8 hour shift.

## END OF PAPER

## MET 1 3P5 Data Sheet

## Energy expenditure

The recommended maximum for mean energy expenditure over 8 hour shift is $5 \mathrm{kcal} / \mathrm{min}$ for males, 4 $\mathrm{kcal} /$ min for females.

## Noise

Sound intensity, sound pressure level (SPL or $\mathrm{L}_{\mathrm{p}}$ ) is measured in dB , relative to a reference pressure. $p_{\text {ref }}=20 \mu \mathrm{~Pa}$ (rms).

$$
L_{p}=10 \log _{10}\left(\frac{p_{\text {rms }}{ }^{2}}{p_{\text {ref }}{ }^{2}}\right)=20 \log _{10}\left(\frac{p_{\text {rms }}}{p_{\text {ref }}}\right) \mathrm{dB},
$$

The Control of Noise at Work regulations are based on the daily personal noise exposure level, that corresponds to an average SPL over an 8 hour shift.
SPL average $=10 \log _{10}\left\{1 / T_{0} \Sigma T_{i}\left(10^{0.15 P_{\mathrm{i}}}\right)\right\}$ Where: $\mathrm{T}_{\mathrm{i}}$ is the time at $\mathrm{SPL}_{\mathrm{i}}$ and $\mathrm{T}_{\mathrm{o}}$ is 8 hours

For daily personal noise exposure level above 80 dBA , hearing protection must be available on request, above 85 dBA it must be used.

## Light

Luminous intensity : candela (cd). The base SI unit. The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency $540 \times 10^{12}$ hertz and that has a radiant intensity in that direction of $1 / 683$ watt per steradian .

Luminous flux, measured in lumens . 1 lumen ( Im ) =1 cd.sr Illuminance - luminous flux shining per unit area on a surface, measured in lux. 1 lux $(\mathrm{lx})=1 \mathrm{~lm} \cdot \mathrm{~m}^{-2}$

Luminance(L) measures the amount of light reflected from a surface Units: $\mathrm{cd} . \mathrm{m}^{-2}$.
Luminance $(\mathrm{L})=$ Illuminance $(\mathrm{E})$. Reflectance $(\mathrm{R}) / \pi$

Visual acuity - capability to discriminate small objects or fine details. $V A=1 / \alpha_{v}$ where $\alpha_{v}$ is measured in arc min. Normal Vision VA=1.

Weber Contrast is expressed by the ratio $\left(\mathrm{L}_{\text {object }}-\mathrm{L}_{\text {background }}\right) / \mathrm{L}_{\text {background }}$

## Learning Curves

Learning Curves have the form $y=k x^{m}$
$y=$ time/unit, $k=$ constant representing the value of the time for the first work cycle,
$x=$ number of work units completed,$m=$ a constant related to the rate of learning. (Note $m$ is negative)

## Statistical Process Control

| Sample <br> Size $=\mathbf{n}$ | X-bar Charts |  |  | R Charts |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{A}_{\mathbf{2}}$ | $\mathbf{A}_{\mathbf{3}}$ | $\mathbf{d}_{\mathbf{2}}$ | $\mathbf{D}_{\mathbf{3}}$ | $\mathbf{D}_{\mathbf{4}}$ |
| 2 | 1.880 | 2.659 | 1.128 | 0 | 3.267 |
| 3 | 1.023 | 1.954 | 1.693 | 0 | 2.574 |
| 4 | 0.729 | 1.628 | 2.059 | 0 | 2.282 |
| 5 | 0.577 | 1.427 | 2.326 | 0 | 2.114 |
| 6 | 0.483 | 1.287 | 2.534 | 0 | 2.004 |
| 7 | 0.419 | 1.182 | 2.704 | 0.076 | 1.924 |
| 8 | 0.373 | 1.099 | 2.847 | 0.136 | 1.864 |
| 9 | 0.337 | 1.032 | 2.970 | 0.184 | 1.816 |
| 10 | 0.308 | 0.975 | 3.078 | 0.223 | 1.777 |

$$
\begin{aligned}
& \text { X-bar chart } \\
& U C L=\overline{\bar{x}}+A_{2} \bar{r} \\
& C L=\overline{\bar{x}} \\
& L C L=\overline{\bar{x}}-A_{2} \bar{r}
\end{aligned}
$$

