Engineering Tripos Part IIA Module 3C1
Manufacturing Engineering Tripos Part I Module 3P1
2011

Question 1.

Take unit depth into the page, principal directions shown, and compressive stress +ve.

(a) Horizontally: $h\,\dfrac{\mathrm{d}\sigma_1}{\mathrm{d}h} + \sigma_1 - p - mk\cot\alpha = 0$ (1)

Vertically: $\sigma_2 + mk\tan\alpha - p = 0$ (2)

von Mises criterion in plane strain: $\sigma_2 - \sigma_1 = \dfrac{2Y}{\sqrt{3}}$

Substitute for σ_2 into eqn. (2): $\sigma_1 + \dfrac{2Y}{\sqrt{3}} + mk\tan\alpha - p = 0$

Then substitute for p in eqn. (1): $h\,\dfrac{\mathrm{d}\sigma_1}{\mathrm{d}h} + \sigma_1 - \left(\sigma_1 + \dfrac{2Y}{\sqrt{3}} + mk\tan\alpha\right) - mk\cot\alpha = 0$

$$\Rightarrow \mathrm{d}\sigma_1 = \frac{Y}{\sqrt{3}}\left(2 + m\tan\alpha + m\cot\alpha\right)\frac{\mathrm{d}h}{h}$$ (note: also substituting $k = \dfrac{Y}{\sqrt{3}}$)

Integrating with boundary conditions $\sigma_1 = 0$ for $h = h_i$ and $\sigma_1 = -\sigma_{draw}$ for $h = h_o$:

$$\int_0^{-\sigma_{draw}} \mathrm{d}\sigma_1 = \frac{Y}{\sqrt{3}}\left(2 + m\tan\alpha + m\cot\alpha\right)\int_{h_i}^{h_o}\frac{\mathrm{d}h}{h}$$

Hence: $\sigma_{draw} = \dfrac{Y}{\sqrt{3}}\left(2 + m\tan\alpha + m\cot\alpha\right)\, ln\left(\dfrac{h_i}{h_o}\right)$

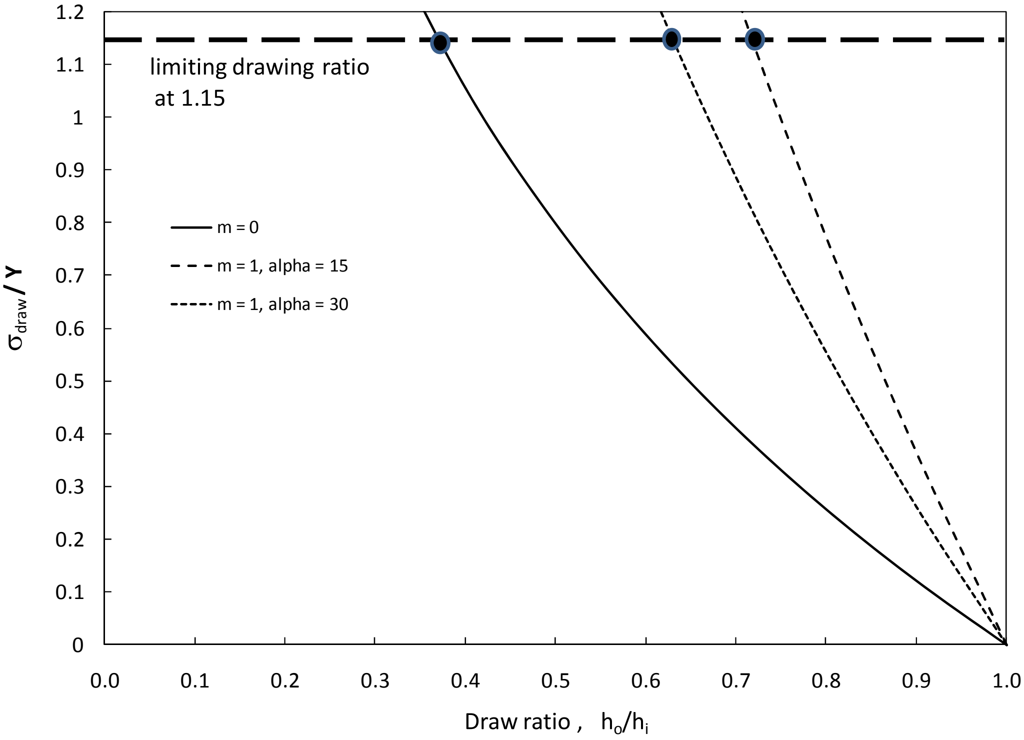
Assumptions:

