Version GRH/3

EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 25 April 2023 9.30 to 11.10

Module 4A10

FLOW INSTABILITY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4A10 data sheet (two pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 One deep layer of inviscid fluid of density ρ_2 flows with uniform horizontal speed U in the x-direction over another deep layer of density ρ_1 which is at rest. There is surface tension γ at the interface $y = \eta(x, t)$ between the two fluids, t denoting time. A small disturbance of the form

$$\eta(x, t) = \hat{\eta}\, e^{i(kx-\omega t)}$$

travels on the interface, where $\hat{\eta}$ is a constant. A linear stability analysis of this system gives the following dispersion relationship for the frequency ω as a function of the horizontal wavenumber k of the disturbance

$$(\rho_1 + \rho_2)\,\omega - \rho_2 U k = \pm \left[(\rho_1 + \rho_2)|k| \left\{k^2\gamma + (\rho_1 - \rho_2)\, g\right\} - \rho_1\rho_2 U^2 k^2\right]^{\frac{1}{2}}$$

Take k to be real throughout.

(a) By writing $\omega = \omega_R + i\,\omega_I$, show that $\omega_I > 0$ corresponds to an exponential growth in time of the disturbance on the interface. [10%]

(b) Using the dispersion relation given above, show that the condition for the exponential growth may be written

$$\frac{\rho_1\rho_2}{\rho_1 + \rho_2} U^2 > \min_k \left\{(\rho_1 - \rho_2)\,\frac{g}{|k|} + |k|\gamma\right\} \qquad [30\%]$$

(c) Comment on the role of the surface tension and the density difference on the stability of the system. [10%]

(d) With reference to the condition in (b) or otherwise, sketch the associated ‘stability loop’ in U-$|k|$ space. Label the regions for linearly stable and unstable flows. Label any other salient features of note. [10%]

(e) Show that as U increases, the first wave to go unstable has $k^2 = g(\rho_1 - \rho_2)/\gamma$. [20%]

(f) Calculate the wind speed necessary to drive waves on the surface of an otherwise stationary freshwater lake. Take the density of fresh water as 1000 kg/m^3, the density of air as 1.25 kg/m^3, and the surface tension to be 0.074 kg/s^2. [20%]

2 (a) A liquid fills the annular gap formed between two long concentric cylinders. The inner cylinder of radius r_1 rotates about its vertical axis with angular velocity Ω_1. The outer cylinder of radius r_2 rotates with angular velocity Ω_2 about the same axis. You may take $\Omega_1 > 0$, $\Omega_2 > 0$ and $r_2 > r_1$. Consider the stability of this Taylor-Couette problem and show that stability for an inviscid fluid requires

$$\frac{d}{dr}\left[(\Omega r^2)^2\right] \geq 0$$

where Ω denotes the angular velocity at a radial distance r from the vertical axis. [35%]

(b) Consider a slender incompressible cylindrical liquid jet of density ρ propagating horizontally in air. You may assume that the diameter d of the jet is sufficiently small that surface tension γ cannot be neglected. The jet is subject to a small amplitude perturbation.

(i) Use dimensional arguments to develop a scaling for the growth rate s of the perturbation.

(ii) By considering sections through the jet perpendicular to its longitudinal axis, discuss with clear physical reasoning the stability of the jet to both axisymmetric and non-axisymmetric disturbances. To support your discussion provide clearly labelled schematics that illustrate the jet for circumferential wavenumbers of $n = 2$ and $n = 4$. [35%]

(c) A fluid subject to a horizontal velocity $u(z)$ is initially stably stratified with a smoothly varying density profile $\rho(z)$, where z denotes the vertical coordinate. Assuming $du/dz > 0$, develop an energy-based argument to show that the flow is unstable for

$$\frac{-g}{\rho_0}\frac{d\rho}{dz} < \frac{1}{4}\left(\frac{du}{dz}\right)^2$$

where g is the acceleration due to gravity and ρ_0 is a reference density. [30%]