R chart
$U C L=D_{4} \bar{r}$
$C L=\bar{r}$
$L C L=D_{3} \bar{r}$

THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$
\Phi(u)=\frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{u}-e^{-\frac{x^{2}}{2}} d x \text { FOR } 0.00 \leq u \leq 4.99 .
$$

| $u$ | . 00 | -1 | -02 | . 03 | . 04 | . 05 | .o6 | . 07 | -08 | -09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | . 5000 | . 5040 | -5080 | . 5120 | . 5160 | -5199 | -5239 | - 5279 | -5319 | 5359 |
| - 1 | -5398 | $\cdot 5438$ | -5478 | -5517 | -5557 | -5596 | -5636 | -5675 | -5714 | - 5753 |
| $\cdot 2$ | -5793 | . 5832 | -5871 | -5910 | -5948 | -5987 | . 6026 | . 6064 | -6103 | -6141 |
| $\cdot 3$. | . 6179 | -6217 | . 6255 | -6293 | .6331 | . 6368 | . 6406 | . 6443 | . 6480 | . 6517 |
| $\cdot 4$ | -6554 | . 6591 | . 6628 | . 6664 | . 6700 | . 6736 | . 6772 | . 6808 | . 6844 | . 6879 |
| -5 | -6915 | -6950 | . 6985 | -7019 | $\cdot 7054$ | $\cdot 7088$ | $\cdot 7123$ | $\cdot 7157$ | 7190 | 7224 |
| $\cdot 6$ | $\cdot 7257$ | $\cdot 7291$ | $\cdot 7324$ | $\cdot 7357$ | $\cdot 7389$ | 7722 | $\cdot 7454$ | $\cdot 7486$ | .7517 | 7549 |
| 7 | .7580 | -7611 | -7642 | -7673 | $\cdot 7703$ | . 7734 | $\cdot 7764$ | $\cdot 7794$ | 7823 | -7852 |
| . 8 | $\cdot 7881$ | $\cdot 7910$ | -7939 | -7967 | -7995 | . 8023 | -8051 | -8078 | .8106 | .8133 |
| '9 | . 8159 | . 8186 | . 8212 | . 8238 | . 8264 | . 8289 | .8315 | -8340 | . 8365 | . 8389 |
| 1.0 | . 8413 | . 8438 | .8461 | . 8485 | . 8508 | .8531 | . 8554 | - 8577 | . 8599 | -8621 |
| I.I | . 8643 | . 8665 | . 8686 | -8708 | . 8729 | . 8749 | -8770 | . 8790 | -8810 | . 8830 |
| 1. | . 8849 | -8869 | -8888 | . 8907 | . 8925 | -8944 | -8962 | . 8980 | -8997 | -90147 |
| $1 \cdot 3$ | -90320 | -90490 | -90658 | -90824 | -90988 | -91149 | -91309 | -91466 | -91621 | -91774 |
| 1.4 | $\cdot 91924$ | -92073 | -92220 | -92364 | -92507 | -92647 | $\cdot 92785$ | -92922 | -93056 | -93189 |
|  | -93319 | -93448 | -93574 | -93699 | . 93822 | -93943 | -94062 | -94179 | . 94295 | -94408 |
| 1.6 | -94520 | . 94630 | -94738 | -94845 | -94950 | -95053 | -95154 | -95254 | -95352 | -95449 |
| 1.7 | -95543 | -95637 | -95728 | -95818 | -95907 | -95994 | -96080 | -96164 | -96246 | : -96327 |
| I. 8 | - 96407 | - 96485 | -96562 | -96638 | -96712 | -96784 | -96856 | -96926 | -96995 | -97062 |
| 1.9 | -97128 | -97193 | $\cdot 97257$ | -97320 | -97381 | -97441 | . 97500 | -97558 | . 97615 | -97670 |
| 2.0 | -97725 | . 97778 | .97831 | . 97882 | -97932 | . 97982 | -98030 | - 98077 | -98124 | . 98169 |