- homogeneous deformation, constant yield stress (i.e. neglect deformation heating)
- principal stresses assumed to be in 1 and 2 directions: correct on the centre-line, and also throughout the element when frictionless. But when sticking friction occurs, the principal axes will deviate from the 1,2 directions as the die wall is approached since the shear stresses become significant compared to the direct stresses. Hence it is an approximation to treat σ_1 and σ_2 as principal stresses
- von Mises yield criterion is itself a model for the real material behaviour

(b) For $m = 0$, expression simplifies to: $\sigma_{draw} = \dfrac{2Y}{\sqrt{3}}\, ln\left(\dfrac{h_i}{h_o}\right)$ i.e. independent of die semi-angle.

For $\alpha = 30^\circ$: $\tan\alpha = 0.577$, $\cot\alpha = 1.732$

Substitute into formula and plot.



Note that yield at exit is given by $\sigma_{draw} = \dfrac{2Y}{\sqrt{3}}$ (von Mises in plane strain), and not $\sigma_{draw} = Y$.

Hence drawing limit reached when $\dfrac{\sigma_{draw}}{Y} = \dfrac{2}{\sqrt{3}} = 1.15$

Solving for h_o/h_i in each case:

For $m = 0$ (independent of α): 0.37

For $m = 1$, $\alpha = 30^\circ$: 0.63

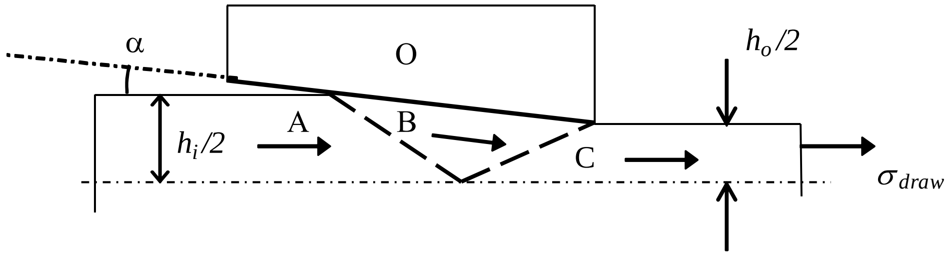
Sticking friction ($m = 1$) significantly increases the limiting draw ratio compared to the frictionless case ($m = 0$).

(c) Consider half of the problem due to symmetry, discretising the workpiece into sliding blocks with planes of intense shear (shown dashed in figure below). Impose unit velocity at outlet, and construct hodograph for relative velocities of A,B,C relative to stationary die O (easiest for specific values of die semi-angle, inlet and outlet thicknesses).

Find lengths of OB, AB, BC and the relative velocities across these interfaces.

Internal work-rate per unit depth = length × relative velocity × $m\,k$ (with m = 1 for AB, BC, and the appropriate value between 0 and 1 for OB).

Equate to external work-rate per unit depth = $\sigma_{draw} \times h_o/2$ × unit velocity



Question 2.

(a)

Grey cast iron is a near-eutectic alloy containing graphite flakes in an iron matrix. Graphite has very low density so the precipitation of graphite is accompanied by expansion, which counterbalances the contraction of the iron as it solidifies. At a specific carbon concentration (just above the eutectic composition) there is zero volume change on solidification. This enhances *castability* because it means that the metal takes the exact shape of the mould with no dimensional change, and no allowance for feeding needs to be made. Added to the advantages of a near-eutectic alloy (minimum melting temperature, so reduced heating costs and minimised problems from reactivity; low solidification temperature range so high fluidity) this makes grey cast iron *highly castable*.

(b)

The graphite flakes act as cracks. Strength is determined by the crack length, according to the relationship $K_{1C} = \sigma\sqrt{\pi a}$. Flake size is inversely proportional to the velocity of the solidification interface. Thicker sections solidify more slowly, so will have bigger flakes and lower strength.
The addition of magnesium poisons the growth of the graphite flakes, blocking the steps which are the favoured sites for the addition of carbon atoms and forcing them to grow by adding atoms onto planar interfaces This changes the precipitate shape from flakes into spheroidal nodules. These do not act as sharp cracks, so the tensile strength and the ductility are increased.

