Version mpj06

EGT3
ENGINEERING TRIPOS   PART IIB

Wednesday 23 April 2014   9.30 to 11

Module 4A10

FLOW INSTABILITY

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
4A10 data sheet (two pages)
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Consider the classical Taylor-Couette problem, which is for a viscous fluid in the narrow gap between two long concentric co-rotating circular cylinders. For this flow, you may assume that

$$-\frac{1}{\rho}\frac{\partial p}{\partial r} = -\frac{u_\theta^2}{r},$$

for the cylindrical polar coordinate system (r, θ, x). Provide a physical argument to establish the Rayleigh criterion, namely that the flow is stable provided

$$\frac{d}{dr}\left(\Omega r^2\right)^2 \geq 0$$

throughout the fluid. All variables have their usual meanings. [25%]

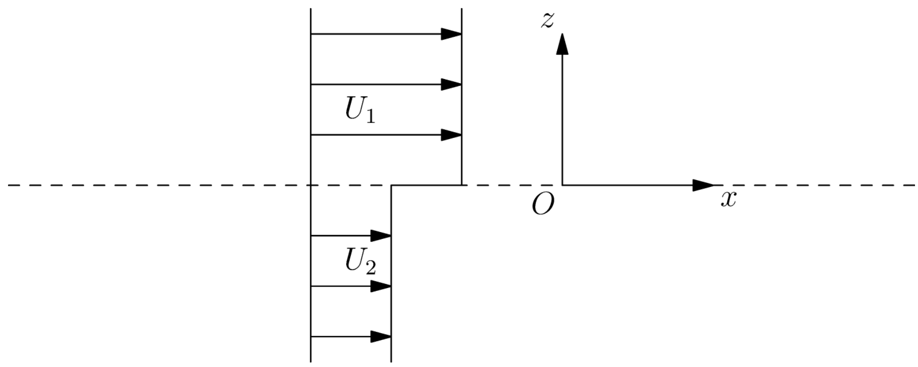

Fig. 1

(b) Consider a two-dimensional inviscid mixing layer with velocity profile

$$\boldsymbol{u} = \begin{cases} U_1 \underline{i} & \text{for } z > 0, \\ U_2 \underline{i} & \text{for } z < 0, \end{cases}$$

as shown in Fig. 1. Investigate the temporal stability of this flow by considering small disturbances to the interface of the vortex sheet of the form

$$\eta'(x,t) = \hat{\eta} e^{ikx+st}$$

to show that the growth rate is governed by a quadratic in s. [75%]

2 (a) A slender circular free jet of an inviscid incompressible liquid of density ρ and surface tension γ has an undisturbed diameter a and discharges into air. Adopting a cylindrical polar coordinate system (r, θ, x) and taking normal modes of the form

$$(\hat{\boldsymbol{u}}(r), \hat{p}(r), \hat{\eta})\, e^{i(kx+n\theta)+st} \tag{1}$$

for the perturbations of velocity, pressure and interface position, respectively, a linear stability analysis yields a growth rate of

$$s^2 = \frac{\gamma}{a^3\rho}\alpha\frac{I_n'(\alpha)}{I_n(\alpha)}\left(1-\alpha^2-n^2\right), \quad \alpha = ka. \tag{2}$$

In (1), k is the axial wave number and n is the circumferential wave number. In (2), $I_n(\alpha)$ and $I_n'(\alpha)$ denote the modified Bessel function of the first kind and its derivative with respect to radius r, respectively.

(i) Evaluate the pressure at the surface of the undisturbed jet using the Laplace result $p - p_\infty = \gamma\nabla\cdot\hat{\boldsymbol{n}}$ ($\hat{\boldsymbol{n}}$ denoting the unit outward normal) and, hence, or otherwise, discuss the physical mechanism by which such a circular jet may break up. Use diagrams to complement your discussion. [15%]

(ii) State the basic requirement on the growth rate for the flow to remain stable. Given $\alpha I_n'(\alpha)/I_n(\alpha)$ is positive and less than unity for all $\alpha \neq 0$, deduce that the circular jet is stable to all non-axisymmetric modes, and unstable to axisymmetric modes whose wavelengths exceed the undisturbed circumference of the jet. [15%]