| $2 \cdot 1$ | . 98214 | . 98257 | . 98300 | -98341 | . 98382 | -98422 | -98461 | . 98500 | -98537 | -98574 |
| 2.2 | -98610 | - 98645 | -98679 | -98713 | -98745 | -98778 | -98809 | - 98840 | 98870 | -98899 |
| $2 \cdot 3$ | -98928 | -98956 | . 98983 | - 920097 | -9 $9^{2} 0358$ | -920613 | $\cdot 9^{2} 0863$ | $9^{2} 1106$ | .$^{2} 1344$ | $\cdot 9^{2} 1576$ |
| $2 \cdot 4$ | -921802 | ${ }^{9}{ }^{2} 2024$ | $9^{92240}$ | $\cdot^{9} 22451$ | -9 $9^{2} 2656$ | $\cdot 9^{2} 2857$ | $9^{9} 3053$ | $\cdot^{-9} 3244$ | -923431 | -923613 |
| $2 \cdot 5$ | -923790 | - $9^{2} 3963$ | ${ }^{9}{ }^{2} 4132$ | - 924297 | -924457 | -9 $9^{2} 4614$ | -9 ${ }^{2} 4766$ | $\cdot 9^{2} 4915$ | $9^{2} 5060$ | -925201 |
| $2 \cdot 6$ | -925339 | - $9^{2} 5473$ | ${ }^{-9} 5604$ | $\cdot^{2} 5731$ | $9^{2} 5855$ | $9^{2} 5975$ | .$^{2} 6093$ | .$^{2} 6207$ | .$^{2} 6319$ | $9^{2} 6427$ |
| 2.7 | $\cdot^{2} 6533$ | - $9^{2} 6636$ | $\cdot{ }^{2} 6736$ | $\cdot^{-9} 6833$ | -926928 | ${ }^{-9} 97020$ | $\cdot^{2} 7110$ | . $9^{2} 7197$ | ${ }^{-9} 7282$ | ${ }^{9} 9^{2} 7365$ |
| 2.8 | $\cdot^{-9} 7445$ | $\cdot^{-9} 7523$ | -927599 | $\cdot 9^{2} 7673$ | -927744 | $\cdot^{2} 7814$ | $\cdot 9^{2} 7882$ | .$^{2} 7948$ | . $9^{2} 8012$ | $\cdot 9^{28074}$ |
| 2.9 | -928134 | $\cdot^{2} 8193$ | - $9^{2} 8250$ | .$^{2} 8305$ | - $9^{2} 8359$ | ${ }^{9} 9^{28411}$ | $\cdot 9^{2} 8462$ | . $9^{2} 8511$ | $\cdot 9^{2} 8559$ | ${ }^{-2} 8605$ |
| 3.0 | -9 $9^{2} 8650$ | -928694 | $\cdot 9^{2} 8736$ | -9 $9^{2} 8777$ | .9 ${ }^{28817}$ | -928856 | ${ }^{-9} 8893$ | - $9^{2} 8930$ | - $9^{2} 8965$ | - $9^{2} 8999$ |
| $3 \cdot 1$ | -930324 | .$^{3} 0646$ | . $9^{3} 0957$ | .$^{3} 1260$ | .$^{3} 1553$ | -93 $9^{1836}$ | $9^{3} 2112$ | $\cdot^{3} 2378$ | $9^{3} 2636$ | - $9^{3} 2886$ |
| $3 \cdot 2$ | -933129 | $\cdot 9^{3} 3363$ | $\cdot 9^{3} 3590$ | - $9^{3} 3810$ | -934024 | -9 $9^{3} 4230$ | ${ }^{-9} 4429$ | $\cdot^{3} 4623$ | ${ }^{-9} 4810$ | $9^{3} 4991$ |
| $3 \cdot 3$ | -935166 | $\cdot^{3} 5335$ | $9^{3} 5499$ | .$^{3} 5658$ | -9358II | - $9^{3} 5959$ | $9^{3} 6103$ | $\cdot 9^{3} 6242$ | ${ }^{-9} 6376$ | $\cdot 9^{3} 6505$ |
| $3 \cdot 4$ | -9 $9^{3} 6631$ | $\cdot 9^{3} 6752$ | -9 $9^{3} 8869$ | -9 $9^{3} 9882$ | $\cdot 9^{3} 7091$ | $\cdot 9^{3} 7197$ | $\cdot 9^{3} 7299$ | $\cdot 9^{3} 7398$ | $\cdot 9^{3} 7493$ | $\cdot 9^{3} 7585$ |