(c)

This Cu-Sn alloy has a large semi-solid region (see phase diagram) and there will be a lot of segregation on solidification. The as-cast structure will have dendrites with relatively pure 'backbones' and impure (high Sn) interdendritic and grain boundary regions. To homogenise the structure, the only option is to heat-treat and to use diffusion to equilibrate the chemical composition across the grains. Recrystallisation is not appropriate because it would require deformation (followed by heat-treatment), destroying the benefit of producing the casting to the correct shape in the first place.
Heat treatment involves heating the casting to above about $0.7T_M$ for a time given by $t = x^2/D$ where the diffusion distance x is approximately half the grain size. Heat treatment time is minimized by

minimizing the diffusion distance, so minimizing the grain size. This is achieved by increasing the number of nuclei on solidification using one or more of the following measures: inoculation; pouring at close to the melting temperature; vibrating the mould to displace dendrite arms.
The temperature determines diffusion rates by $D = D_0 \exp - Q/RT$
It is chosen as a trade-off between reduced process time against increased costs associated with heating to a higher temperature.

(d)
Extrude polythene through a die, which causes some chain alignment by *extension flow* [sketch of molecules being grabbed by die walls]. The thread is then cold-drawn, resulting in further chain alignment by *draw-strengthening*.
The modulus will be increased by increasing the chain-length: the chain-ends are sources of weakness in the structure, and there are fewer such sites with long molecules. The highest modulus is obtained with ultra-high molecular weight polythene.

Question 3.

(a)

Quenching and tempering a low alloy steel cylindrical bar:

- austenitise to dissolve C in solution (typically 850°C):

Austenitisation temperature (process variable) must be high enough and for long enough to fully dissolve C in solution, without grain growth. The temperature must be above the lower limit of austenite field (material variable).

- quench to room temperature, with objective of forming a minimum fraction of martensite (e.g. > 50%) at the centre of the component.

Imposed cooling rate depends on heat flow, governed by quenching medium, e.g. air, oil, water (process variable), the diameter and length of the bar (design variable), thermal properties of the steel (material parameter – though assumed constant across all CCT diagrams).

The amount of martensite formed depends on the hardenability of the steel (material characteristic, dependent on composition).

- temper at intermediate temperature to precipitate carbides, restoring useful toughness in combination with high strength:

Tempering temperature and time (process variables) determine extent of precipitation and exact combination of hardness and toughness achieved. Hardness also influenced by type of precipitate formed (iron carbide or alloy carbides) which depends on alloy composition (material).

(b)
Arc welding a medium carbon steel plate:
- traverse heat source over joint region, usually with filler, fusing joint with a molten pool solidifying a short distance behind the heat source. The surrounding heat-affected zone is subjected to a temperature history with a peak below melting, but high enough to modify the microstructure. Peak temperature falls with distance from the fusion boundary.

Thermal cycle (particularly peak temperature and cooling rate) governed by heat flow, depending on: power and traverse speed of arc (process variables), thickness and geometry of joint (design), thermal properties of the steel (material).

The phases formed in the heat-affected zone depend on peak temperature and cooling rate: the first determines whether austenite is formed, the second on the phases formed on cooling. If a maximum hardness is prescribed, this reflects the objective of avoiding martensite formation (and thus forming a brittle weldment). This depends on the hardenability of the steel (material characteristic).

(c)

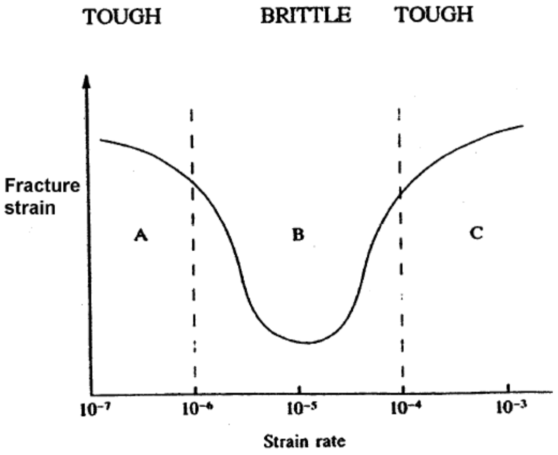
Thermal residual stresses can result when only part of a component is heated, or when different parts are cooled at different rates (e.g. surface and centre). When material is heated locally (e.g. by flame, resistance heating, induction heating, laser), the hot region expands. The hot region is constrained by the cooler surrounding material, and so is put into compression. However, being hot, its yield stress is reduced, so it can plastically deform relatively easily (and it may also creep). Where the material can flow to depends on the geometry. When the region cools, there is contraction, and the surrounding regions are put into tension.