3 A cylinder with radius a_1 moving horizontally at speed V_1 is placed concentrically inside a cylinder with radius a_2 moving horizontally at speed V_2, as shown in Fig. 1. For small displacements of the cylinders, the velocity potential of the inviscid incompressible flow between the cylinders is

$$\phi = c_1 r \cos\theta + \frac{c_2}{r}\cos\theta \quad \text{where} \quad c_1 = \frac{V_2 a_2^2 - V_1 a_1^2}{a_2^2 - a_1^2} \quad \text{and} \quad c_2 = \frac{(V_2 - V_1) a_1^2 a_2^2}{a_2^2 - a_1^2}$$

(a) By considering frames of reference moving at speeds V_1 and V_2, or otherwise, show that this flow satisfies the necessary boundary conditions. [20%]

(b) The outer cylinder is held stationary while the inner cylinder is vibrated with a sufficiently small displacement that the above velocity potential is accurate and that the unsteady Bernoulli equation can be written

$$p = p_\infty - \rho \frac{\partial \phi}{\partial t}$$

Derive an expression for the added mass of the inner cylinder and sketch this as a function of a_2/a_1. Explain this variation on a physical basis. [50%]

(c) Describe, with the aid of sketches, how and why the added mass and added damping will vary with a_2/a_1 and with the viscosity of the fluid. [30%]

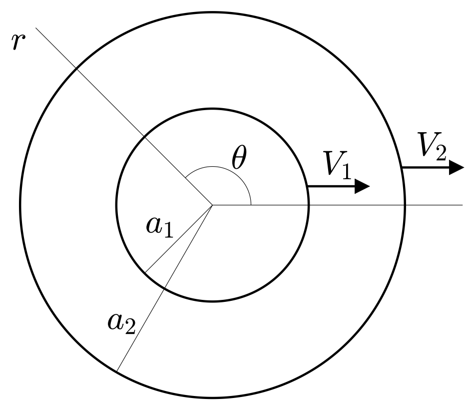

Fig. 1

4 A hydrofoil with chord c, mass per unit length m, and lift coefficient $C_L(\theta)$, is attached to a wall by two identical translational spring/damper systems as shown in Fig. 2(a). The springs have stiffness k and damping coefficient b. The two systems are separated by a distance d. This arrangement is placed in water of density ρ flowing at velocity U. This can be modelled by a single translational spring with stiffness $k_y = 2k$ and damping coefficient $b_y = 2b$, combined with a single torsional spring with stiffness $k_\theta = kd^2/2$ and damping coefficient $b_\theta = bd^2/2$, as shown in Fig. 2(b). The springs attach to the hydrofoil on the elastic axis. The aerodynamic centre of pressure is a distance c_a upstream of the elastic axis. The centre of mass is a distance S_x/m downstream of the elastic axis. For vertical displacement, y, and angular displacement relative to the flow direction, θ, the translational and torsional equations of motion are

$$\begin{aligned} m\ddot{y} - S_x\ddot{\theta} + b_y\dot{y} + k_y y &= F_y \\ I_\theta\ddot{\theta} - S_x\ddot{y} + b_\theta\dot{\theta} + k_\theta\theta &= F_\theta \end{aligned}$$

where I_θ is the moment of inertia about the elastic axis, F_y (positive upwards) is the flow-induced vertical force and F_θ (positive clockwise) is the flow-induced moment resolved around the elastic axis. F_y and F_θ can be modelled with

$$\begin{aligned} F_y &= \frac{1}{2}\rho U^2 c \left.\frac{\partial C_L}{\partial \theta}\right|_{\theta=0} \theta = q\theta \\ F_\theta &= c_a F_y \end{aligned}$$