| $3 \cdot 5$ | $\cdot 9^{3} 7674$ | -937759 | $\cdot 9^{3} 7842$ | ${ }^{1} 97922$ | -937999 | -938074 | $9^{3} 8146$ | -938215 | $\cdot 9^{3} 8282$ | $\cdot 9^{3} 8347$ |
| 3.6 | -938409 | $9^{3} 8469$ | -9 ${ }^{3} 8527$ | -9 ${ }^{3} 8583$ | $9^{3} 8637$ | $9^{3} 8689$ | $9^{3} 8739$ | $9^{3} 8787$ | -9 $9^{3834}$ | $9^{3} 8879$ |
| 3.7 | ${ }^{-9} 8922$ | $9^{9} 8964$ | $\cdot 9^{4} 0039$ | -940426 | -940799 | $9^{4} 1158$ | ${ }^{-9} 1504$ | $\cdot 9^{4} 1838$ | $\cdot{ }^{4} 215159$ | ${ }^{-9}{ }^{4} 2468$ |
| 3.8 | -942765 | $\cdot{ }^{4} 3052$ | $\cdot{ }^{4} 3327$ | $\cdot{ }^{4} 43593$ | - 943848 | ${ }^{9} 9^{4} 4094$ | ${ }^{-9} 4331$ | ${ }^{-9} 44558$ | $\cdot{ }^{4} 4777$ | ${ }^{-9} 4988$ |
| 3.9 | -945190 | . 945385 | $\cdot{ }^{4} 5573$ | $\cdot{ }^{4} 5753$ | $\cdot{ }^{4} 5926$ | . 946092 | $\cdot 9^{4} 6253$ | $\cdot 9^{4} 6406$ | $\cdot 9^{4} 6554$ | $\cdot 9^{46696}$ |
| $4 \cdot 0$ | $\cdot 946833$ | -946964 | ${ }^{-9} 97090$ | .947211 | ${ }^{-9} 7327$ | -947439 | - 947546 | ${ }^{-9} 7649$ | ${ }^{-9} 7748$ | $\cdot{ }^{4} 7843$ |
| $4 \cdot 1$ | $\cdot 9^{4} 7934$ | . 948022 | -948106 | $\cdot 9^{4} 8186$ | $\cdot 9^{48263}$ | -948338 | $\cdot{ }^{4} 88409$ | $\cdot{ }^{4} 84777$ | -948542 | $\cdot{ }^{4} 8605$ |
| $4 \cdot 2$ | -948665 | - 948723 | $\cdot 9^{4} 8778$ | $\cdot 9^{4} 8832$ | $\cdot 9^{4} 8882$ | -948931 | $\cdot 9^{4} 8978$ | -950226 | $9^{5} 0655$ | $9^{5} 1066$ |
| $4 \cdot 3$ | - $9^{5} 1460$ | -9 ${ }^{5} 1837$ | -952199 | -9 $9^{5} 2545$ | $\cdot 9^{5} 2876$ | -953193 | -9 $9^{5} 3497$ | -9 $9^{5} 3788$ | $9^{5} 4066$ | .$^{5} 4332$ |
| 4.4 | -9 ${ }^{5} 4587$ | -954831 | .95 5065 | -95 5288 | - $9^{5} 5502$ | -9'5706 | $9^{5} 5902$ | -956089 | -9 ${ }^{5} 668$ | $9^{5} 6439$ |
|  | -956602 | -956759 | -956908 | -957051 | -957187 | -9 ${ }^{5} 7318$ | -95 7442 | -9 ${ }^{5} 7561$ | $9^{5} 7675$ | .$^{5} 7784$ |
| $4 \cdot 6$ | -957888 | -957987 | -95 ${ }^{5081}$ | -9 ${ }^{5} 8172$ | -9 ${ }^{5} 8258$ | -9 $9^{5} 340$ | -9 ${ }^{5} 8419$ | -9 $9^{5} 8494$ | -9 ${ }^{5} 8566$ | $9^{5} 8634$ |
| 4.7 | -958699 | -9 $9^{5} 8761$ | -958821 | -958877 | -9 $9^{5931}$ | -958983 | .960320 | . $9^{6} 0789$ | -9 $9^{6} 1235$ | $\cdot 9^{6} 1661$ |
| $4 \cdot 8$ | -9 $9^{6} 2967$ | - $9^{6} 2453$ | - $9^{6} 2822$ | -9 $9^{6} 3173$ | -9 $9^{6} 3508$ | -9 $9^{6} 3827$ | .$^{6} 4131$ | . $9^{6} 4420$ | -964696 | .$^{6} 4958$ |
| 4.9 | . $9^{6} 5208$ | . $9^{6} 5446$ | . $9^{6} 5673$ | -965889 | -966094 | -96289 | -9 $9^{6} 6475$ | -966652 | . 96682 I | $\cdot 9^{6} 698 \mathrm{I}$ |