Tensile stresses are generally of the order of the material yield stress, and are balanced by compressive stresses elsewhere. Residual stresses can cause problems with distortion, and where tensile can lead to crack initiation and growth, especially under cyclic loading (fatigue cracks) and during cooling from the welding process itself. In certain environments they may lead to stress corrosion cracking.

Residual stresses can be reduced by pre-heating the part before welding, reducing the cooling rate, stress relief by post-weld heat treatment, and also by appropriate design of the welding process (including selection of welding method, process parameters and welding sequence).

(d)

This is an example of strain-rate sensitivity in stress corrosion cracking. At low strain-rates, the metal surface can re-passivate. At high strain-rates, the rate of crack growth is limited by the rate at which the reactive agent (in this case, chloride ions) can be transported to the crack tip. There is therefore a critical strain-rate range for SCC as shown in the sketch below.



Question 4

(a)
Production of a ceramic component by a powder route involves: production of powder (by chemical route or crushing/grinding); blending with any additives (e.g. sintering aids, grain growth inhibitors, lubricant, binder, alloying additives); pressing in a shaped die or mould; sintering at around 0.7 – 0.9 T_m. In uniaxial pressing the powder is compressed in the die in one direction (along its axis) by one or more punches and removed in the same direction (optional diagram). In cold isostatic pressing the die is a deformable rubber mould which is compressed by external oil pressure by means of a piston (diagram) – in this case the compression is isotropic.

(b)
The presence of the internal step means that in order to produce a constant density of compact at least two stepped punches would have to be used in uniaxial pressing – adding complexity to the design and operation of the press tool. Also, the external groove cannot be formed in uniaxial pressing as it would not be possible to remove the part from the die, and it would have to be formed either by machining of the compact before sintering (i.e. green machining) or after sintering (by a slower, but potentially more precise, grinding process). Isostatic pressing, in contrast, would allow the whole external shape of the component to be formed in a single process step, although a green machining process would be needed to form the stepped bore (since a 'waisted' mandrel could not be removed from the green component). Choice between the two routes would depend on part size, production volume, dimensional precision (tolerances) needed especially on the external groove, and the properties required.

(c)
High tensile stress along the axis would lead to high stress at the internal corners of the internal step and the external groove – both then being potential sites for fracture initiation. Possible methods to ensure that the component does not fail in this way: modify design (if possible) by introduction of radii at the these corners to reduce stress concentration; ensure that particle size in powder is as small as possible to minimize size of maximum flaws due to inclusions, porosity etc.; reduce grain size by using grain growth inhibitor and/or reducing sintering time/temperature; achieve minimum porosity by addition of hot isostatic pressing (HIP) stage after sintering.

(d)
Joint is to be made between alumina and steel. Fusion welding methods are out of the question as they require both components to be melted: possible methods are brazing using a suitable fusible filler alloy or adhesive bonding (e.g. with a thermoset polymer such as epoxy resin). There would be problems with joint integrity for both methods due to the different thermal expansion coefficients of the materials – steel expands more than alumina. With a suitably ductile brazing alloy, brazing would potentially allow higher operating temperature than adhesive bonding. Exposure to wet conditions in the case of adhesive bonding could lead to degradation of the adhesive over time. Friction welding and diffusion bonding are also possible candidates – but would need research.

Examiner's comments:

q.1 Quite a few candidates assumed that they could use the Tresca criterion instead of von Mises and simply apply a numerical factor to get the answer. Marks were lost by candidates who failed to comment on the assumptions in part (a). Several assumed that in plane strain $\sigma_3 = 0$. Nearly all assumed that at the limiting draw ratio $\sigma_d = Y$, despite this being plane strain and a von Mises material.

q.2 A popular and straightforward question with some good answers. There was confusion over the microstructure of grey cast iron, with some candidates ignorant of the existence of graphite and referring to needles or flakes of silicon, or FeS.

q.3 Popular question which demanded wide knowledge across the course. Some misunderstanding of the processes which occur during tempering of martensite. Weaker candidates were unable to identify and explain SCC in part (d)

q.4 Reasonable recall of the process steps in part (a) although some thought that heat was applied during pressing.