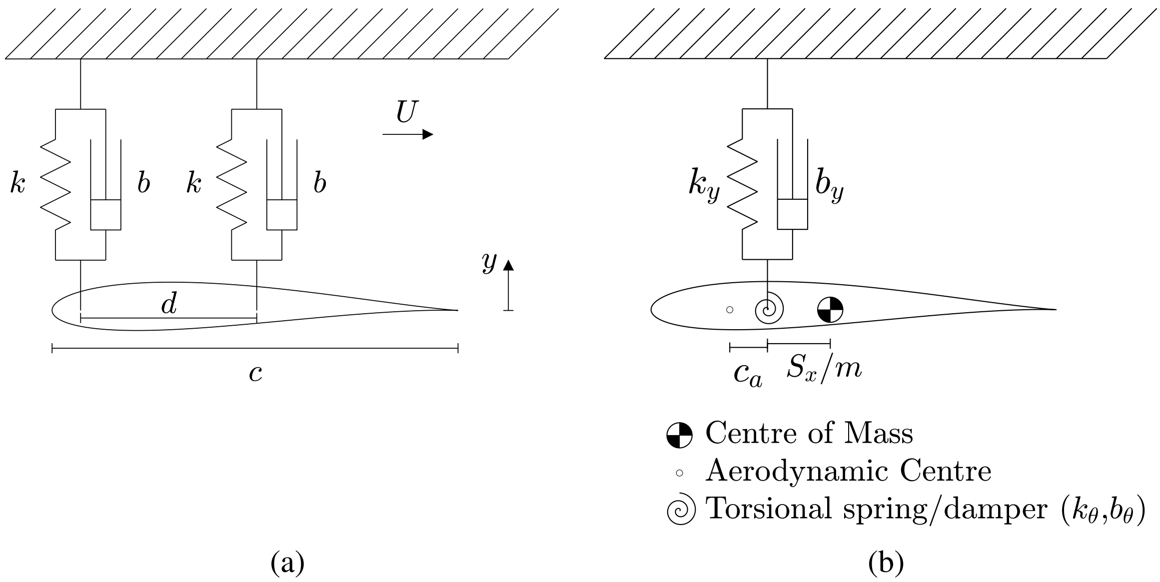

Fig. 2

(a) Assuming that perturbations to y and θ are proportional to $\exp(st)$, with t denoting time, and setting $b = 0$, show that [30%]

$$s^2 = \frac{-C_2 \pm (C_2^2 - 4C_0C_4)^{1/2}}{2C_0}$$

$$\text{where} \quad \begin{aligned} C_0 &= mI_\theta - S_x^2 \\ C_2 &= mk_\theta + I_\theta k_y - qc_a m - qS_x \\ C_4 &= k_y k_\theta - qc_a k_y \end{aligned}$$

(b) Find the condition on C_0, C_2, and C_4 at which the system will start to oscillate from rest (flutter). [10%]

(c) Without further calculations, describe how the condition in (b) will change when the damping coefficient, b, is positive. How could the centre of mass be moved to make the system more susceptible to flutter? [20%]

(d) When fluttering with $b \neq 0$, energy is dissipated in the dampers. Where does this energy come from? What happens to the drag on the hydrofoil when it starts to flutter? [10%]

(e) A hydrofoil board is a short surf board attached to an underwater hydrofoil as shown in Fig. 3. With reference to your previous answers, but without further calculations, describe how a surfer could propel the board forwards in stagnant water. [30%]

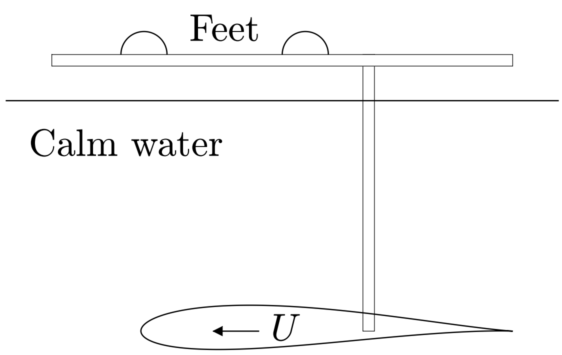

Fig. 3

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