Example: $\Phi(3.57)=\cdot 9^{3} 8215=0.9998215$.

FRACTILES OF THE $t$ DISTRIBUTION. $t_{1-p}=-t_{p}$.

| $P$ | Probability in per cent |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f$ | 60 | 70 | 80 | 90 | 95 | $97 \cdot 5$ | 99 | $99 \cdot 5$ | 99.9 | 99.95 |
| 1 | $\cdot 325$ | $\cdot 727$ | I. 376 | 3.078 | 6.314 | 12.71 | $31 \cdot 82$ | 63.66 | 318.3 | $636 \cdot 6$ |
| 2 | -289 | . 617 | 1.061 | I. 886 | 2.920 | $4 \cdot 303$ | 6.965 | 9.925 | 22.33 | $31 \cdot 60$ |
| 3 | -277 | -584 | $\cdot 978$ | 1.638 | $2 \cdot 353$ | $3 \cdot 182$ | $4 \cdot 5+1$ | 5.84 I | 10.22 | $12.9+$ |
| 4 | -271 | -569 | '941 | 1.533 | $2 \cdot 132$ | 2.776 | 3.747 | $4 \cdot 604$ | 7•173 | 8.610 |
| 5 | -267 | -559 | . 920 | 1.476 | 2.015 | 2.571 | $3 \cdot 365$ | 4.032 | $5 \cdot 893$ | 6.859 |
| 6 | - 265 | -553 | -906 | 1.440 | 1.943 | 2.447 | $3 \cdot 143$ | $3 \cdot 707$ | $5 \cdot 208$ | 5.959 |
|  | -263 | -549 | -896 | 1.415 | I. 895 | $2 \cdot 365$ | 2.998 | $3 \cdot 499$ | 4.785 | $5 \cdot 405$ |
| 8 | -262 | -546 | . 889 | I-397 | I. 860 | $2 \cdot 306$ | $2 \cdot 896$ | $3 \cdot 355$ | $4 \cdot 501$ | $5 \cdot 041$ |
| 9 | -261 | -543 | . 883 | 1.383 | I. 833 | $2 \cdot 262$ | 2.821 | 3.250 | $4 \cdot 297$ | $4 \cdot 781$ |
| 10 | - 260 | - 542 | . 879 | 1.372 | I.812 | $2 \cdot 228$ | $2 \cdot 764$ | 3.169 | $4 \cdot 144$ | 4.587 |
| II | . 260 | - 540 | . 876 | I. 363 | 1.796 | $2 \cdot 201$ | 2.718 | 3.106 | 4.025 | $4 \cdot 437$ |
| 12 | -259 | - 539 | . 873 | 1.356 | 1.782 | $2 \cdot 179$ | 2:681 | 3.055 | 3.930 | $4 \cdot 318$ |
| 13 | - 259 | . 538 | . 870 | I.350 | 1.771 | $2 \cdot 160$ | $2 \cdot 650$ | 3.012 | 3.852 | 4.221 |
| 14 | - 258 | . 537 | . 868 | I. 345 | 1.761 | $2 \cdot 145$ | 2.624 | 2.977 | 3.787 | 4. 140 |
| 15 | - 258 | . 536 | . 866 | 1.341 | 1.753 | $2 \cdot 131$ | 2.602 | 2.947 | 3.733 | 4.073 |
| 16 | - 258 | - 535 | . 865 | I. 337 | 1.746 | $2 \cdot 120$ | $2 \cdot 583$ | 2.921 | $3 \cdot 686$ | 4.015 |
| 17 | -257 | -534 | . 863 | I. 333 | 1.740 | $2 \cdot 110$ | $2 \cdot 567$ | 2.898 | 3.646 | 3.965 |
| 18 | - 257 | -534 | . 862 | I 330 | 1.734 | $2 \cdot 101$ | 2.552 | 2.878 | $3 \cdot 611$ | 3.922 |
| 19 | - 257 | -533 | .86I | I-328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 | 3.883 |