(b) A planar free jet of an inviscid incompressible liquid of density ρ and surface tension γ has a width of $2a$ and discharges into air. The jet flows in the x-direction and has free surfaces at $z = \pm a$. Show that the stability of the jet is governed by:

$$\nabla^2 p' = 0, \quad -\infty < x,\ y < \infty,\ -a < z < a,$$

for a perturbation p' of the steady state, with boundary conditions

$$\rho\frac{\partial^2 p'}{\partial t^2} = \pm\gamma\left(\frac{\partial^2}{\partial x^2}\right)\frac{\partial p'}{\partial z} \quad \text{at} \quad z = \pm a.$$

Take normal modes of the form $p' \propto e^{ikx+st}$ to show that, for the upper surface,

$$\frac{a^3\rho}{\gamma}s^2 = -\alpha^3\tanh(\alpha), \quad \alpha = ka,$$

and, hence, explain why the jet is stable.

Hint: $\rho\partial\boldsymbol{u}'/\partial t = -\nabla p'$ and $\nabla\cdot\boldsymbol{u}' = 0$. Also $p' = -\gamma\nabla\cdot\hat{\boldsymbol{n}}$, $\hat{\boldsymbol{n}} = \pm\left(-\partial\eta'/\partial x, 0, 1\right)$ and $w' = \pm\partial\eta'/\partial t$, at $z = \pm a$. [70%]

3 Air with density ρ and speed U flows over a bridge deck of mass m, height D, and stiffness k, as shown in Fig. 2.

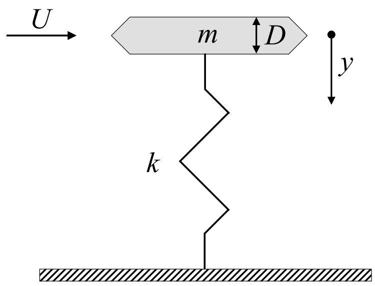

Fig. 2

The damping factor, ζ, is small and the undamped natural frequency is ω. The bridge deck remains horizontal and its vertical displacement, y, about equilibrium satisfies:

$$m\ddot{y} + p\dot{y} + ky = qU^2 c_y$$

where $p = 2m\zeta\omega$ and $q = \rho D/2$. The vertical force coefficient of the bridge deck, c_y, is related to the angle of attack, α, by

$$c_y = -\alpha + r\alpha^3 - s\alpha^5$$

where r and s are positive constants. The total energy of the system is $E = m\dot{y}^2/2 + ky^2/2$.

(a) Show that the bridge deck is stable to soft excitation at all values of U. [20%]

(b) By considering oscillations of the form $y = a\cos\omega t$, show that the change of energy in one cycle, ΔE, is

$$\Delta E = -c_1 a^2 - c_2 U a^2 + c_3 \frac{a^4}{U} - c_4 \frac{a^6}{U^3}$$

where $c_1 = \pi p\omega$, $c_2 = \pi q\omega$, $c_3 = 3\pi q r\omega^3/4$, $c_4 = 5\pi q s\omega^5/8$, all of which are positive numbers. You may assume without proof that $\int_0^{2\pi/\omega} \sin^2 \omega t \,\mathrm{d}t = \pi/\omega$, $\int_0^{2\pi/\omega} \sin^4 \omega t \,\mathrm{d}t = 3\pi/(4\omega)$, and $\int_0^{2\pi/\omega} \sin^6 \omega t \,\mathrm{d}t = 5\pi/(8\omega)$. [40%]

(c) Find the lowest value of U at which the bridge is unstable to hard excitation and comment on the influence of the damping. With the aid of sketches, describe the vertical motion of the bridge above this value of U when it is subjected to (i) a small impulse and (ii) a big impulse. [40%]

4 (a) Two identical chimneys stand next to each other. Describe and explain the flow-structure interactions that may occur, the problems they may cause, and possible strategies to reduce them. [40%]

(b) With reference to phase velocity and group velocity, and with the aid of sketches of $\omega(k)$, where ω is the angular frequency and k is the wavenumber, describe what is meant by *temporal stability analysis*, *spatial stability analysis*, *spatio-temporal stability analysis*, *absolute instability* and *convective instability*. Give an example of a flow that contains absolute instability and describe how it behaves in the absence of forcing. [60%]

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4A10, 2014, Answers

Q1 –

Q2 –

Q3

(a) $U_{crit} = 4c_1c_4/(c_3^2 - 4c_2c_4)$

Q4 –