| 20 | -257 | $\cdot 533$ | . 860 | I.325 | 1.725 | 2.086 | $2 \cdot 528$ | 2.845 | 3.552 | $3 \cdot 850$ |
| 21 | - 257 | $\cdot 532$ | . 859 | 1-323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | 3.819 |
| 22 | -256 | -532 | -858 | I-321 | 1.717 | 2.074 | $2 \cdot 508$ | 2.819 | 3.505 | 3.792 |
| 23 | $\cdot 256$ | $\cdot 532$ | . 858 | 1-319 | 1.714 | 2.069 | $2 \cdot 500$ | 2.807 | 3.485 | 3.767 |
| 24 | -256 | -531 | . 857 | 1.318 | 1.711 | 2.064 | 2.492 | $2 \cdot 797$ | 3.467 | 3.745 |
| 25 | -256 | -531 | . 856 | 1.316 | 1.708 | 2.060 | 2.485 | $2 \cdot 787$ | 3.450 | 3.725 |
| 26 | $\cdot 256$ | -531 | . 856 | $1 \cdot 315$ | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 | $3 \cdot 707$ |
| 27 | -256 | -531 | . 855 | I-314 | 1.703 | 2.052 | 2.473 | 2.77 x | $3 \cdot 421$ | 3.690 |
| 28 | . 256 | . 530 | . 855 | I.313 | I.701 | 2.048 | 2.467 | $2 \cdot 763$ | $3 \cdot 408$ | 3.674 |
| 29 | -256 | . 530 | . 854 | I.3II | I. 699 | 2.045 | $2 \cdot 462$ | $2 \cdot 756$ | 3.396 | 3.659 |
| 30 | - 256 | -530 | -854 | 1.310 | 1. 697 | 2.042 | $2 \cdot 457$ | $2 \cdot 750$ | $3 \cdot 385$ | 3.646 |
| 40 | -255 | -529 | . 851 | 1-303 | r.684 | 2.021 | 2.423 | $2 \cdot 704$ | $3 \cdot 307$ | 3.551 |
| 50 | -255 | -528 | . 849 | I. 298 | r. 676 | 2.009 | 2.403 | 2.678 | 3.262 | 3.495 |
| 60 | -254 | . 527 | . 848 | I. 296 | r.671 | 2.000 | $2 \cdot 390$ | 2.660 | 3.232 | $3 \cdot 460$ |
| 80 | -254 | $\cdot 527$ | . 846 | 1-292 | r. 664 | 1.990 | 2.374 | 2.639 | 3.195 | 3.415 |
| 100 | -254 | $\cdot 526$ | . 845 | I. 290 | 1.660 | 1.984 | $2 \cdot 365$ | 2.626 | 3.174 | $3 \cdot 389$ |
| 200 | - 254 | $\cdot 525$ | . 843 | I. 286 | I. 653 | 1.972 | $2 \cdot 345$ | 2.601 | $3 \cdot 131$ | $3 \cdot 339$ |
| 500 | -253 | $\cdot 525$ | . 842 | I. 283 | I. 648 | 1.965 | 2.334 | 2.586 | 3.106 | 3.310 |
| $\infty$ | -253 | - 524 | . 842 | I-282 | I. 645 | I. 960 | $2 \cdot 326$ | $2 \cdot 576$ | 3.090 | 3.291 |
| $2(\mathrm{I}-\mathrm{P}$ ) | 80 | 60 | 40 | 20 | 10 | 5 | 2 | I | 0.2 | $0 \cdot 1$ |

Example: $P\{t<2.086\}=97.5 \%$ for $f=20$.
$P\left\{|t|>t_{P}\right\}=2(1-P) . P\{|t|>2.086\}=5 \%$ for $f=20$.
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